Adaptive Partitioning for Gabor Wavefield Extrapolation

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Outline

- Forming an adaptive partition of unity (POU) using the lateral position error criterion
- 2D imaging examples of the Marmousi dataset
- 3D Impulse test in a homogeneous medium
- Conclusions
- Acknowledgements

Gabor Wavefield Extrapolation

Key concepts

- Spatial windows are used to localize the wavefield to regions of roughly constant velocity.
- Within each window a constant-velocity extrapolation phase shift, with a spilt step Fourier correction, is applied.

Phase shift in window "j“ (Ω_j) is given by:

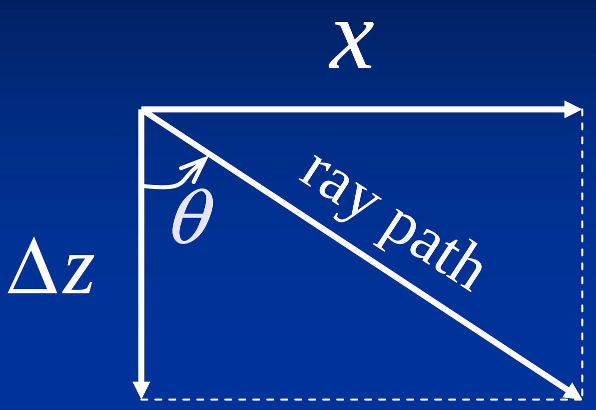
$$\phi_j(x, k_x, \omega) = \omega \Delta z \left(\frac{1}{v(x)} - \frac{1}{v_j} \right) + \Delta z \sqrt{\frac{\omega^2}{v_j} - k_x^2}$$

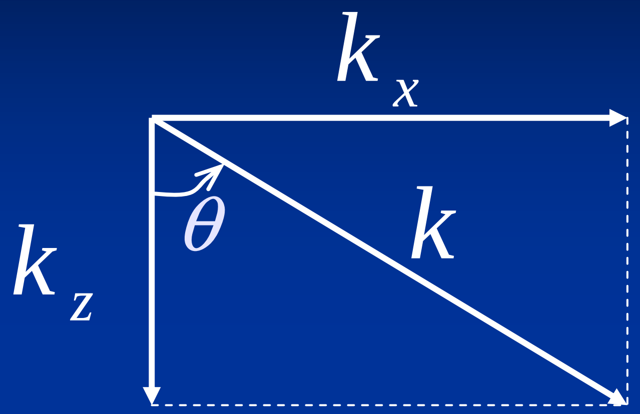
Adaptive Partitioning Criteria

- Lateral velocity gradient exceeds threshold (Grossman et al., 2003)
- Extrapolator phase error with respect to the GPSPI approximation (Ma and Margrave, 2005)
- Lateral position error with respect to the GPSPI approximation

We have tested all three of these methods;
We introduce the third one here.

Estimate Lateral Position Errors





$$x = \Delta z \tan\theta$$

$$\omega / v = k = k_x / \sin\theta$$

$$\delta x = \Delta z \sec^2\theta \frac{\partial\theta}{\partial v}\delta v$$

$$\frac{\partial\theta}{\partial v} = \frac{k_x}{\omega}\sec\theta$$

Lateral Position Error

$$\delta v = \frac{\cos^3\theta}{\sin\theta}\frac{\delta x}{\Delta z}v, \quad \theta \in [0,90]$$

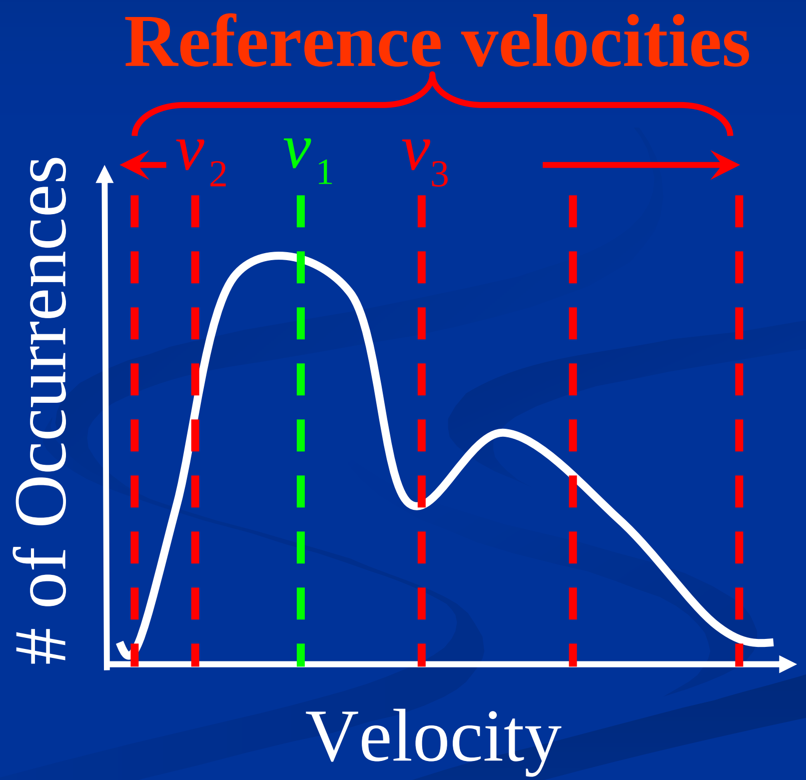
Choose the Reference Velocities

$$\delta v_j = \frac{\cos^3\theta}{\sin\theta}\frac{\delta x}{\Delta z} v_j = a v_j, \quad j = 1, 2, \ldots$$

$$v_1 + \frac{1}{2}\delta v_1 = v_3 - \frac{1}{2}\delta v_3 = v_3\left(1 - \frac{1}{2}a\right)$$

$$v_1 - \frac{1}{2}\delta v_1 = v_2 + \frac{1}{2}\delta v_2 = v_2\left(1 + \frac{1}{2}a\right)$$

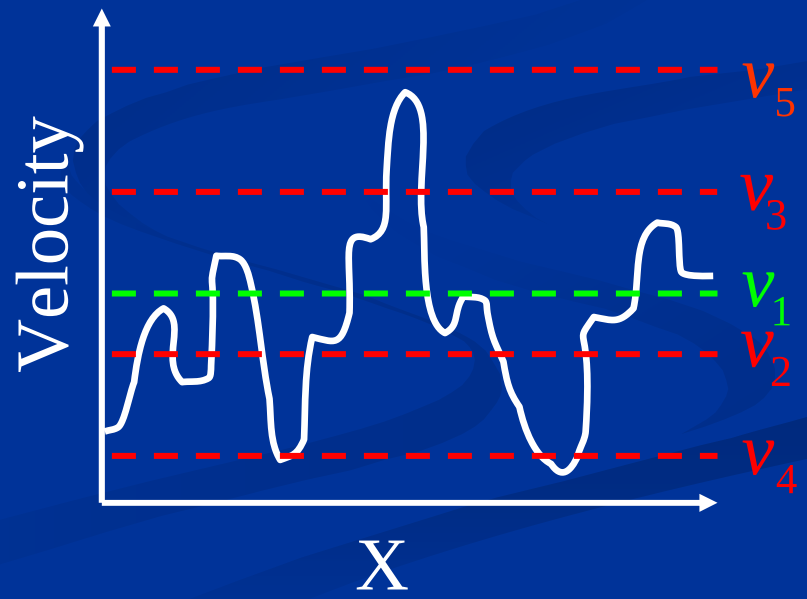
$$v_2 = \frac{2v_1 - \delta v_1}{2+a}, \quad v_3 = \frac{2v_1 + \delta v_1}{2-a}$$



Build Indicator Functions

- For each reference velocity define an indicator function:

$$I_j(x) = \begin{cases} 1, & |v(x) - v_j| = \min \\ 0, & \text{otherwise} \end{cases}$$



Build Indicator Functions

- For each reference velocity define an indicator function:

$$I_j(x) = \begin{cases} 1, & \left| v(x) - v_j \right| = \min \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{j \in \mathbb{Z}} I_j(x) = 1$$

j ↓

I_1	0	0	…	0	0	0	0
I_2	0	0	…	0	1	0	0
I_3	0	1	…	0	0	1	1
I_4	0	0	…	1	0	0	0
I_5	1	0	…	0	0	0	0

x →

Create Partitions

- Define a smallest "atomic window"
- Build the POU by a normalized convolution:

$$\Omega_j(x) = \left(I_j \bullet \Theta\right)(x)$$

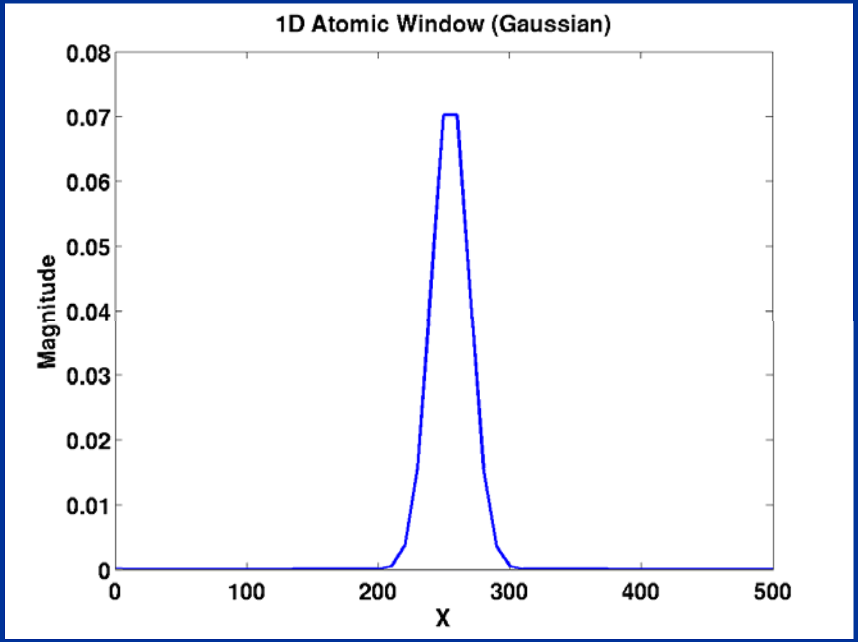
$$\Theta = \text{atomic window}$$

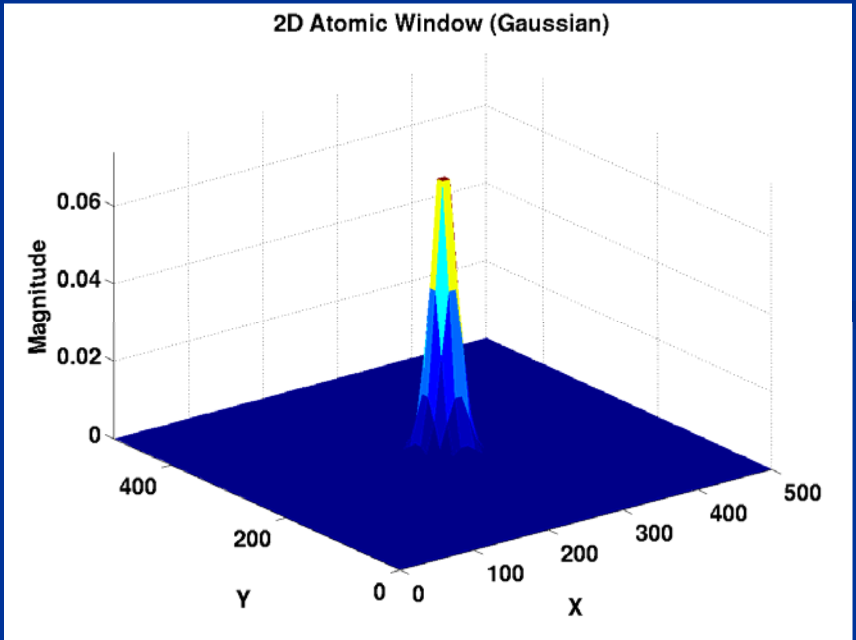
The POU is satisfied automatically works in any number of dimensions;

$$\text{2D}: \Omega_j(x) = \left(I_j \bullet \Theta\right)(x) \quad \text{Partitioning in 1D}$$

$$\text{3D}: \Omega_{ij}(x, y) = \left(I_{ij} \bullet \Theta\right)(x, y) \quad \text{Partitioning in 2D}$$

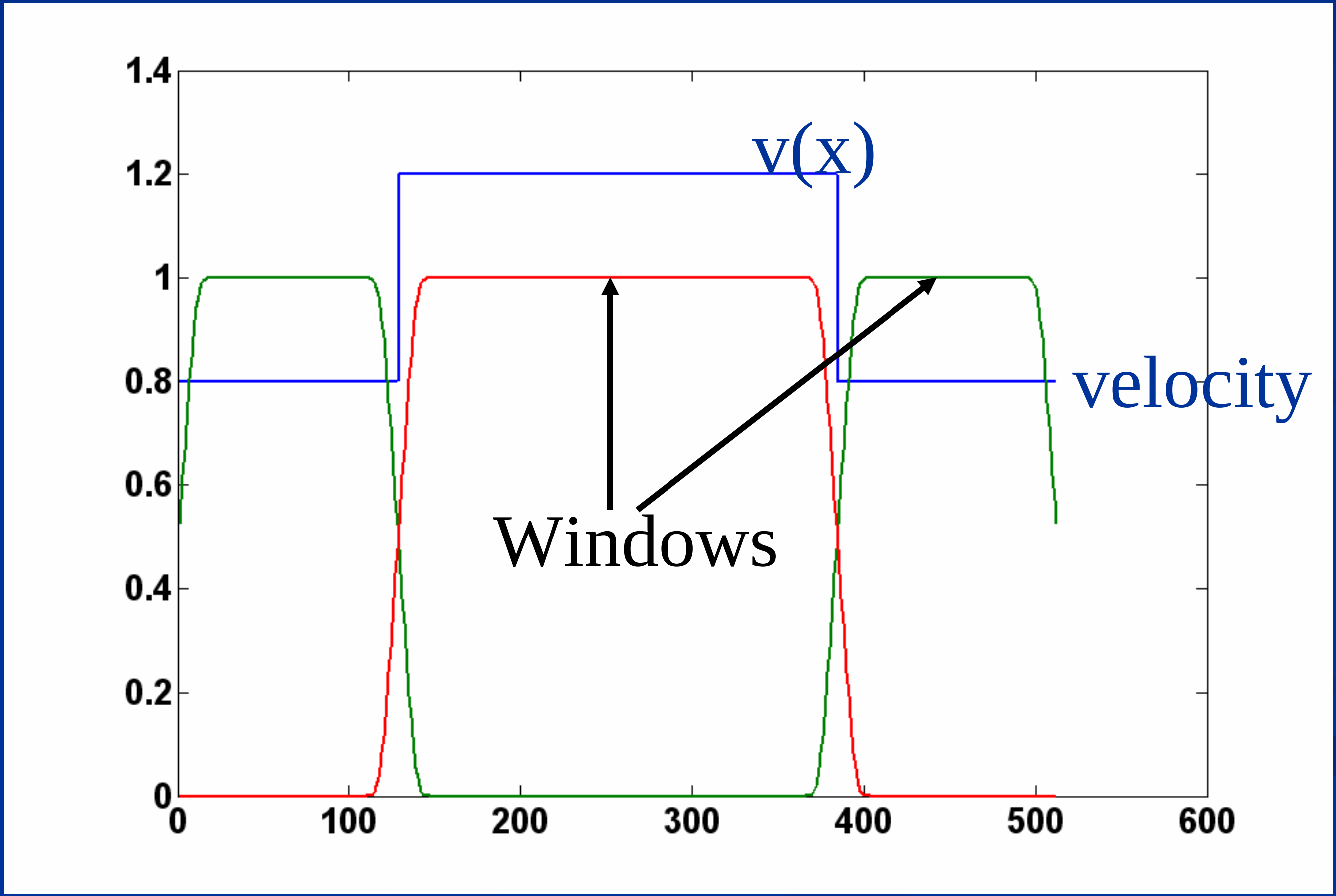
Atomic Windows in 1D and 2D





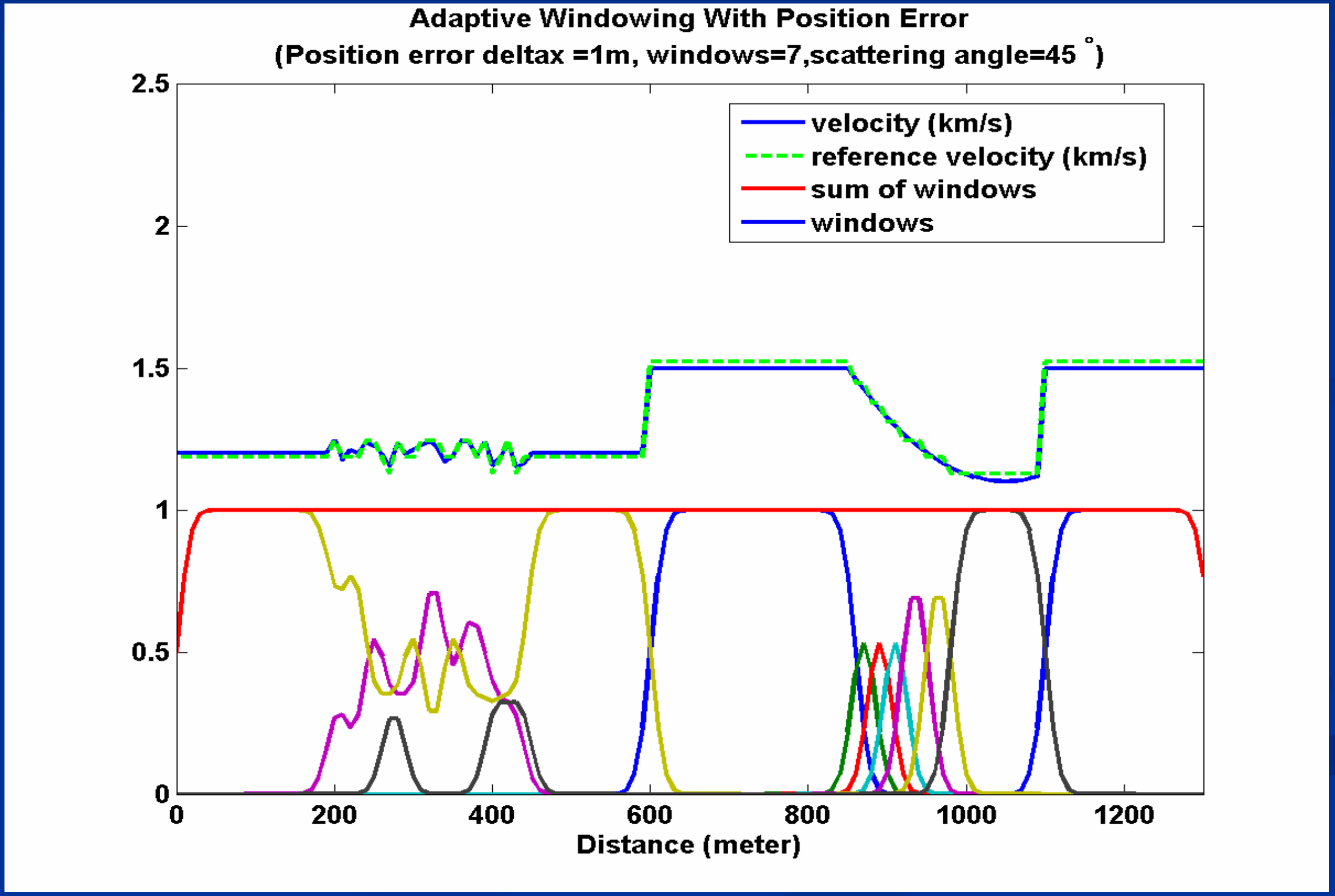
1D Position-error Partitioning for 2D Gabor Imaging

Example: v(x) is a step bump function and two partitions are chosen.

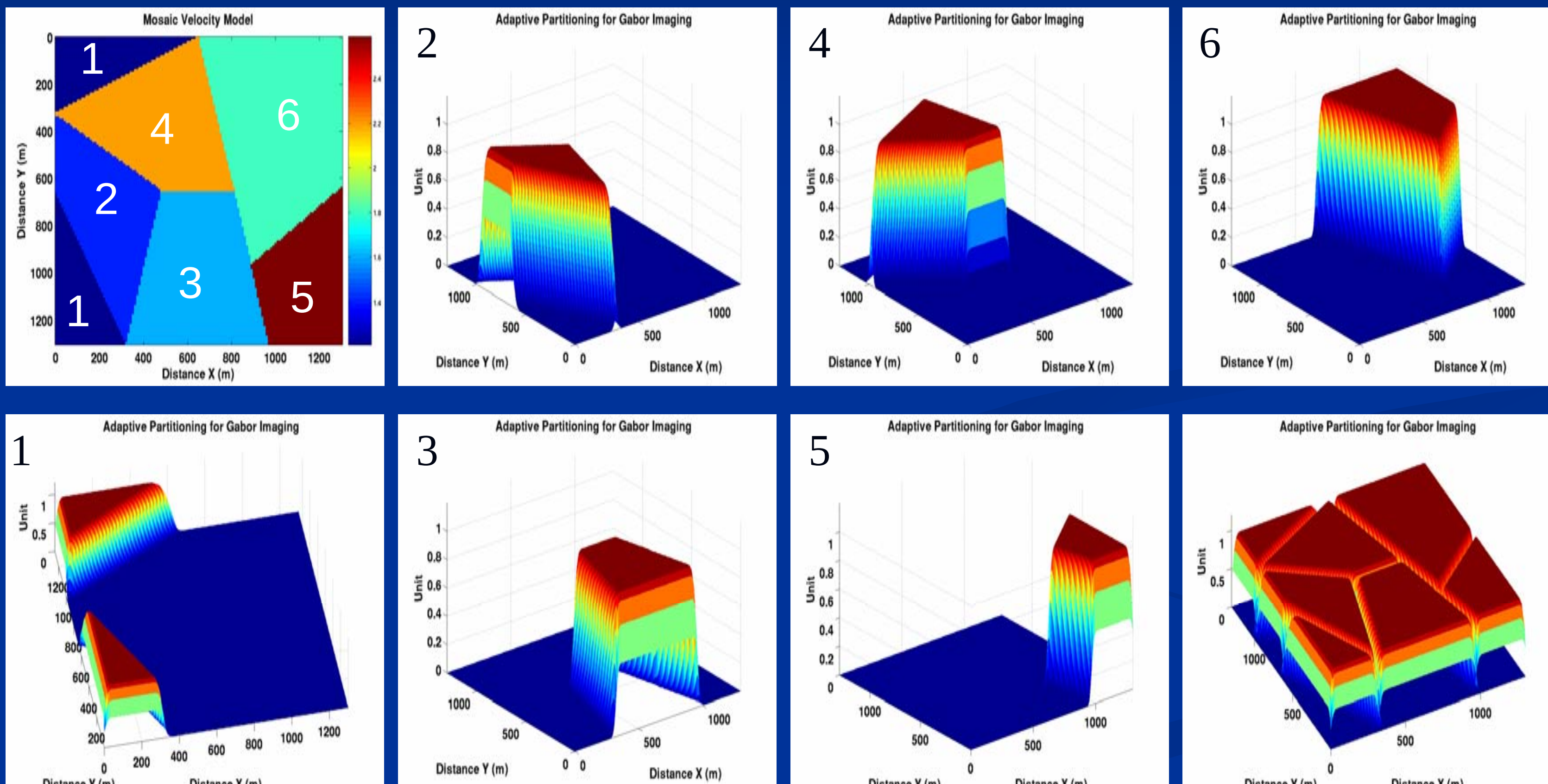


1D Position-error Partitioning for 2D Gabor Imaging

Example: v(x) is a complex velocity profile and seven partitions are chosen.



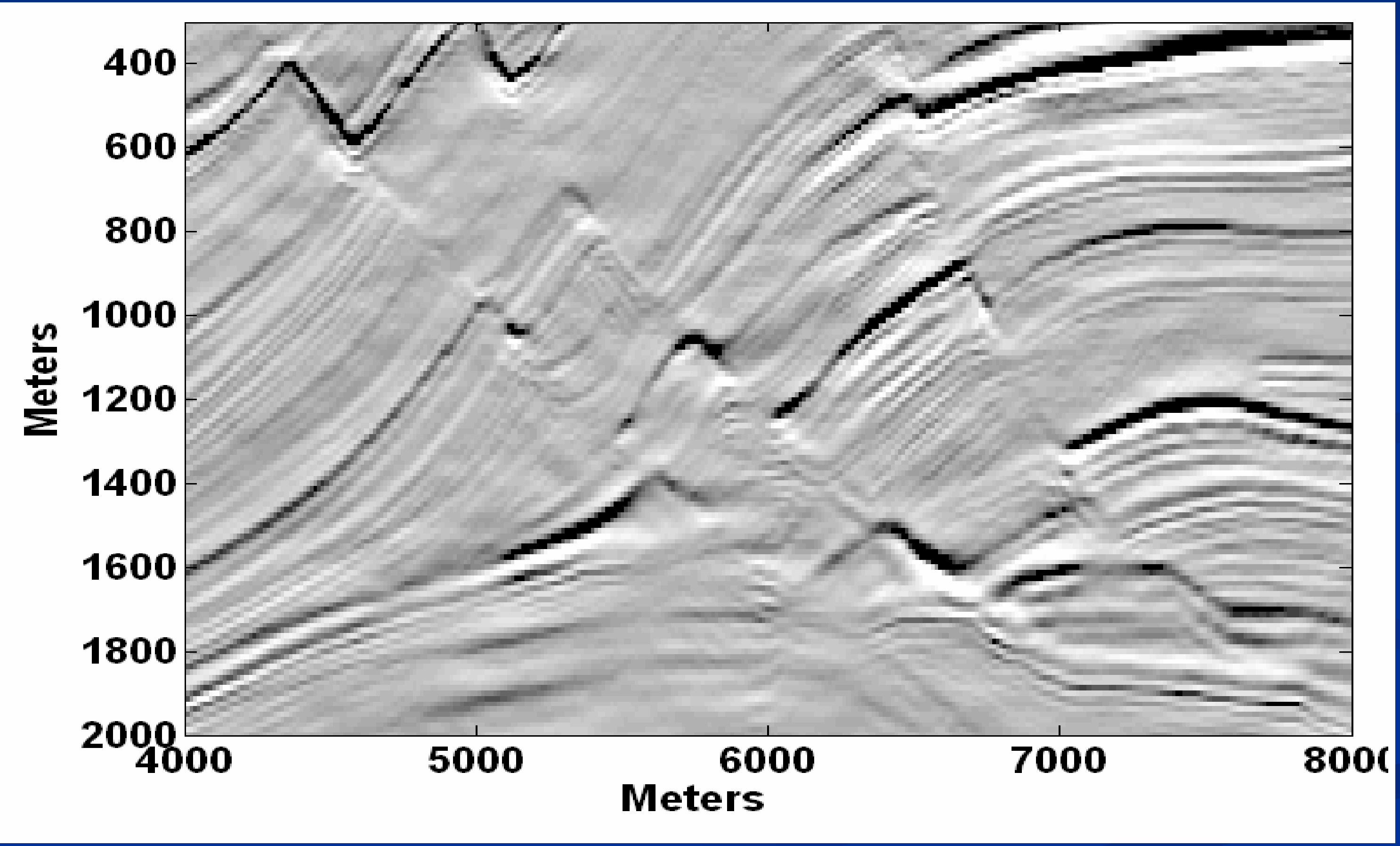
2D Position-error Partitioning for 3D Gabor Imaging



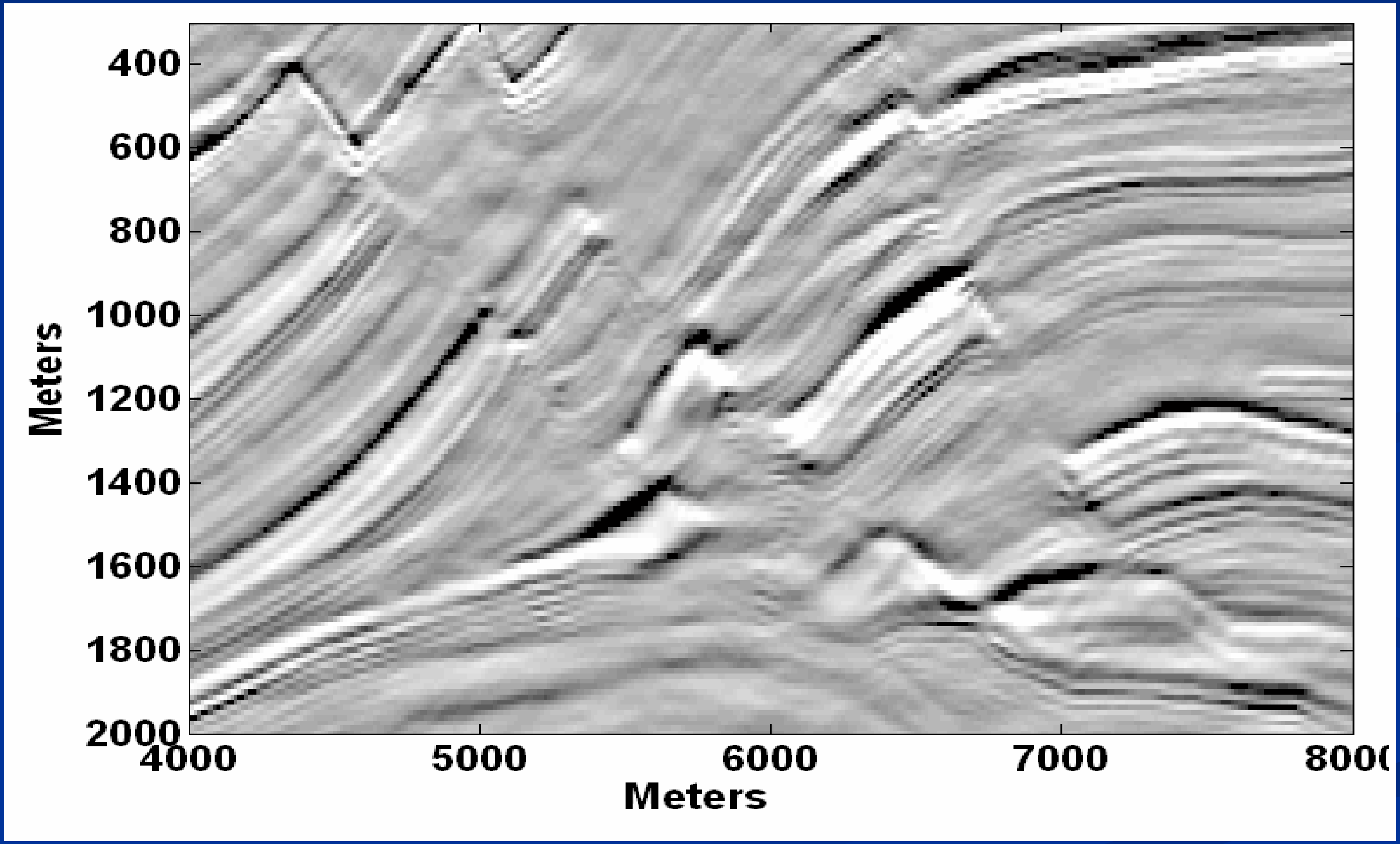
Marmousi Synthetic Data Sets

- Pre-stack depth migration on shot records
- Number of shots: 240
- Each shot record is imaged with deconvolution Imaging condition

Gabor Imaging Enlargement

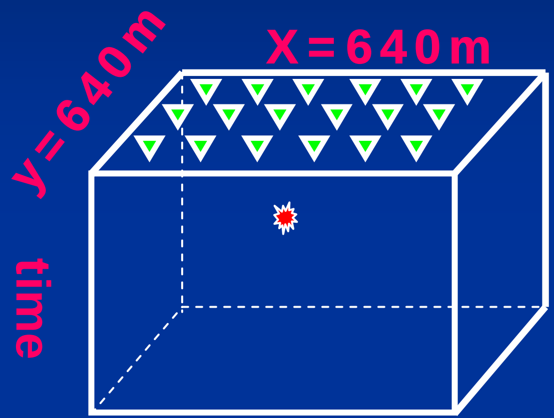


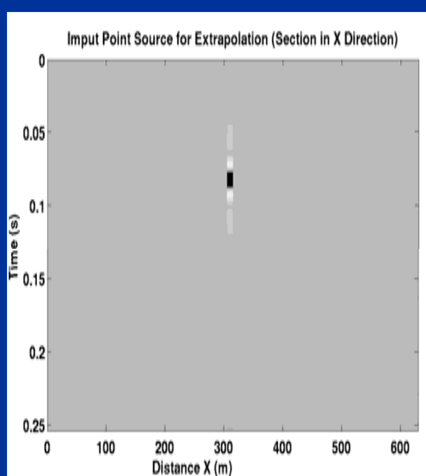
FOCI Imaging Enlargement

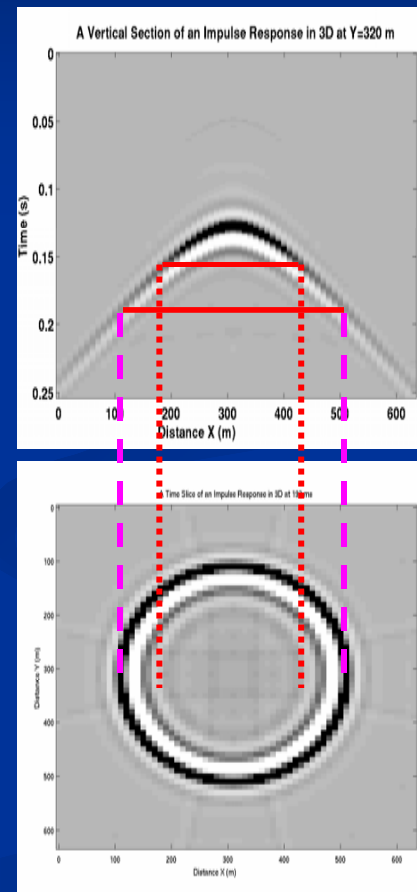


(Margrave et al, 2006)

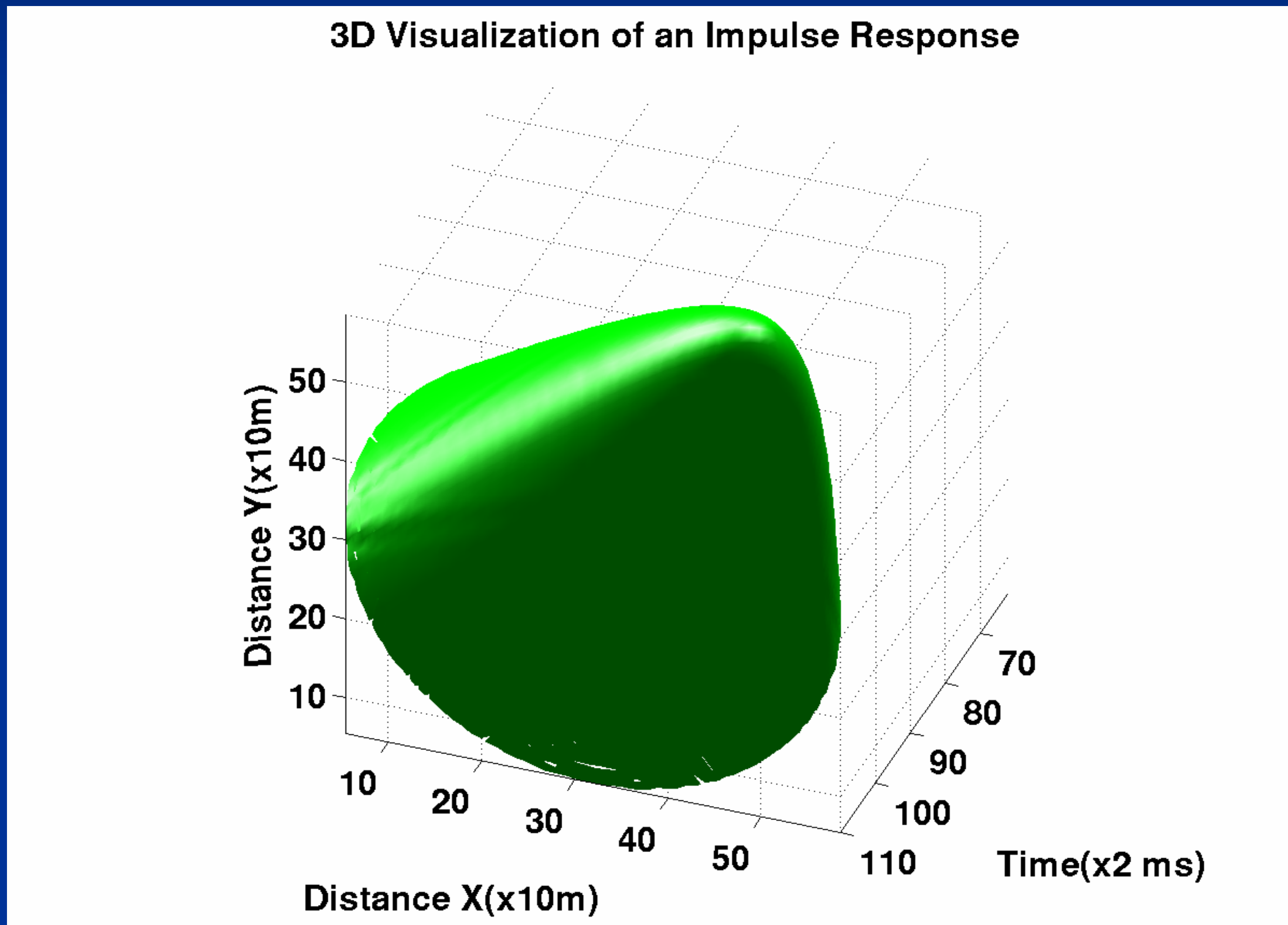
3D Impulse Response Test







3D Visualization of the Impulse Response



Conclusions

- Gabor imaging method is an effective depth migration tool for complex velocity structures
- Adaptive partitioning scheme enhances Gabor imaging method and allows it have a trade-off between accuracy and runtime
- Gabor method easily extends to 3D

Acknowledgements

- CREWES and the sponsors
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- David Henley, Chad Hogan
- Kevin Hall

Slides in the Following Reserved for Possible Questions

Approximation of GPSPI

– Gabor Wavefield Extrapolation

$$\psi_P(x,\Delta z,\omega) = \frac{1}{2\pi}\int_{\mathbb{R}} \hat{\psi}(k_x,0,\omega)\,\hat{W}\left(k = \frac{\omega}{v(x)}, k_x, \Delta z\right)\exp(-ik_x x)\,\mathrm{d}k_x$$

$$\hat{W}\left(k = \frac{\omega}{v(x)}, k_x, \Delta z\right) \approx \sum_{j\in\mathbb{Z}} \Omega_j(x)\, S_j(x)\, \hat{W}\left(k_j = \frac{\omega}{v_j}, k_x, \Delta z\right)$$

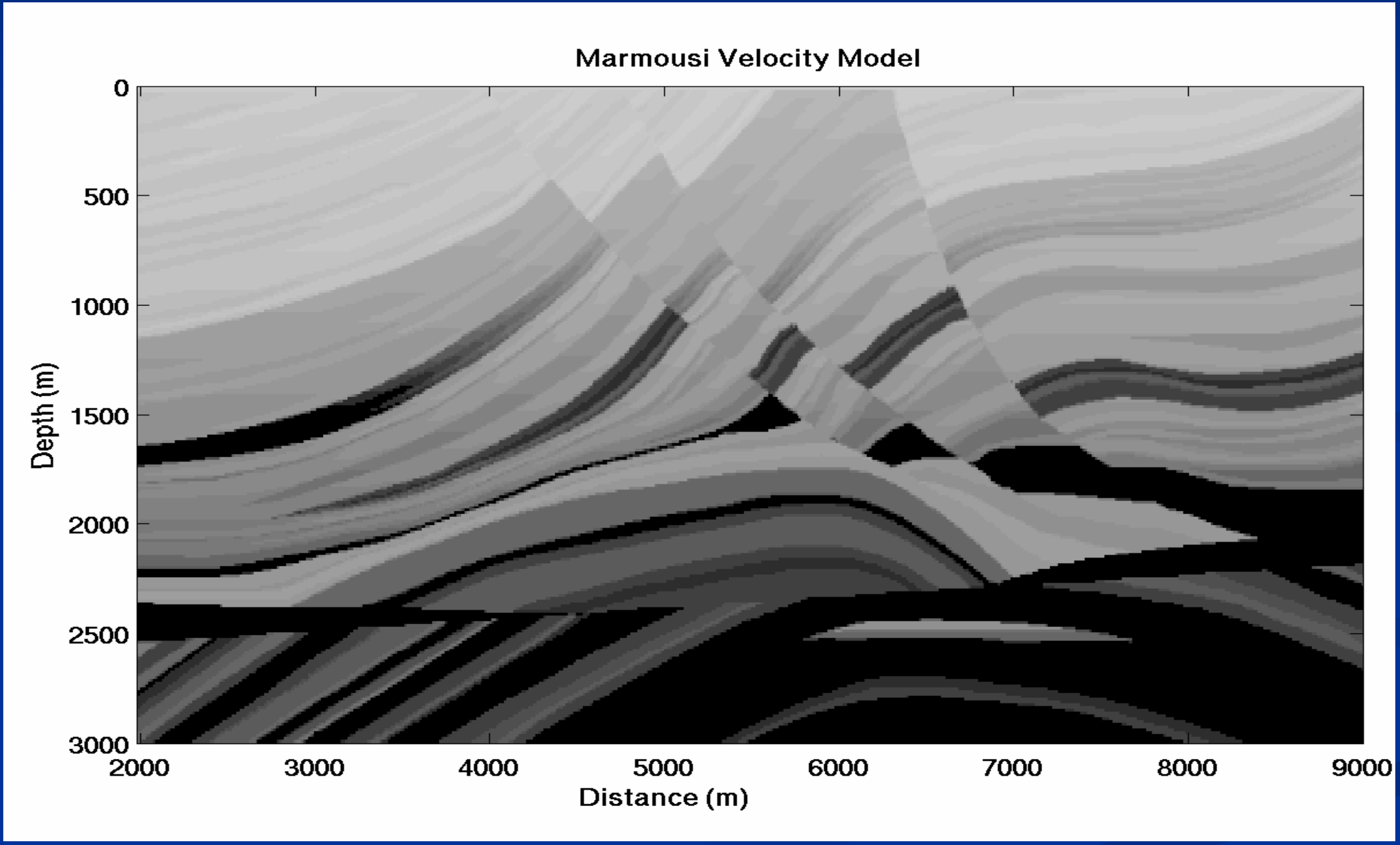
Partitioning Windows

Fourier Split-step Operator

Locally Constant Extrapolator

$$\psi_P(x,\Delta z,\omega) \approx \sum_{j\in\mathbb{Z}} \Omega_j(x)\, S_j(x) \times \frac{1}{2\pi}\int_{\mathbb{R}} \hat{\psi}(k_x,0,\omega)\,\hat{W}\left(k_j = \frac{\omega}{v_j}, k_x, \Delta z\right)\exp(-ik_x x)\,\mathrm{d}k_x$$

Marmousi Velocity Model



Gabor Imaging Result using Lateral Position Error of 2.5 m