P-impedance, S-impedance and density from linear AVO inversion: Application to a VSP dataset from Alberta

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Outline

- The linear AVO inversion description
- Methodology
- damped SVD method
- A synthetic surface seismic example
- AVO inversion of the Red Deer VSP data
- Conclusions
- Acknowledgments

Inversion Problem

Pre-stack reflection AVO data → *Aki-Richards approximations* → (I, J, ρ)

Inverse problem: to estimated Physical properties
(I:P-impedance, J:S-impedance, ρ:density)

Physical property:

- **Imaging subsurface structure**
  - **Directly detecting changes in the subsurface**
  - **An aid in the interpretation of seismic reflection data**

Compressional data → **PP inversion**

Converted P-Sv data → **PS inversion**

{ **Compressional data**, **Converted data** } → **Joint inversion**

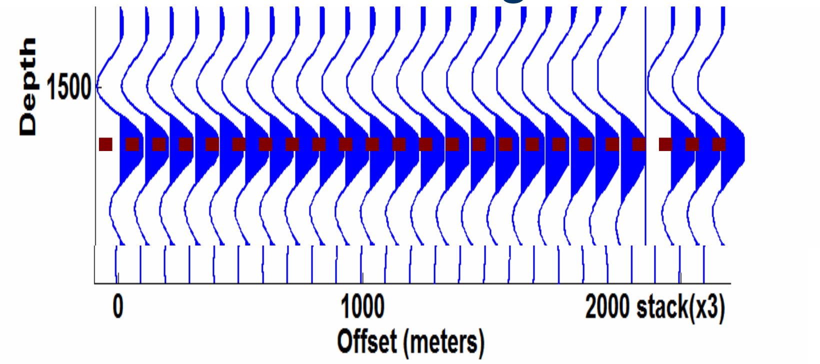
Methodology

Aki-Richards

$$R_{PP} = A\frac{\Delta I}{I} + B\frac{\Delta J}{J} + C\frac{\Delta\rho}{\rho}$$

$$R_{PS} = E\frac{\Delta J}{J} + D\frac{\Delta\rho}{\rho}$$

Data = CCP gather



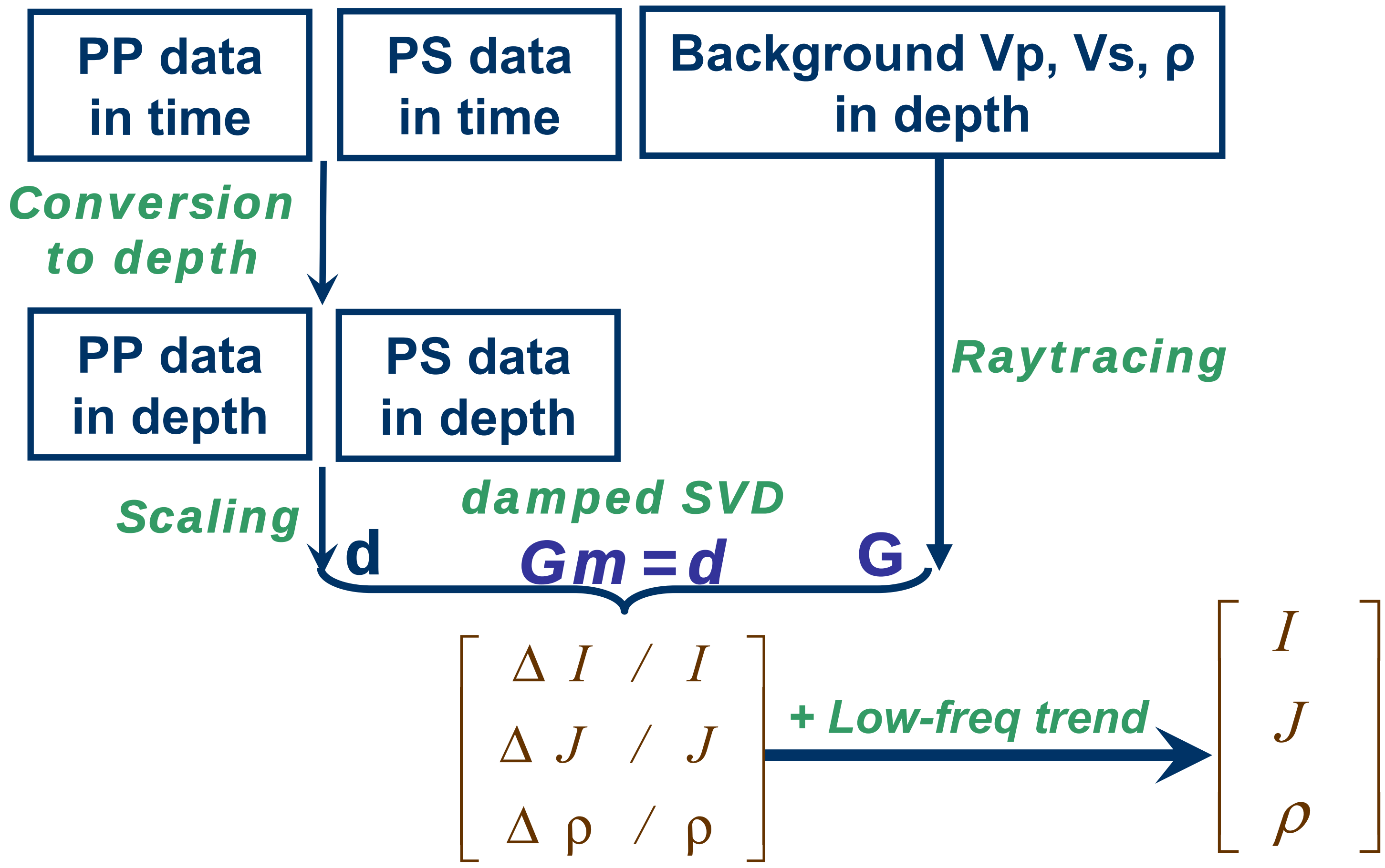
sample depth

$$\begin{bmatrix} A_1 & B_1 & C_1 \\ \vdots & \vdots & \vdots \\ A_1 & B_1 & C_1 \\ 0 & E_1 & D_1 \\ \vdots & \vdots & \vdots \\ 0 & E_m & D_m \end{bmatrix}_{2m \times 3} \times \begin{bmatrix} \Delta I / I \\ \Delta J / J \\ \Delta \rho / \rho \end{bmatrix} = \begin{bmatrix} R_{PP1} \\ \vdots \\ R_{PPm} \\ R_{PS1} \\ \vdots \\ R_{PSm} \end{bmatrix}$$

m = offset

$$Gm = d$$

Workflow: Joint inversion



Inverse problem

- **Least-squares** method
  minimizing the prediction error

$$Gm = d$$

- **Ill-posed**: **a small change in data will cause the large change in solution.**
- **3-parameter linear inversion**: **ill-posed problem**
  - » **Nonlinearity of the problem**
  - » **Limited data acquisition aperture**
- **Singular Value Decomposition**, SVD

$$m^{est} = G_g^{-1} d$$

Generalized inverse

$$G_g^{-1} = V \Lambda^{-1} U^T$$

(Lay, 1996)

$$\Lambda = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$\sigma_1 \geq \sigma_2 \geq \sigma_3 > 0$$

σ_i = **non-zero singular value of G**

$= +\sqrt{\text{eigenvalues of } G^T G}$

SVD analysis

$$\text{Condition number} = \frac{\text{Largest singular value}}{\text{Smallest singular value}}$$

A matrix is ***well-posed*** when its condition number is not far from 1 (Jin et al., 2000).

Damped SVD

$$G_g^{-1} = V\Lambda(\Lambda^2 + \varepsilon^2 I)^{-1}U^T$$

$$\varepsilon^2 = Damping\ factor$$

Damping factor = ε percent of the largest singular value

Small damping factor is desired, with large ε the model parameter will not be resolved correctly

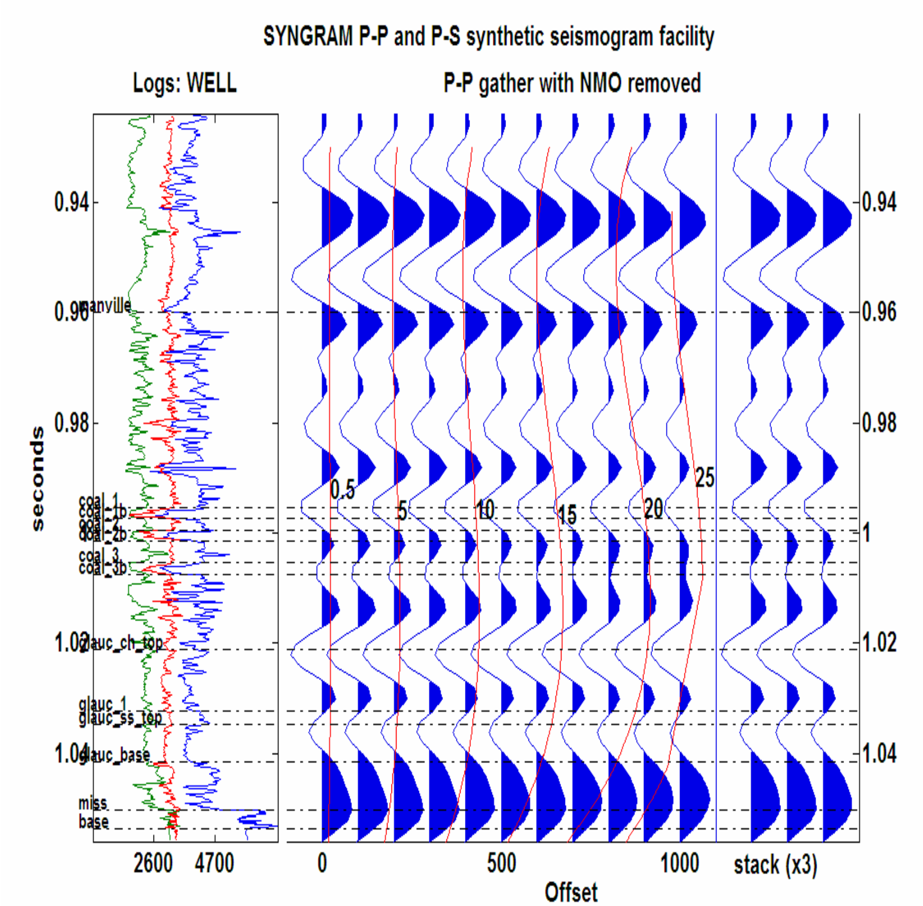
Model parameter resolution matrix: $R = G_g^{-1}G$

$$m^{est} = G_g^{-1}d = (G_g^{-1}G)m$$

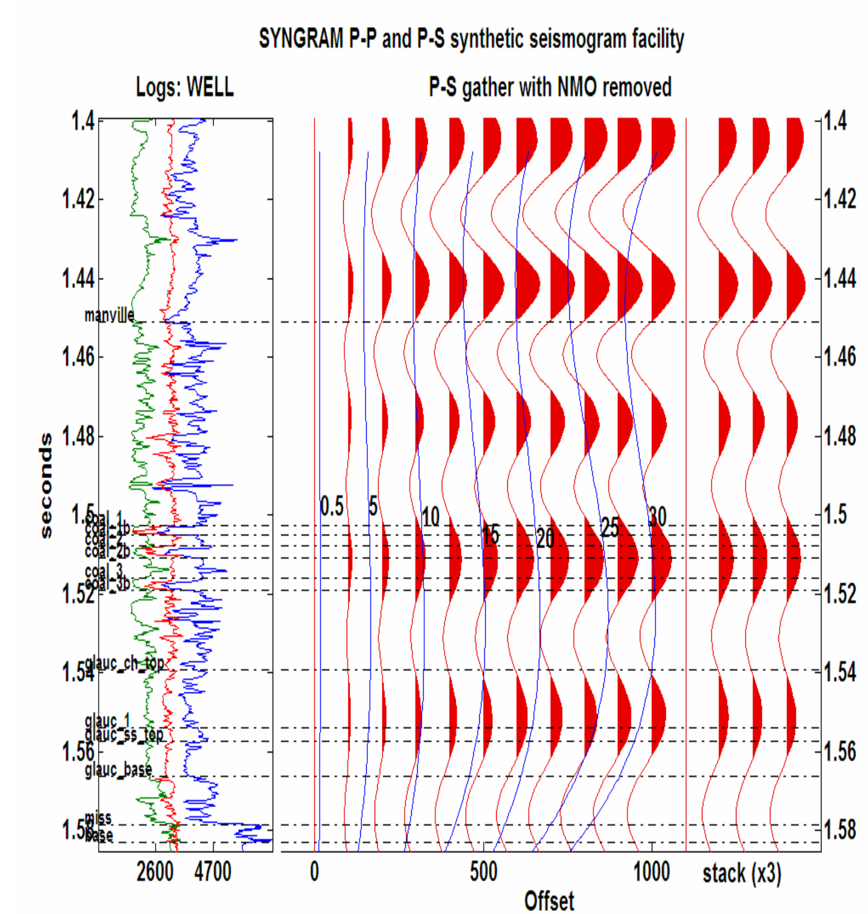
Perfect resolution: $G_g^{-1}G = I$

AVO inversion testing

SYNGRAM, CREWES software



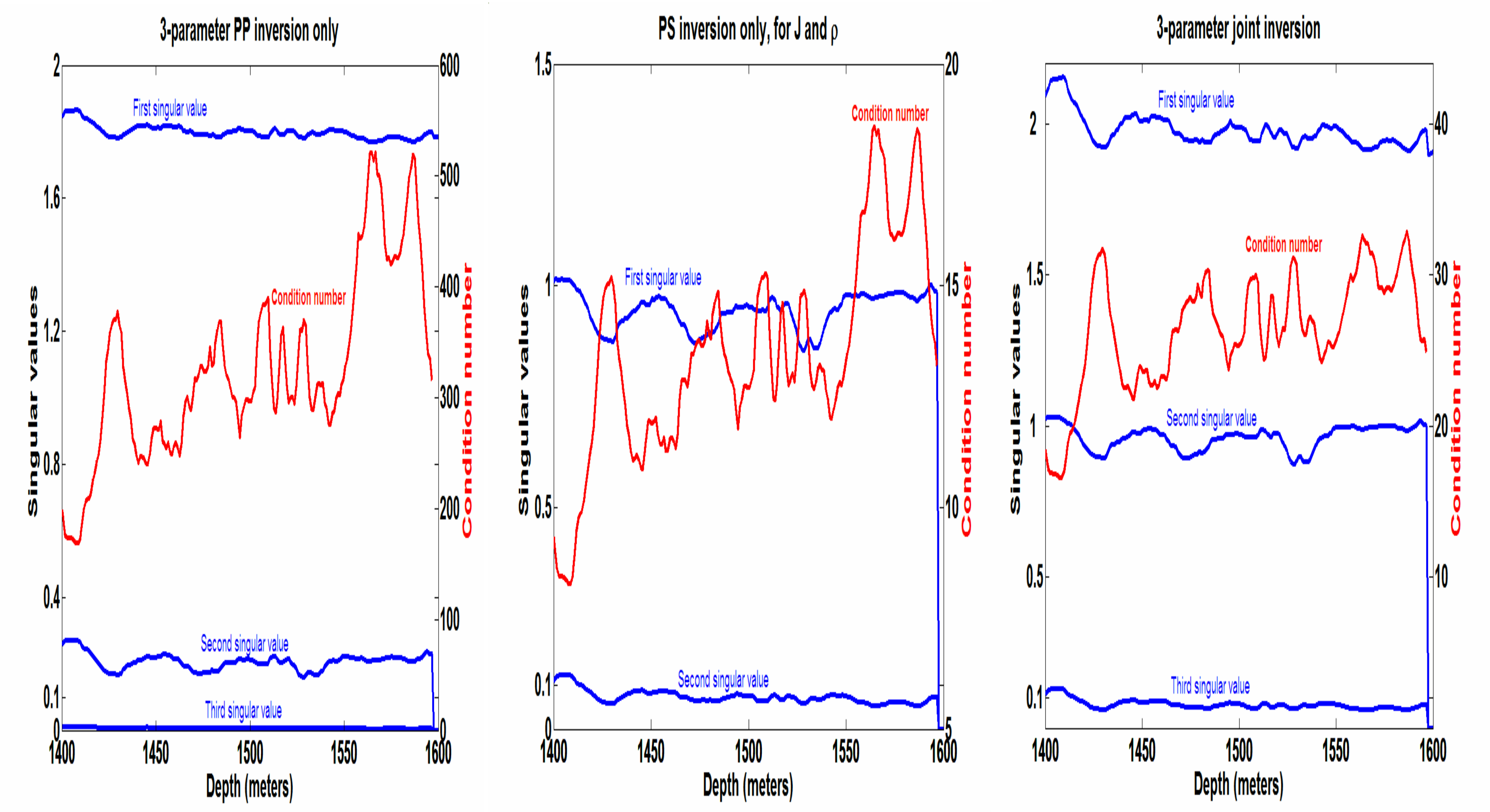
Zero-phase wavelet 5-10-80-100



Zero-phase wavelet 3-7-57-70

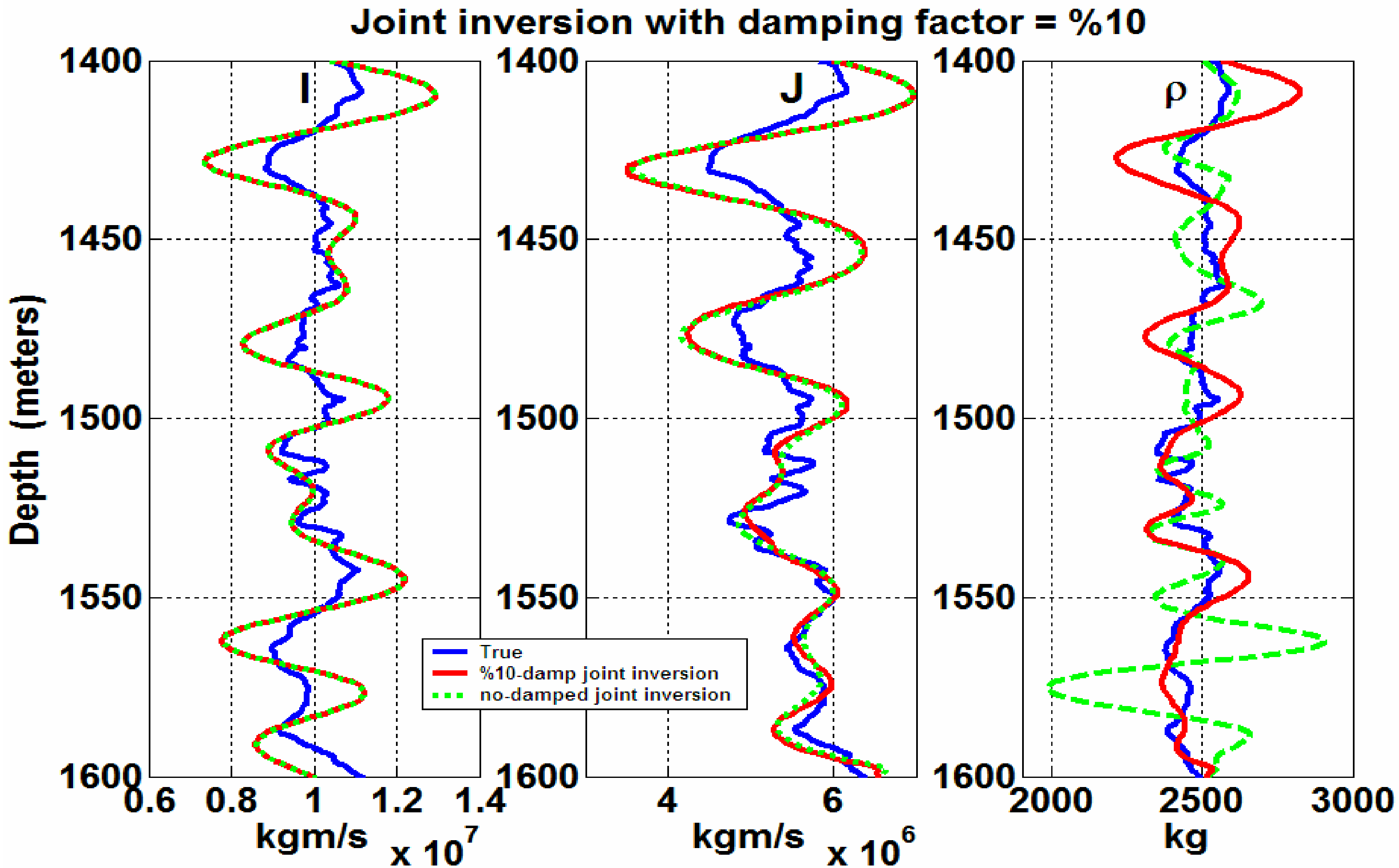
Real log from Blackfoot field, owned and operated by Encana, south-eastern Alberta, Canada

Condition number

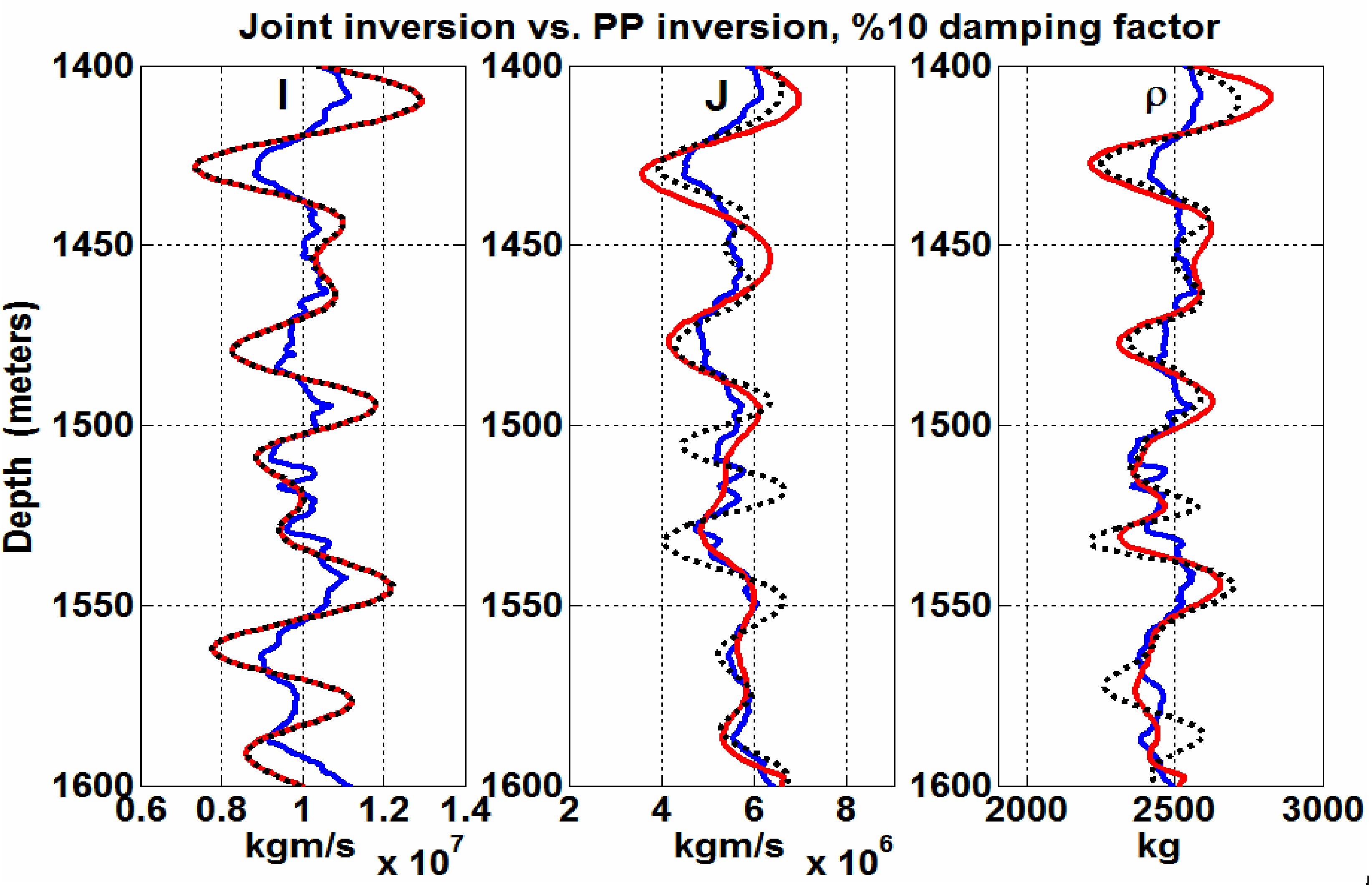


The ratio is important

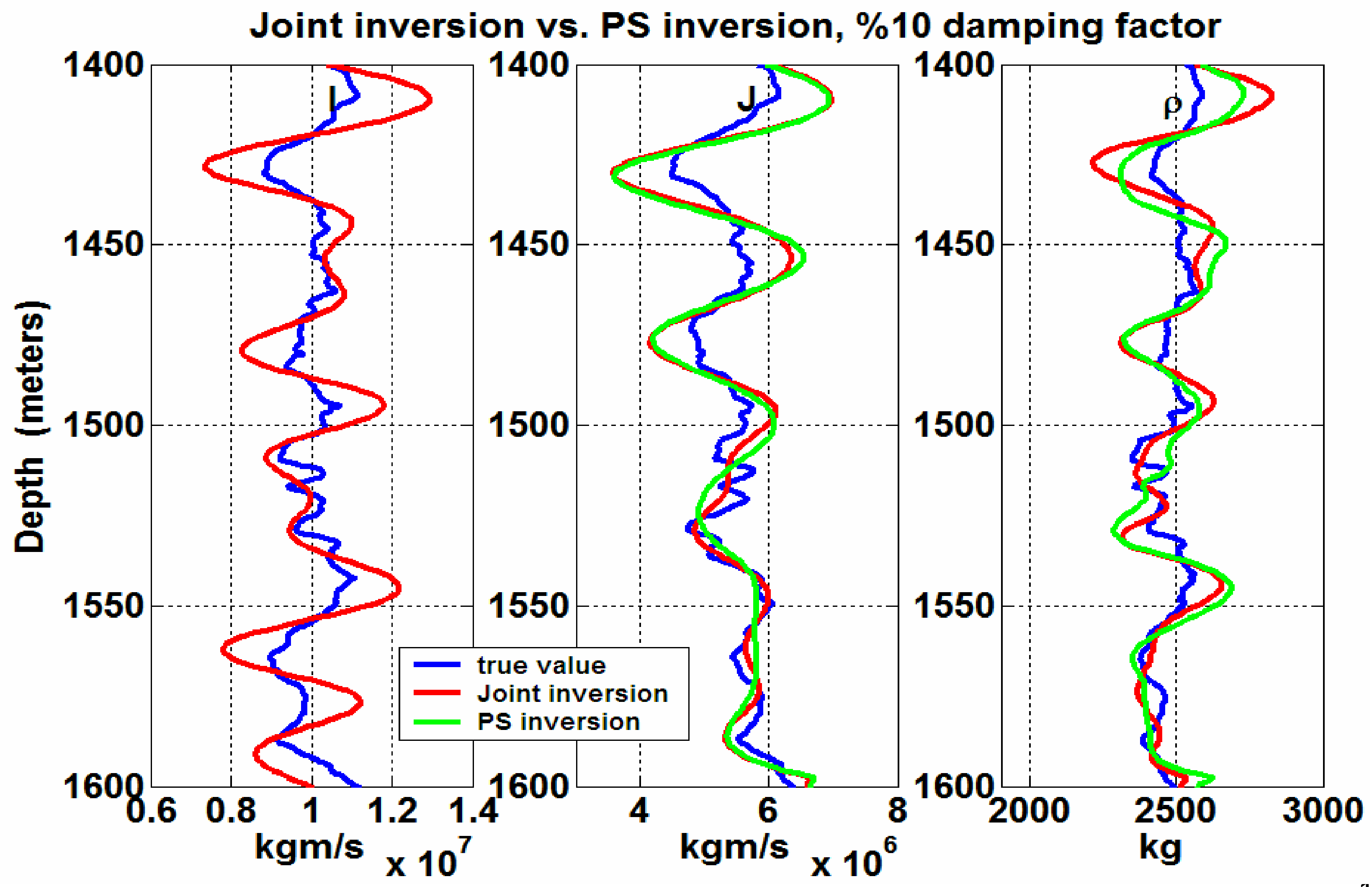
Joint inversion, 10% damping



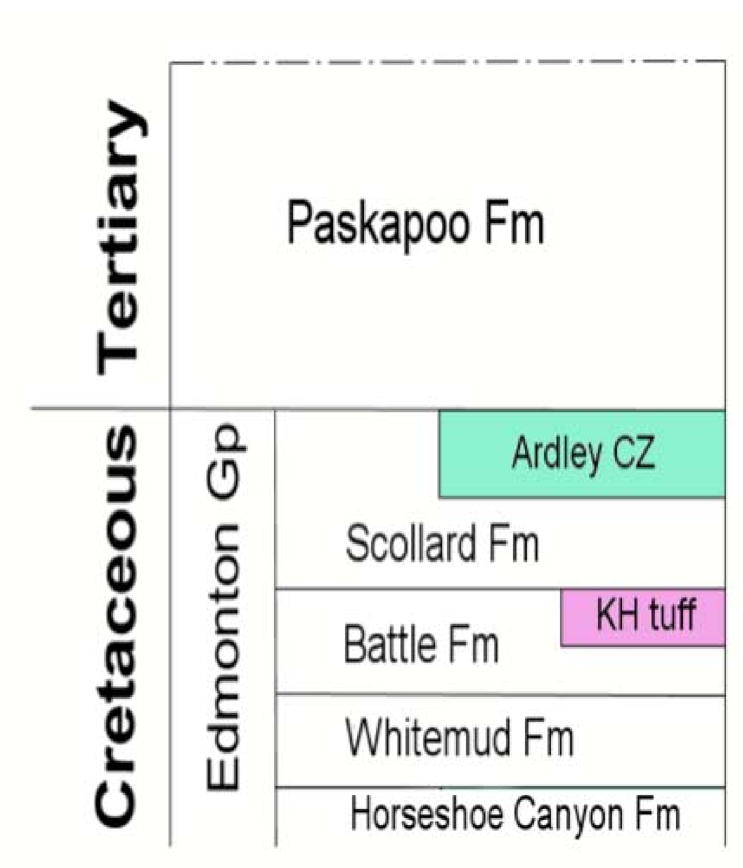
Joint inversion vs. PP inversion



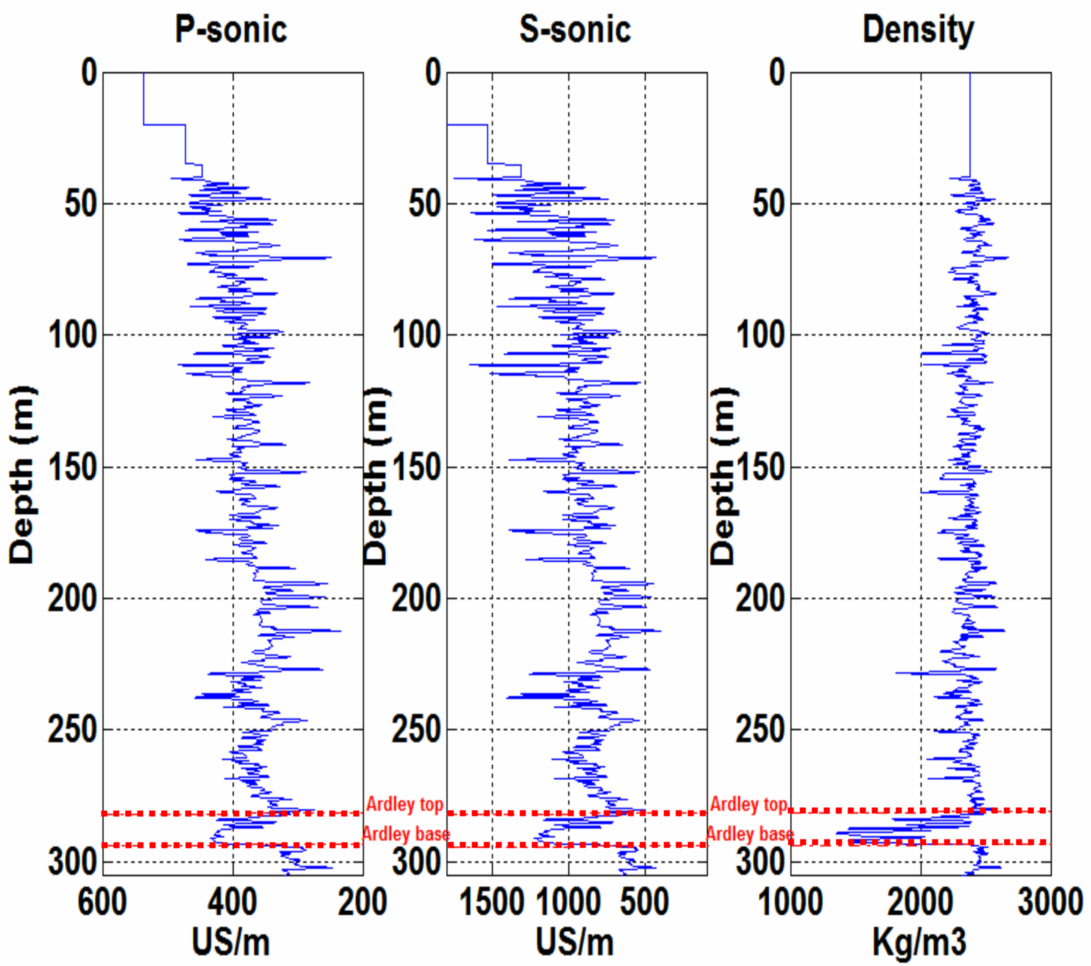
Joint inversion vs. PS inversion



Study area: Cygnet 9-34 in Red Deer, Alberta

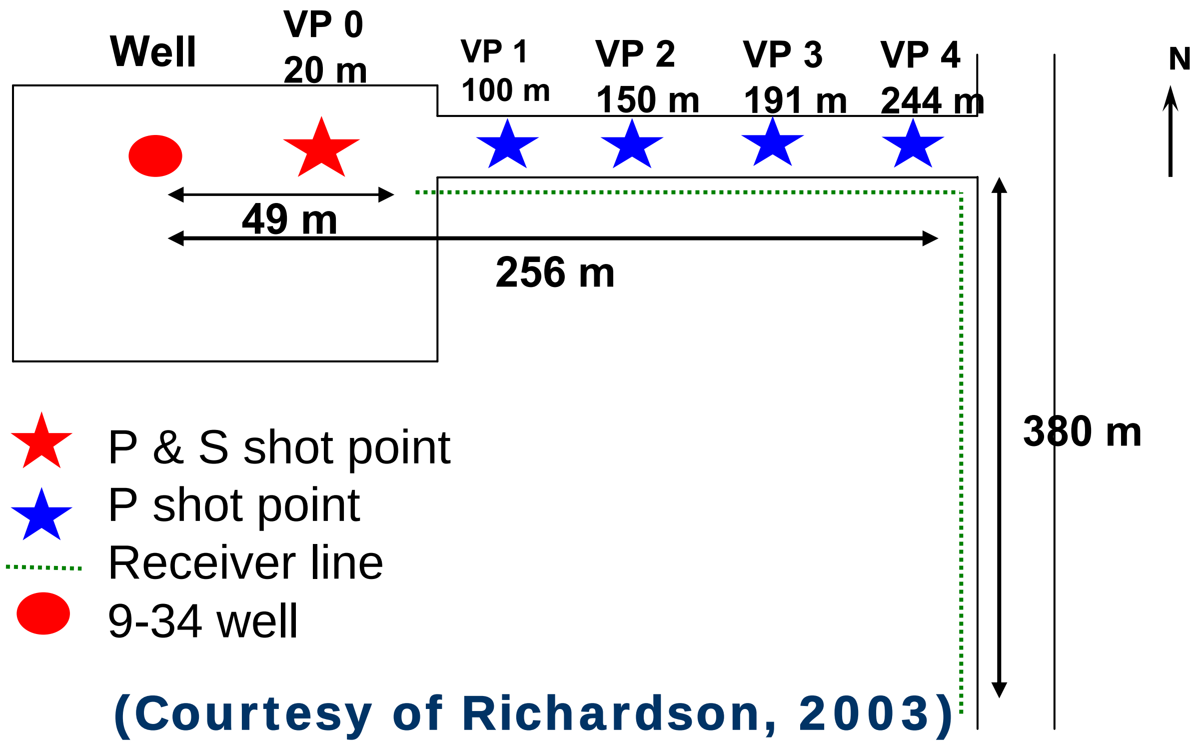


(Courtesy of Richardson, 2003)



Well logs: Schlumberger Canada

Survey geometry



(Courtesy of Richardson, 2003)

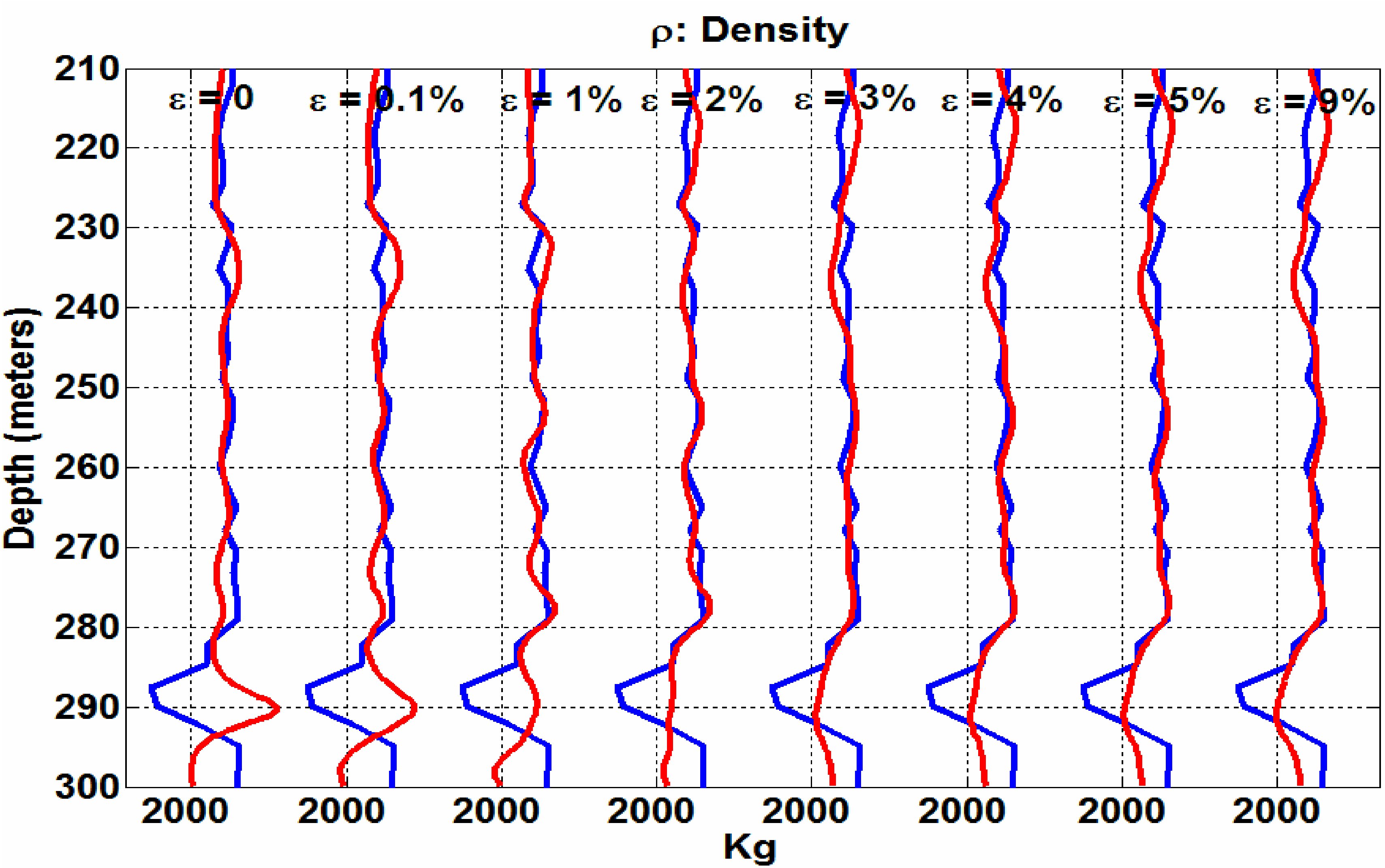
Source: Compressional vibroseis, with sweep 8-250 HZ

AVO inversion input: deconvolved upgoing wavefield

Ardley coal seams are laterally continues with no lateral variation (Beaton, 2003)

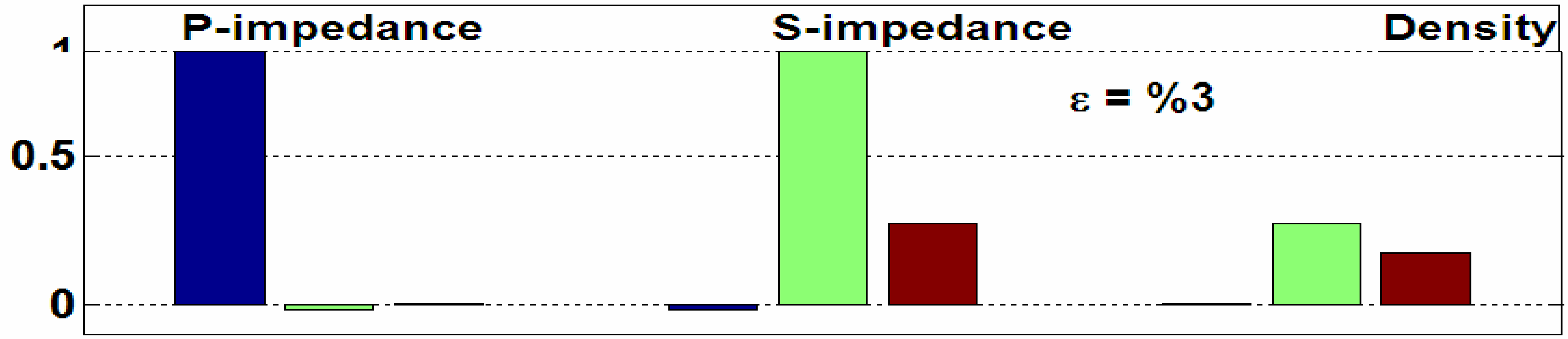
CCP gather = Common shot gather

Joint inversion, offset 3



Resolution matrix

Density relation for Red Deer area

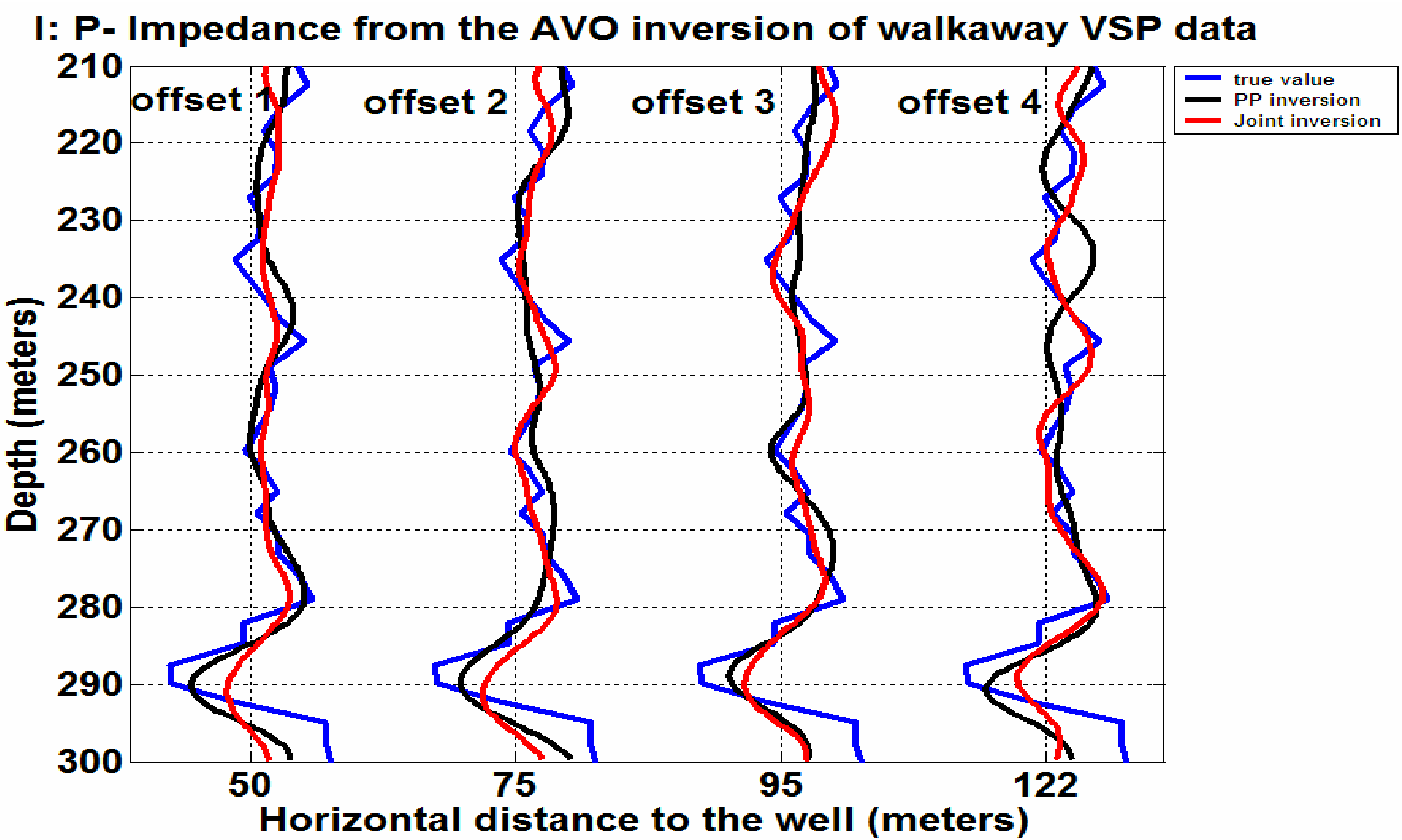


Resolution

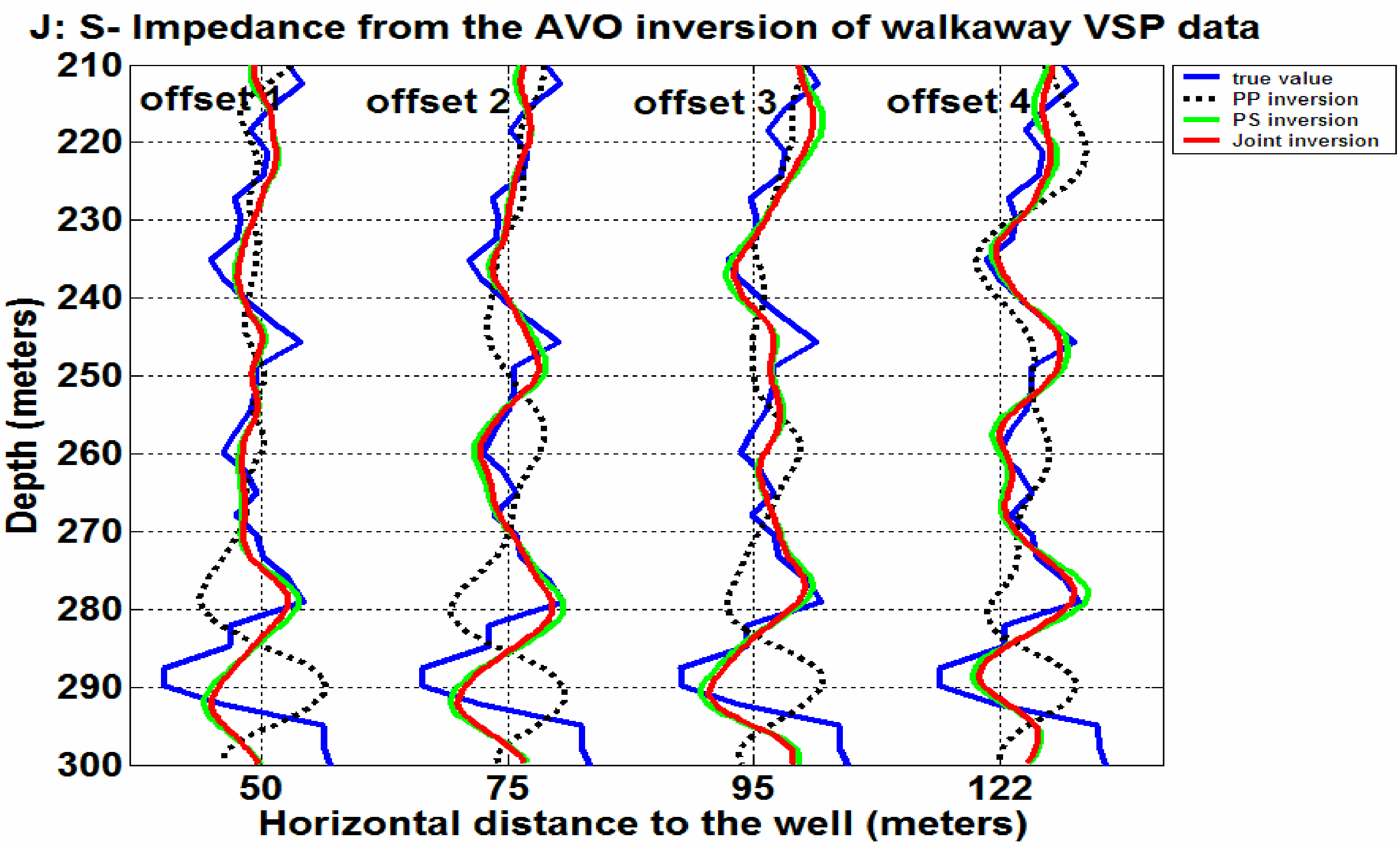
$$\begin{bmatrix} \frac{\Delta I}{I}^{est} \\ \frac{\Delta J}{J}^{est} \\ \frac{\Delta \rho}{\rho}^{est} \end{bmatrix} = \begin{bmatrix} 0.98 & 0.016 & 0.003 \\ 0.016 & 0.86 & 0.27 \\ 0.003 & 0.27 & 0.17 \end{bmatrix} \begin{bmatrix} \frac{\Delta I}{I}^{true} \\ \frac{\Delta J}{J}^{true} \\ \frac{\Delta \rho}{\rho}^{true} \end{bmatrix}$$

$$\frac{\Delta \rho}{\rho}^{est} = 0.25 \frac{\Delta J}{J}^{true}$$

I estimate

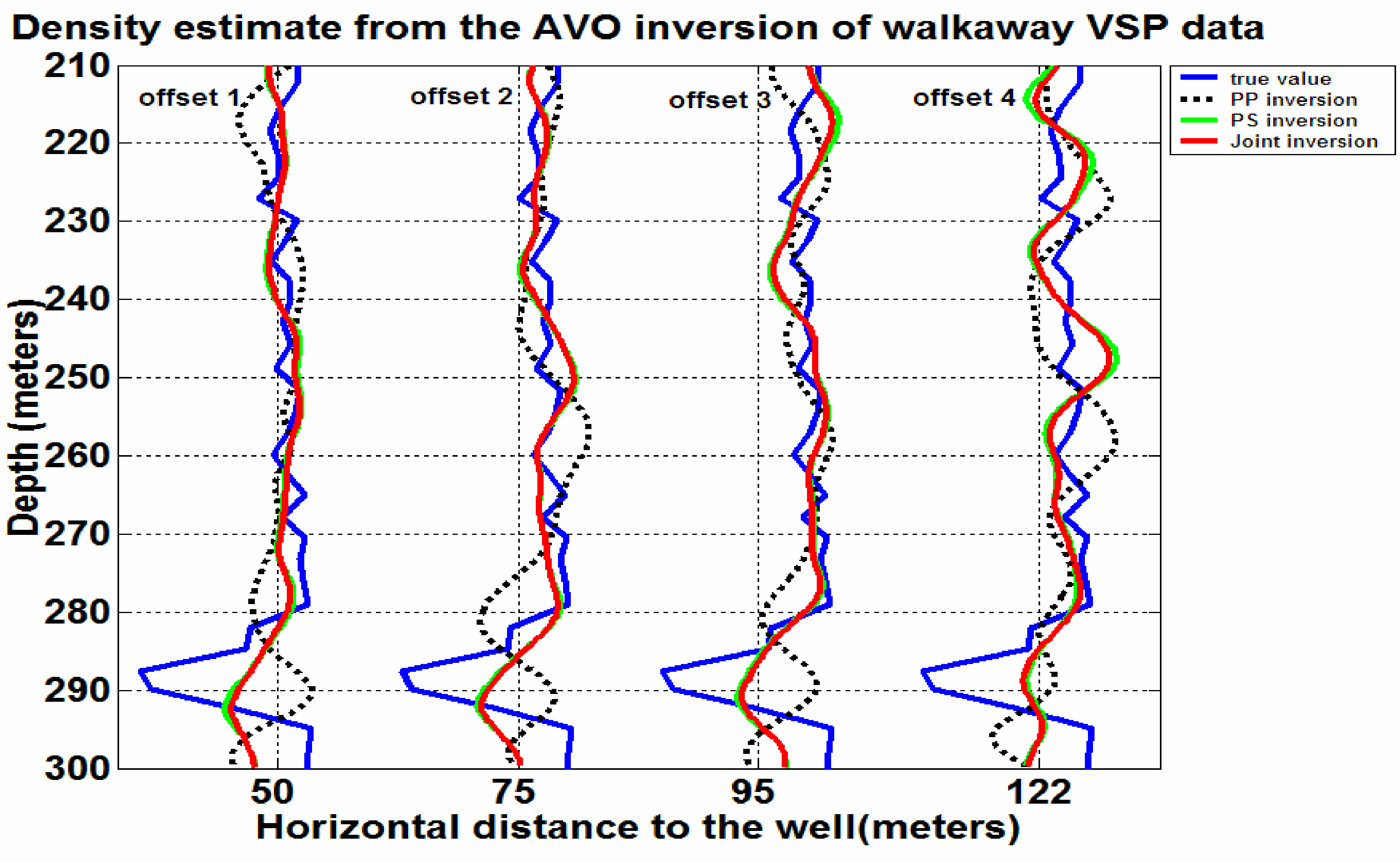

I: P- Impedance from the AVO inversion of walkaway VSP data
offset 1
offset 2
offset 3
offset 4
true value
PP inversion
Joint inversion
Depth (meters)
210
220
230
240
250
260
270
280
290
300
50
75
95
122
Horizontal distance to the well (meters)

J estimate



J: S- Impedance from the AVO inversion of walkaway VSP data
offset 1
offset 2
offset 3
offset 4
true value
PP inversion
PS inversion
Joint inversion
Depth (meters)
210
220
230
240
250
260
270
280
290
300
50
75
95
122
Horizontal distance to the well (meters)

Density estimate



Conclusions

- A favorable density estimate is obtained from linear AVO inversion.
- A good compressional impedance estimate can be obtained from the AVO inversion of PP data.
- Good shear impedance and density can be obtained from the AVO inversion of converted data.
- Converted waves data provides information on density not obtainable from compressional data.

Red Deer case study

- The shear wave velocity contributes more to improving the density estimate.
- Possible discontinuity in coal properties at the lateral distance between 95-125 m to the well.

Acknowledgment

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