# LOW-FREQUENCY WAVES

## Ray-tracing and eikonal solutions


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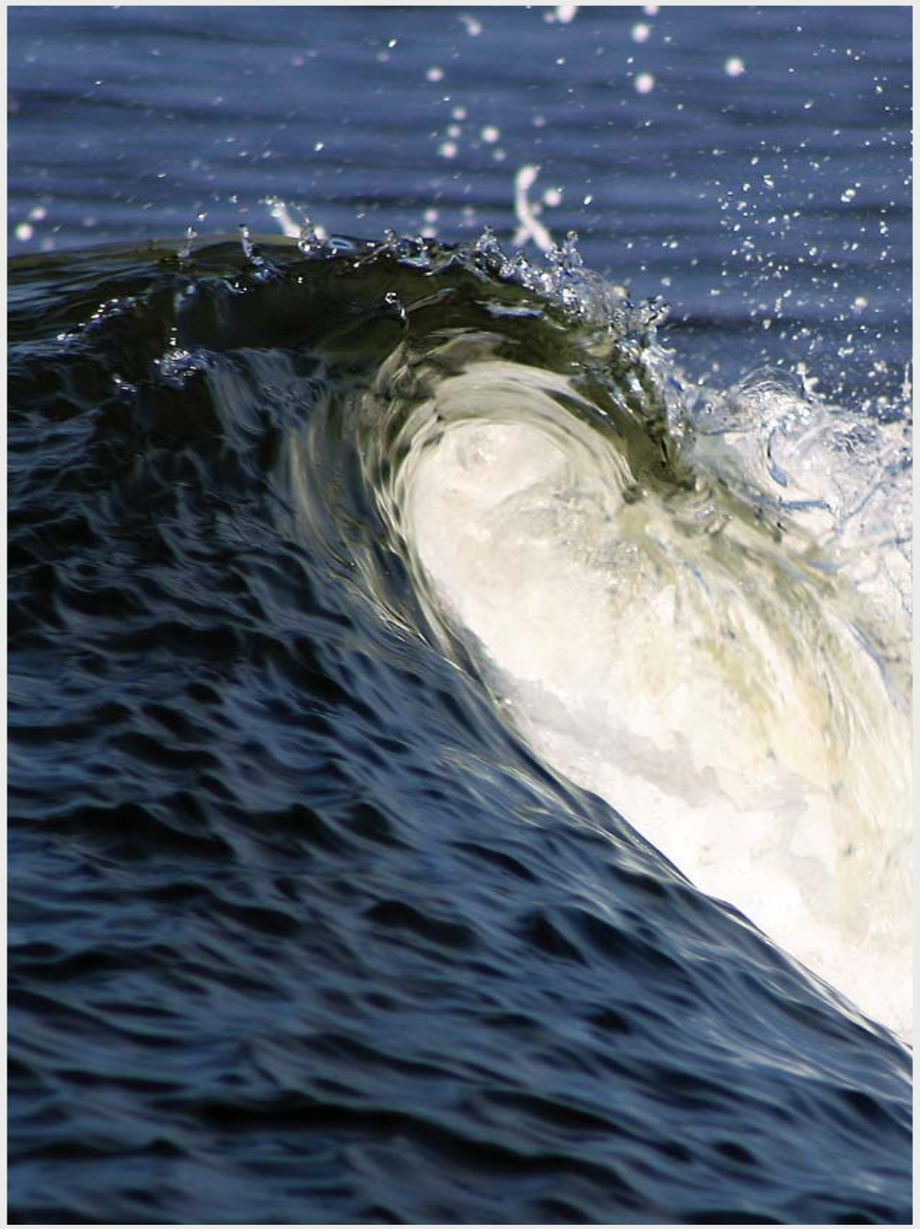
# HIGH-FREQUENCY MODELLING

- High-frequency solutions to wave equations
  - eikonal equation and ray tracing
  - hypereikonal equation
- What about low frequency?



# WAVE EQUATION

$$\nabla^2\Psi(\vec{x},t) - \frac{1}{v^2(\vec{x})}\partial_t^2\Psi(\vec{x},t) = 0$$

Helmholtz equation:

$$\nabla^2\psi(\vec{x},\omega) + \frac{\omega^2}{v^2(\vec{x})}\psi(\vec{x},\omega) = 0$$

$$\psi(\vec{x},\omega) = A(\vec{x},\omega)e^{i\omega\phi(\vec{x})}$$

# EIKONAL EQUATION

$$\left((\partial_j \phi)^2 - \frac{1}{v^2}\right) - \frac{i}{\omega}\left(\frac{2}{A}\partial_j A \partial_j \phi + \partial_j^2 \phi\right) - \frac{1}{\omega^2 A}\partial_j^2 A = 0$$

Transport equation

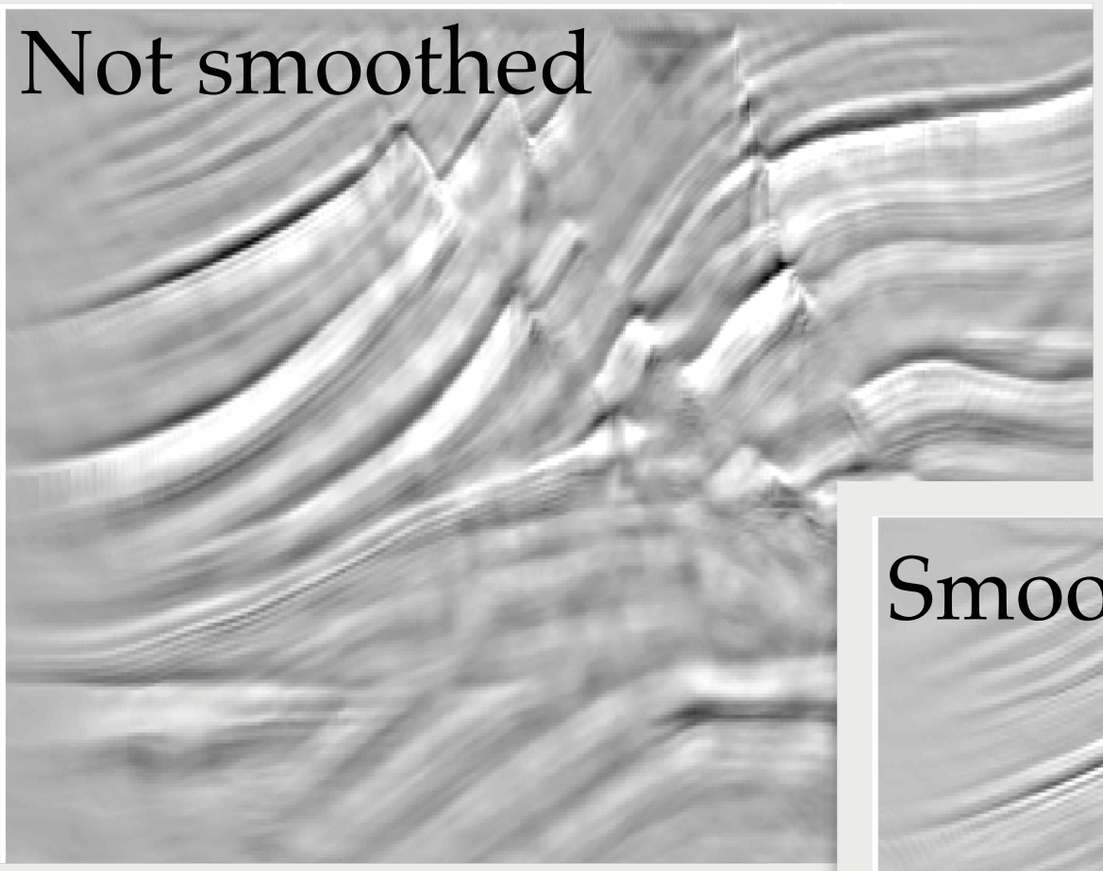
$$2\nabla A \cdot \nabla \phi + A\nabla^2 \phi = 0$$

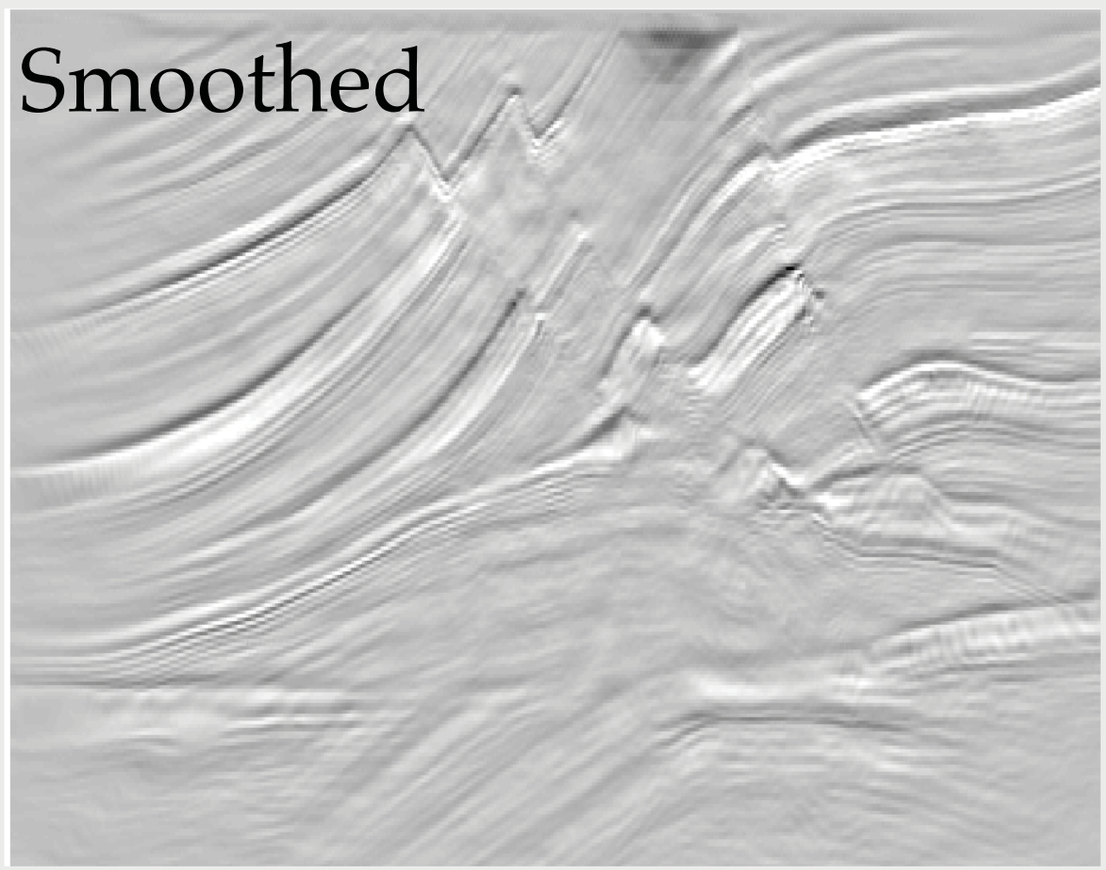
Eikonal equation

$$|\nabla \phi(\vec{x})|^2 = \frac{1}{v(\vec{x})^2}$$

# EIKONAL EQUATION

- Gives us travel times
- Can be very stable and reliable, even where ray-tracing fails miserably.
- Can be difficult to adapt to multiple arrival situations.




# SMOOTHING VELOCITY MODELS

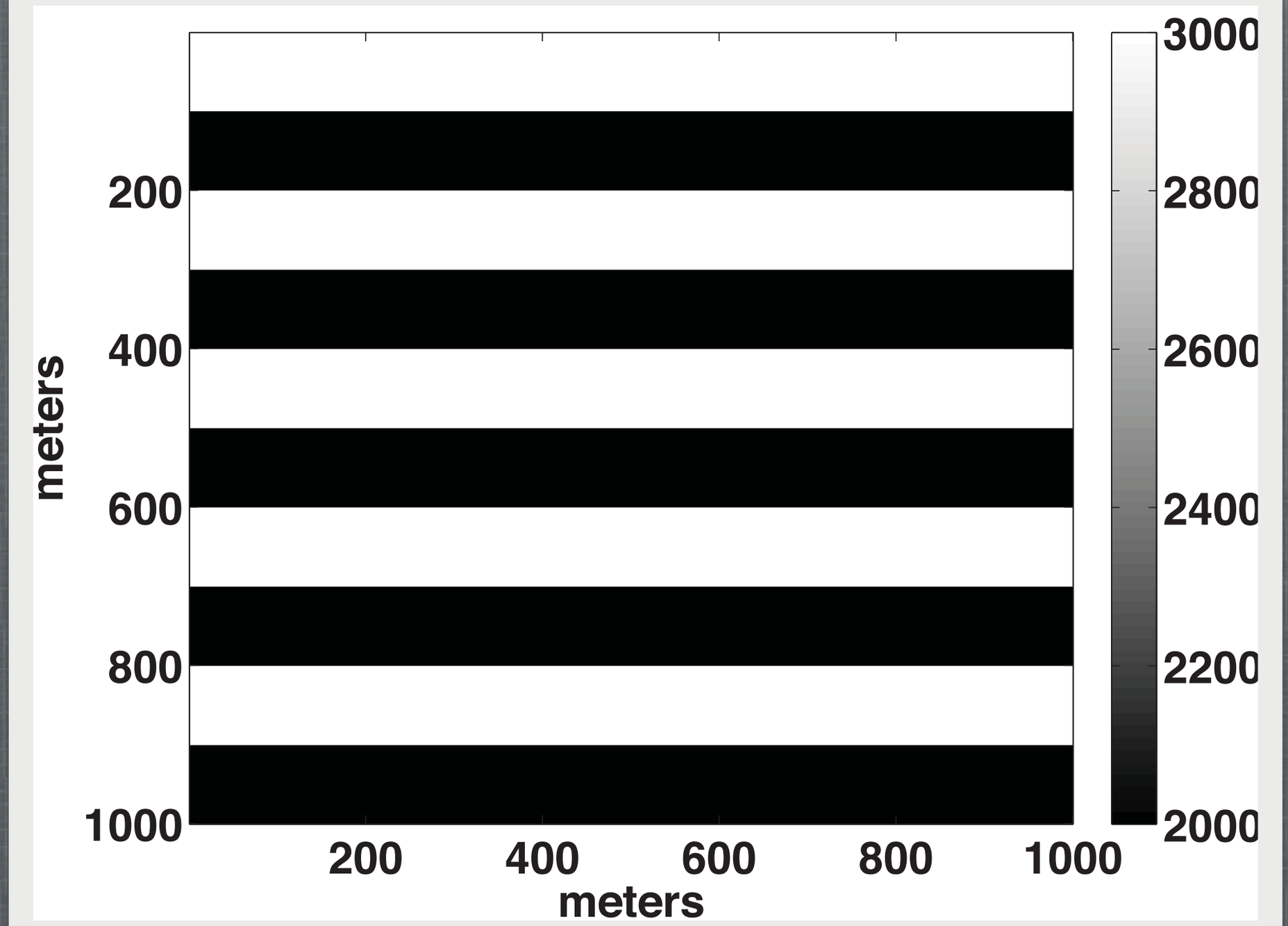
*It really does improve the image (at least for GPSPI)*

# FREQUENCY-DEPENDENT SMOOTHING?

- GPSPI shows a big improvement.
- Biondo Biondi (SEP 1999) showed that the hypereikonal equation leads to a smoothing of the velocity model.

$$\left((\partial_j \phi)^2 - \frac{1}{v^2}\right) - \frac{i}{\omega}\left(\frac{2}{A}\partial_j A \partial_j \phi + \partial_j^2 \phi\right) - \frac{1}{\omega^2 A}\partial_j^2 A = 0$$

$$S^2(\vec{x},\omega) = S_0^2(\vec{x}) + \frac{1}{\omega^2}\frac{\nabla^2 A(\vec{x},\omega)}{A(\vec{x},\omega)}$$



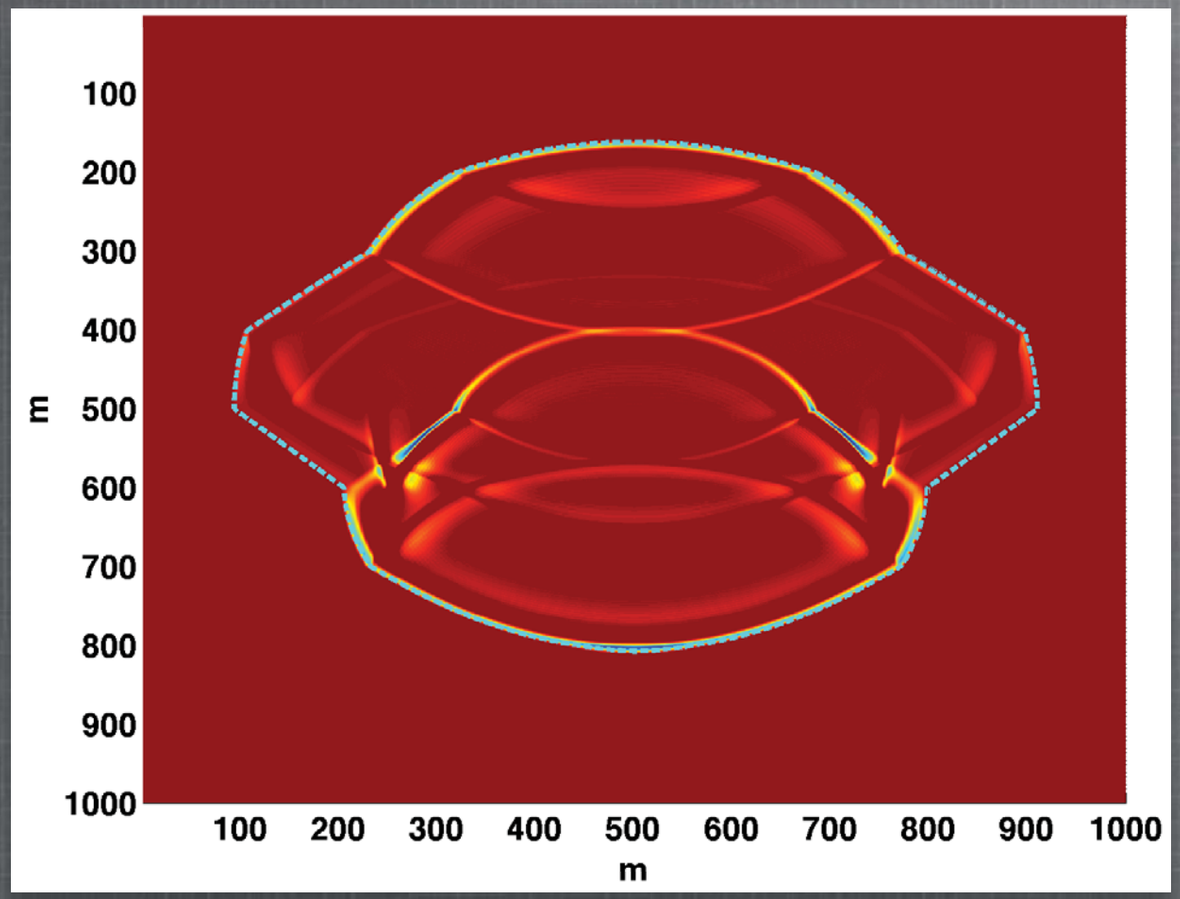

STRIPED VELOCITY MODEL

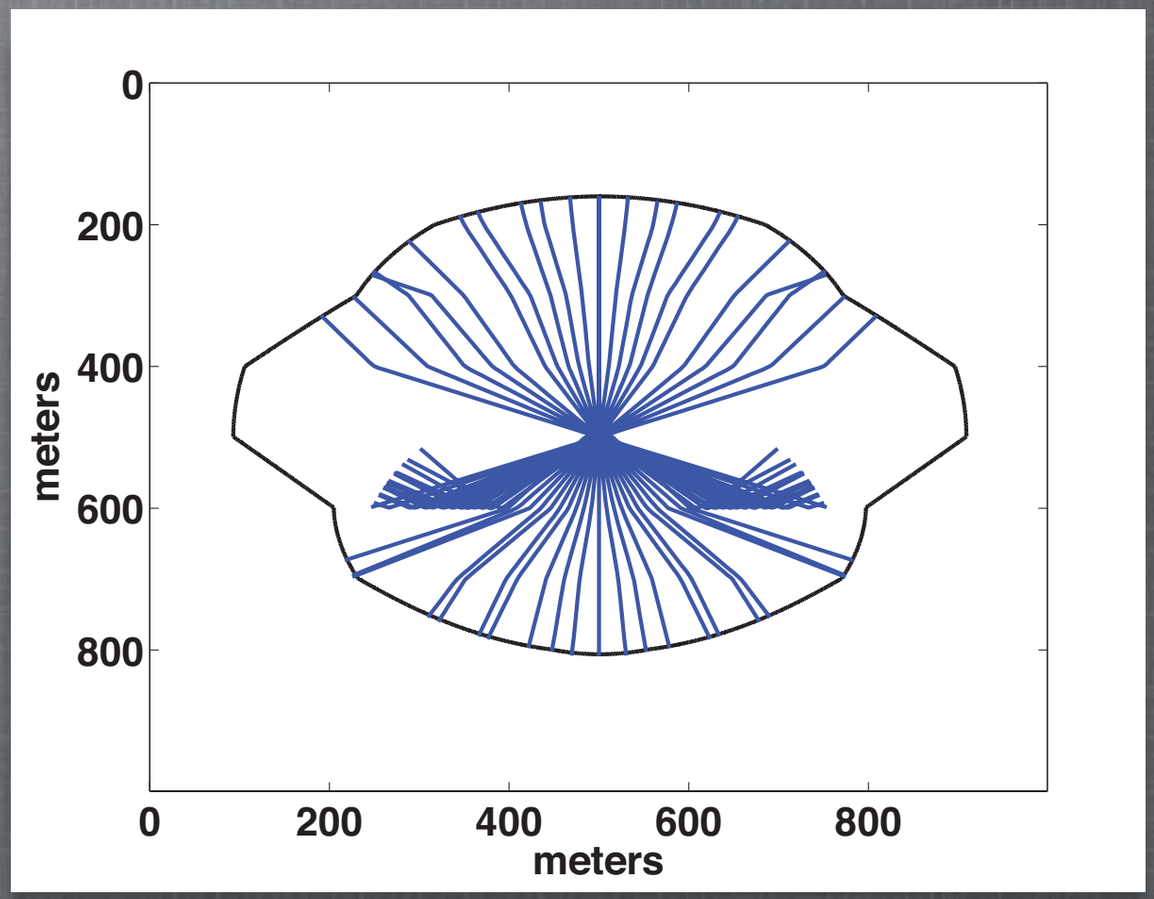
*(piecewise smooth)*


# TRYING OUT THE VARIOUS WAVE METHODS

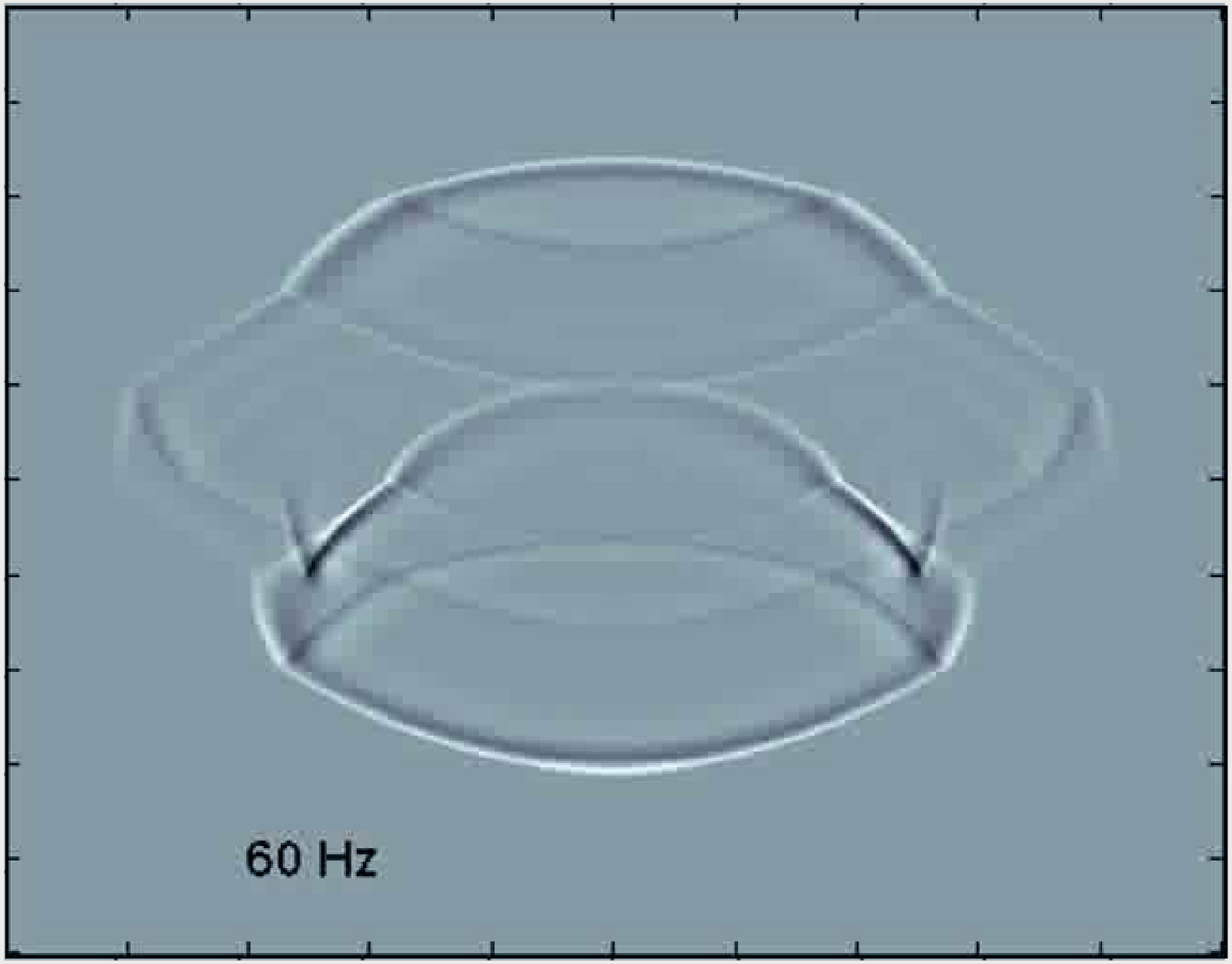
*Eikonal and ray-tracing solutions through variously smoothed media*

# FULL WAVEFIELD SOLUTION



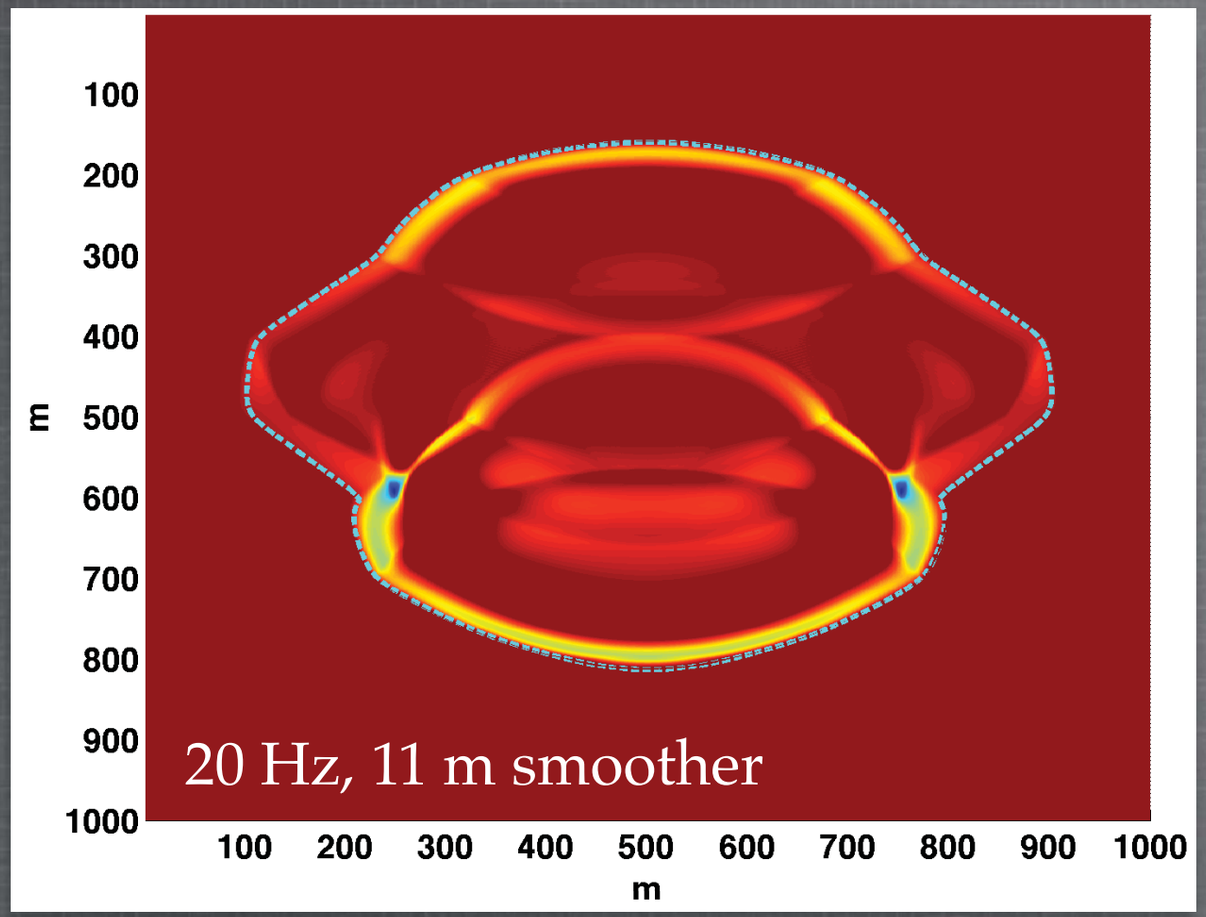




# FULL WAVEFIELD VS. EIKONAL & RAY-TRACING




## VARYING FREQUENCIES

*(actually, varying the dominant frequency of the source)*


100
200
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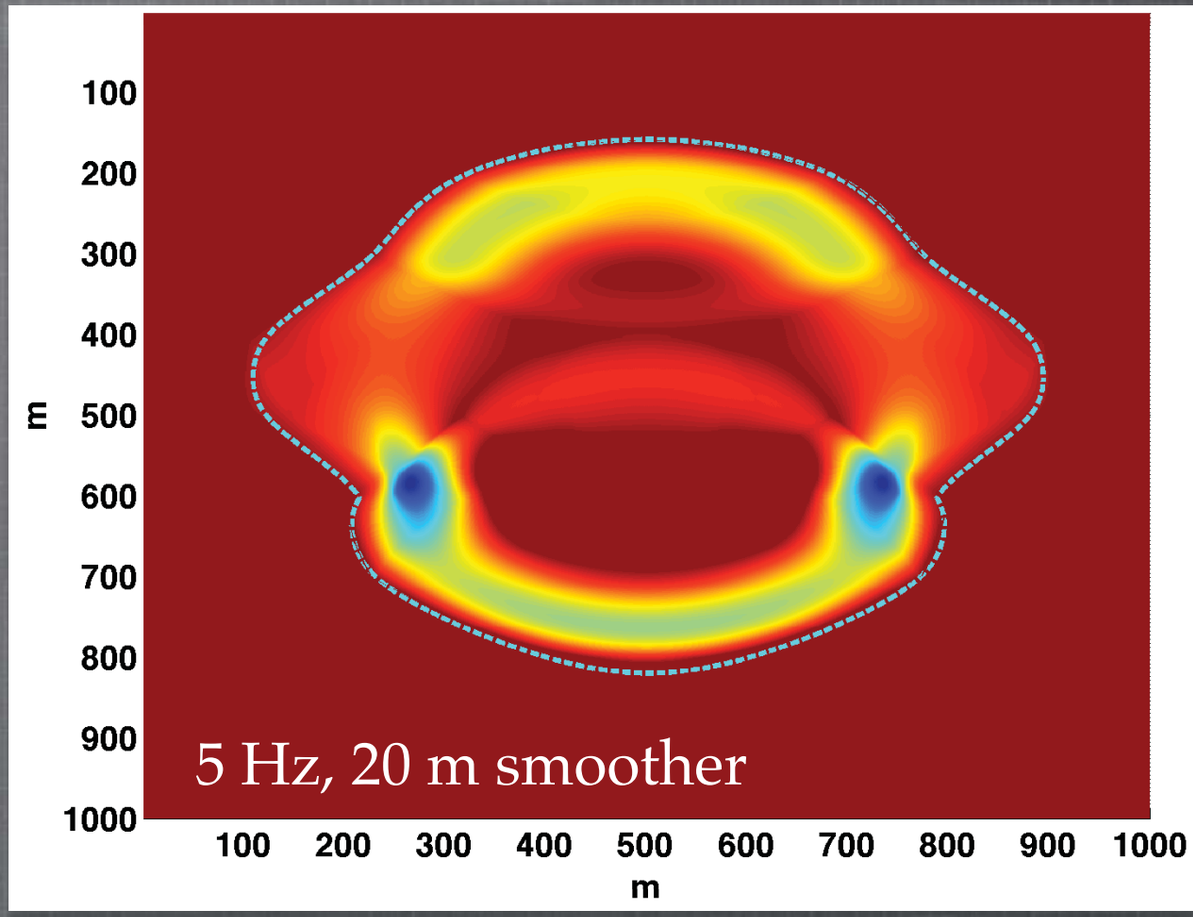
400
500
600
700
800
900
1000
m
100 200 300 400 500 600 700 800 900 1000
m
20 Hz, 11 m smoother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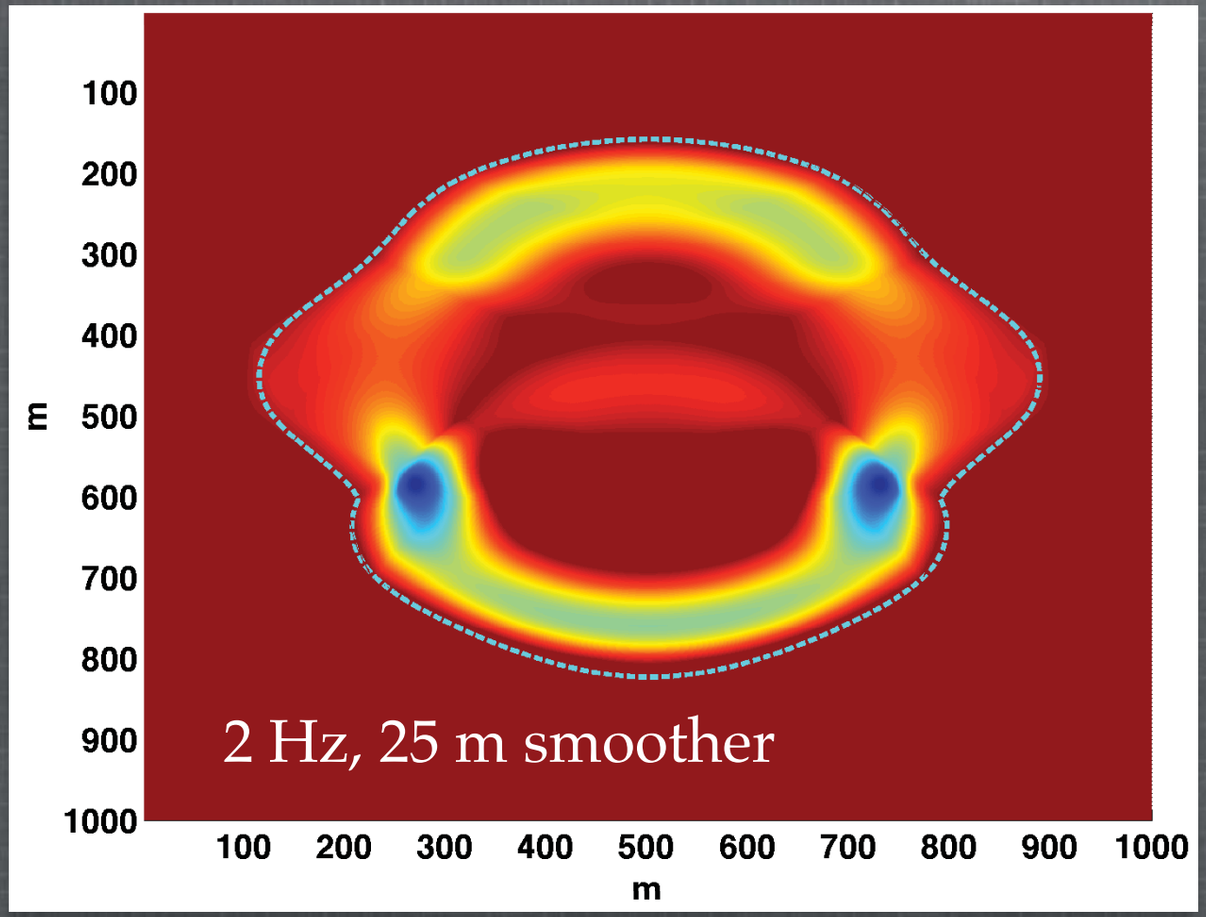
m
100 200 300 400 500 600 700 800 900 1000
m
5 Hz, 20 m smoother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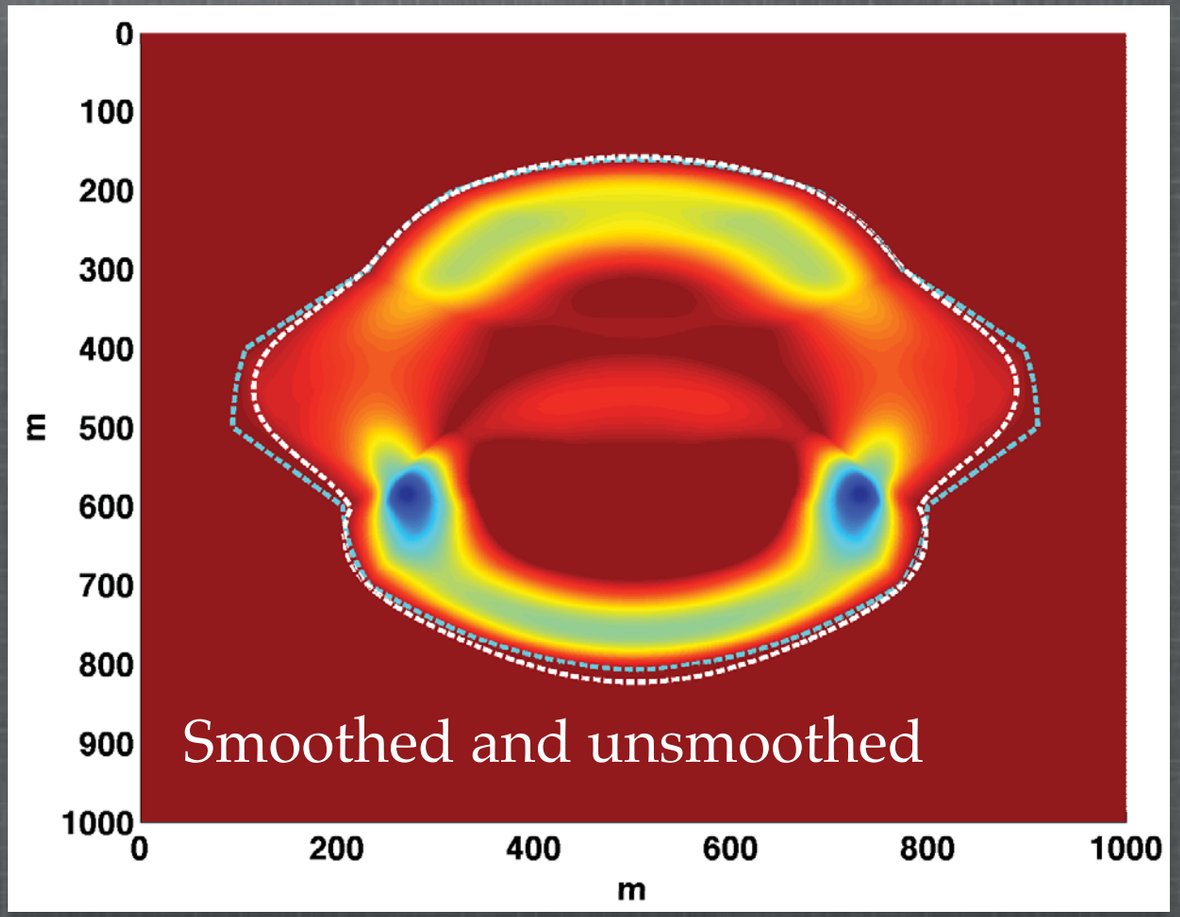
m
100 200 300 400 500 600 700 800 900 1000
m
2 Hz, 25 m smoother


0
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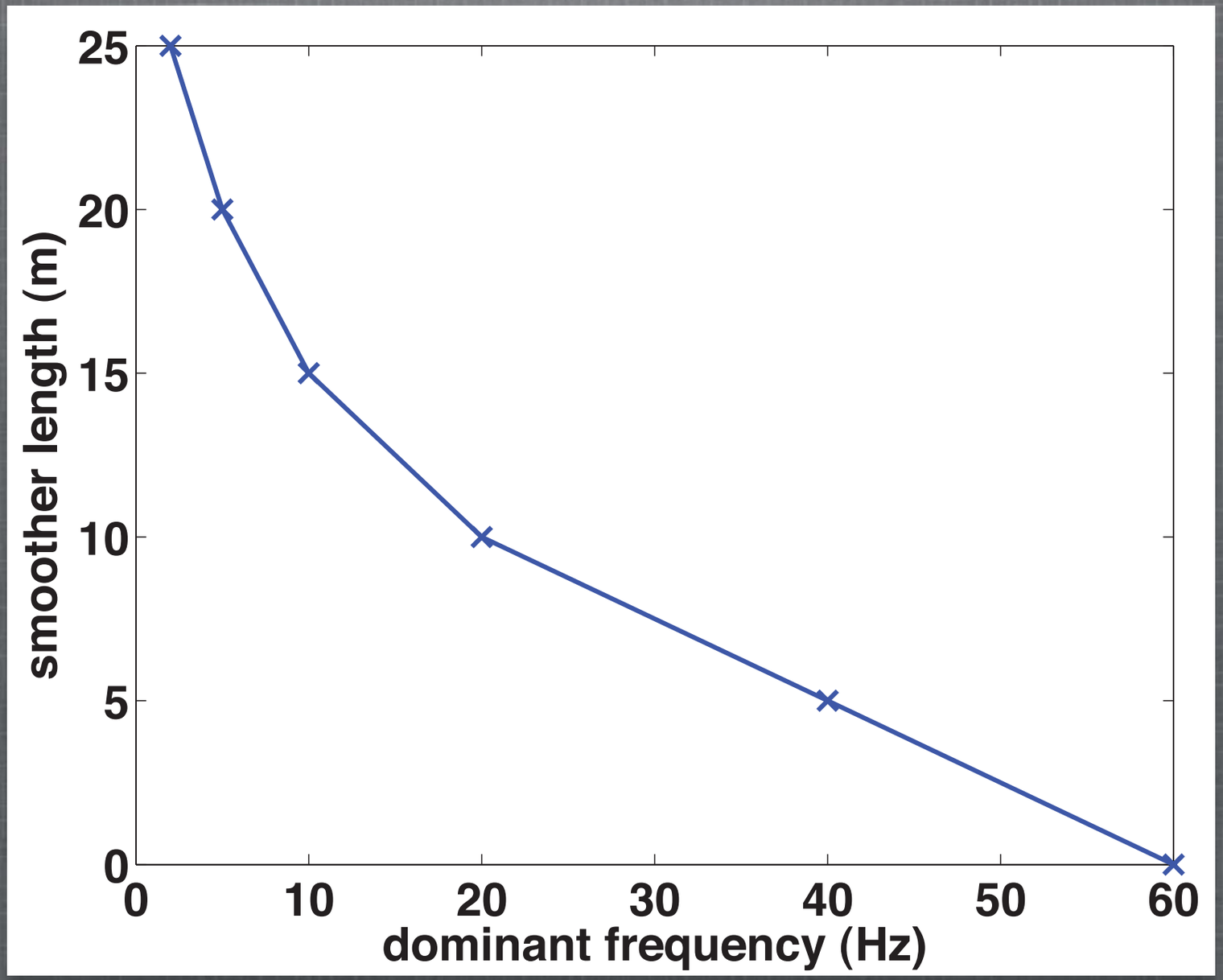
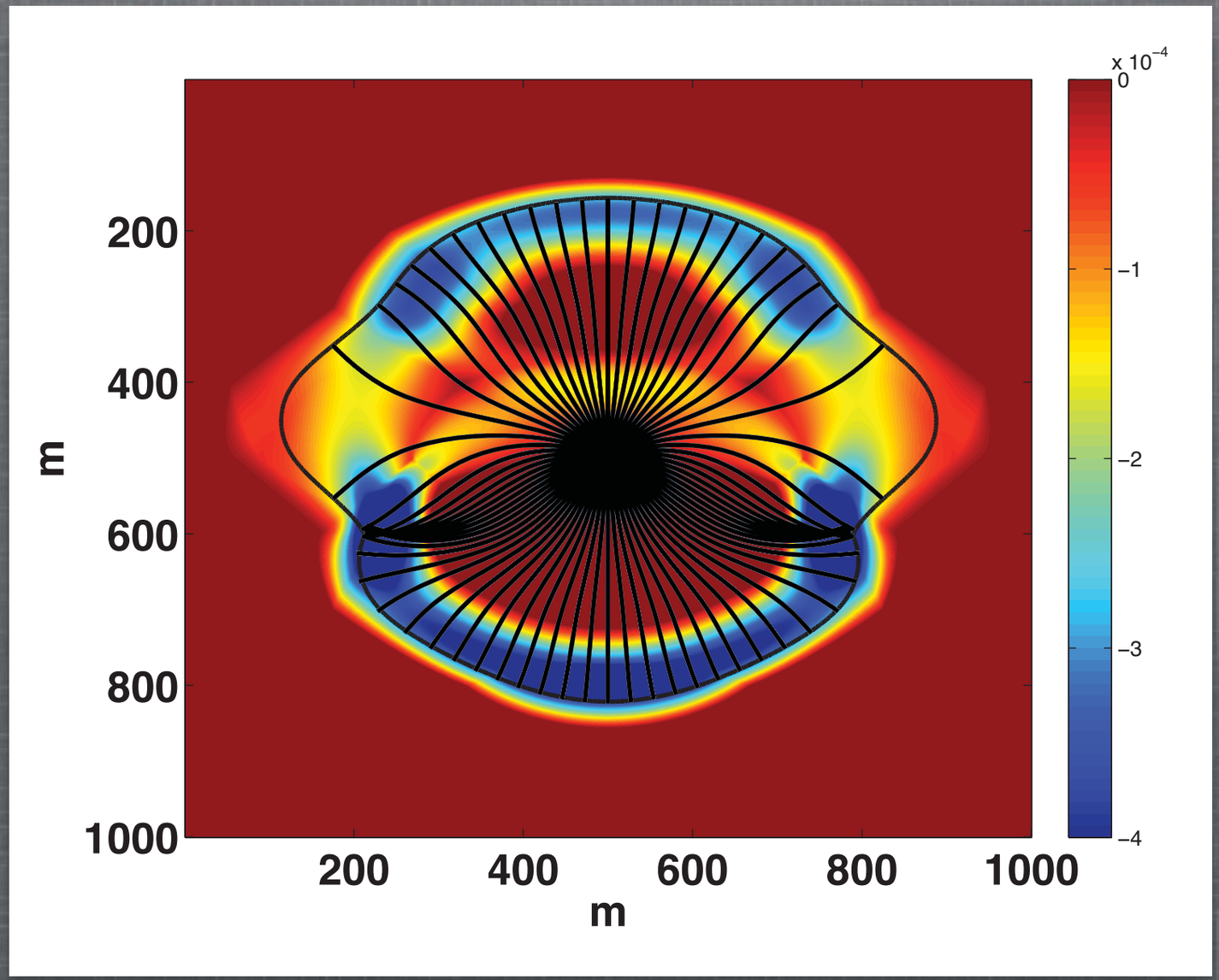
m
0 200 400 600 800 1000
m
Smoothed and unsmoothed

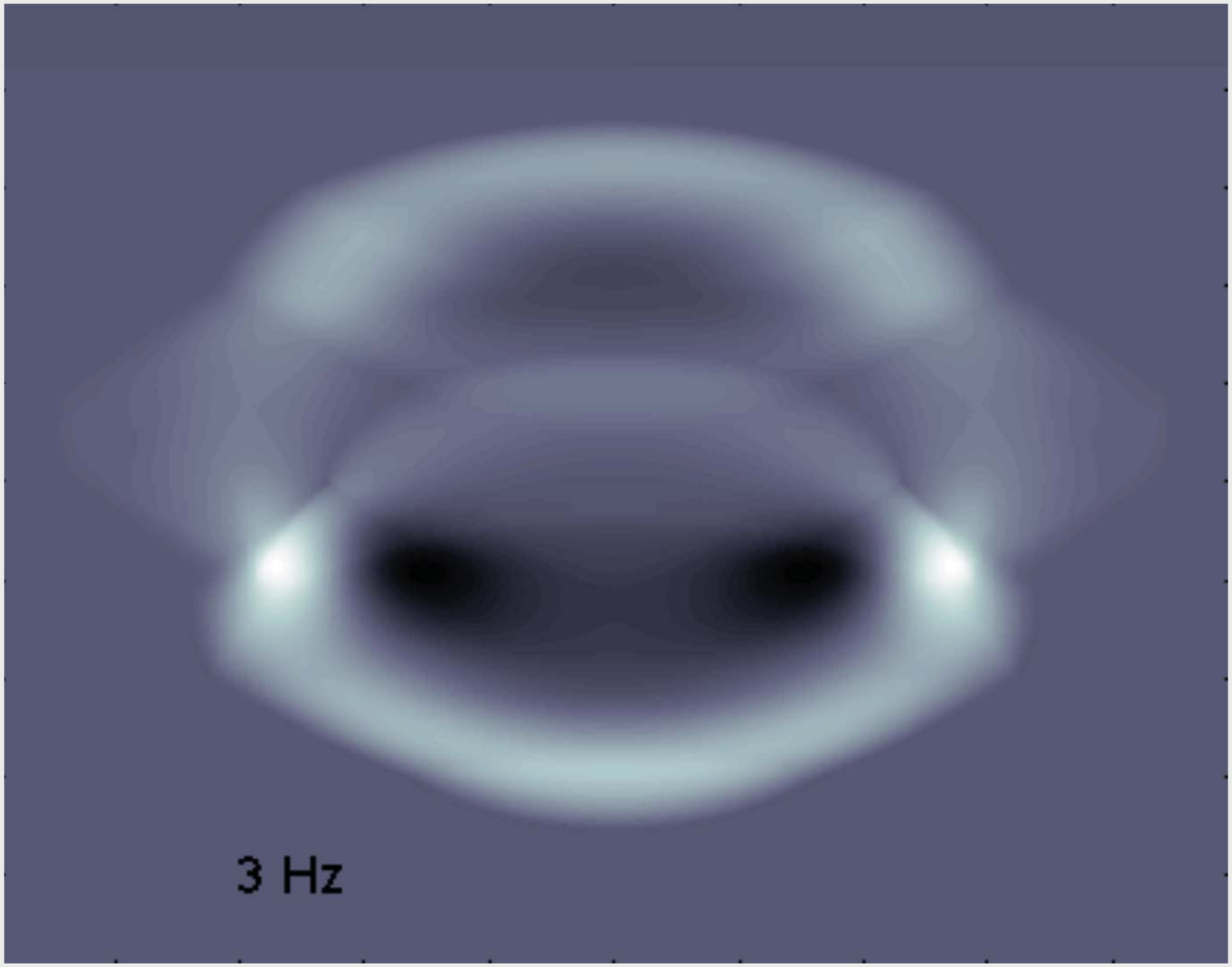



# SMOOTHER LENGTH VS. FREQUENCY

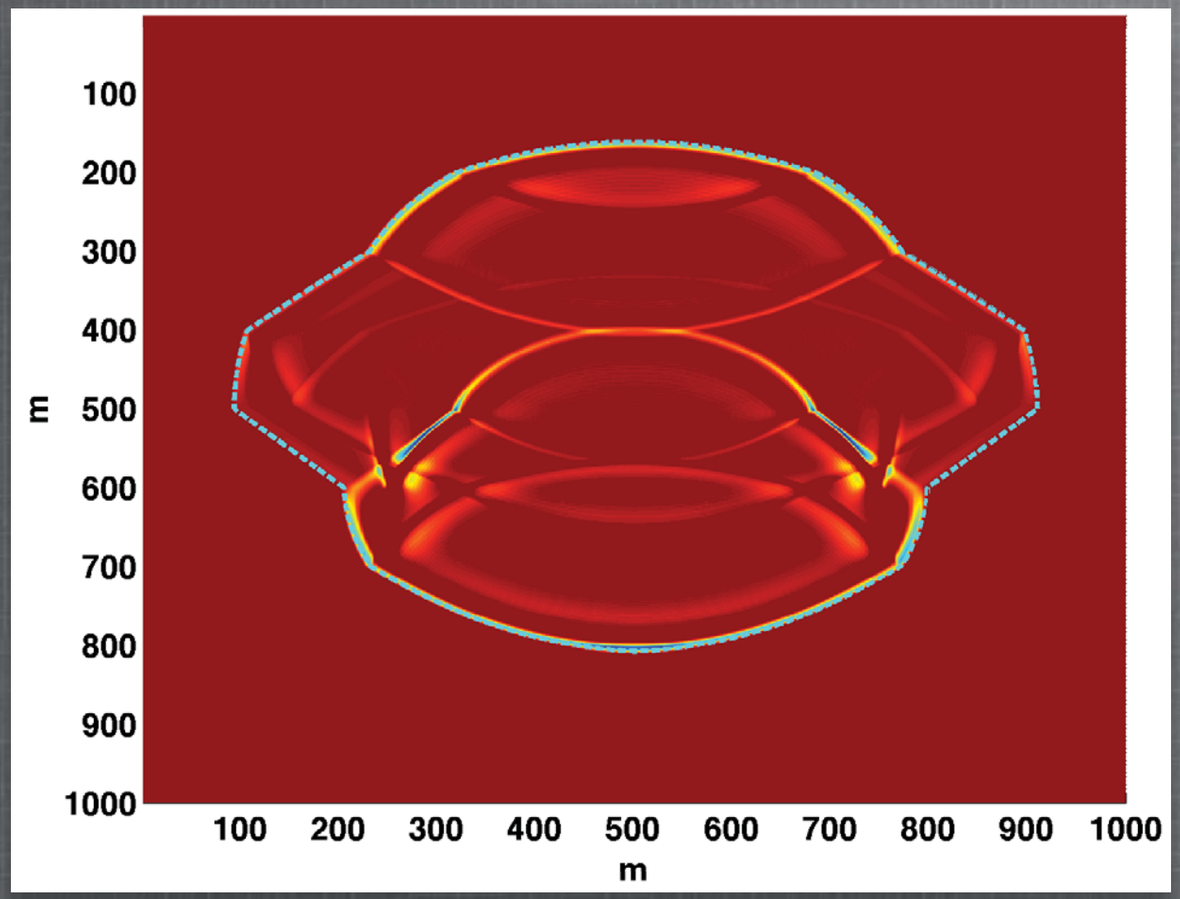
*An empirical relationship*

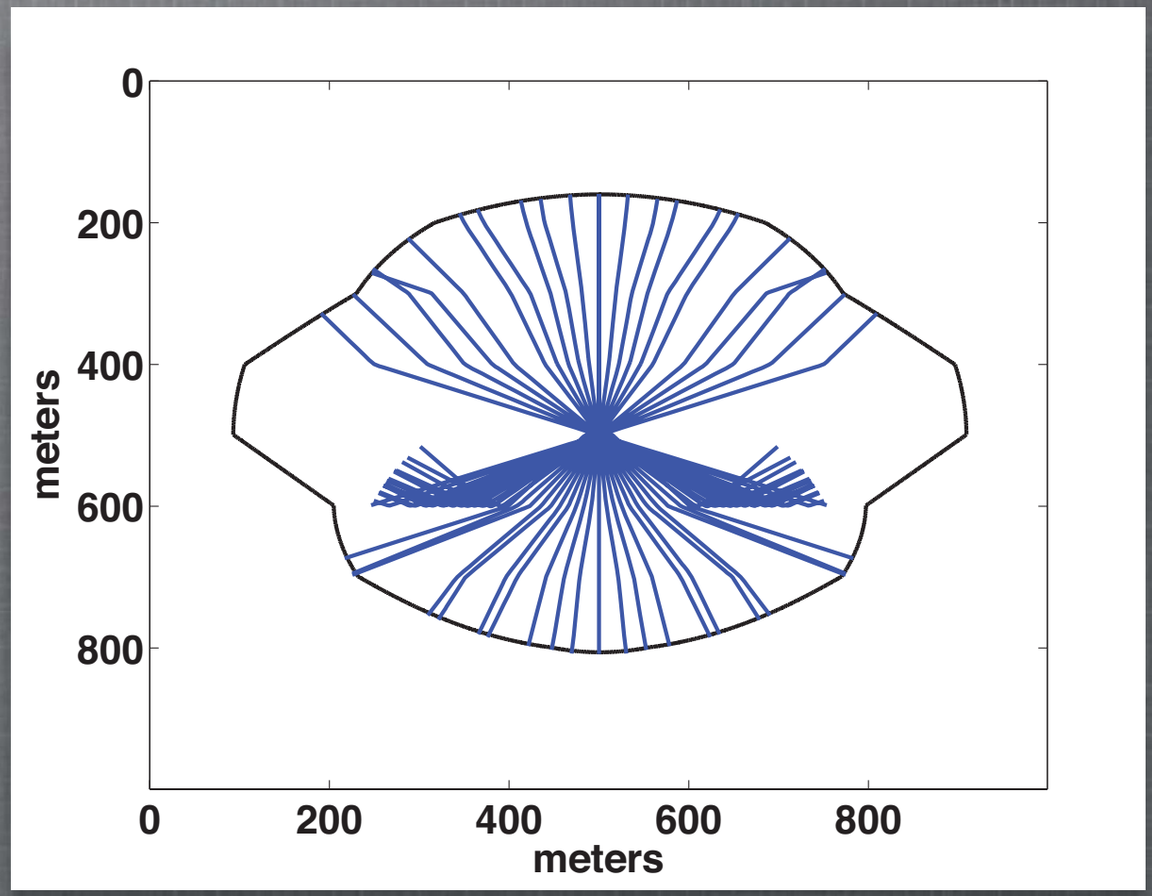



2 HZ WAVEFIELD, RAYS MATCHING EIKONAL SOLUTION




LOWEST FREQUENCY

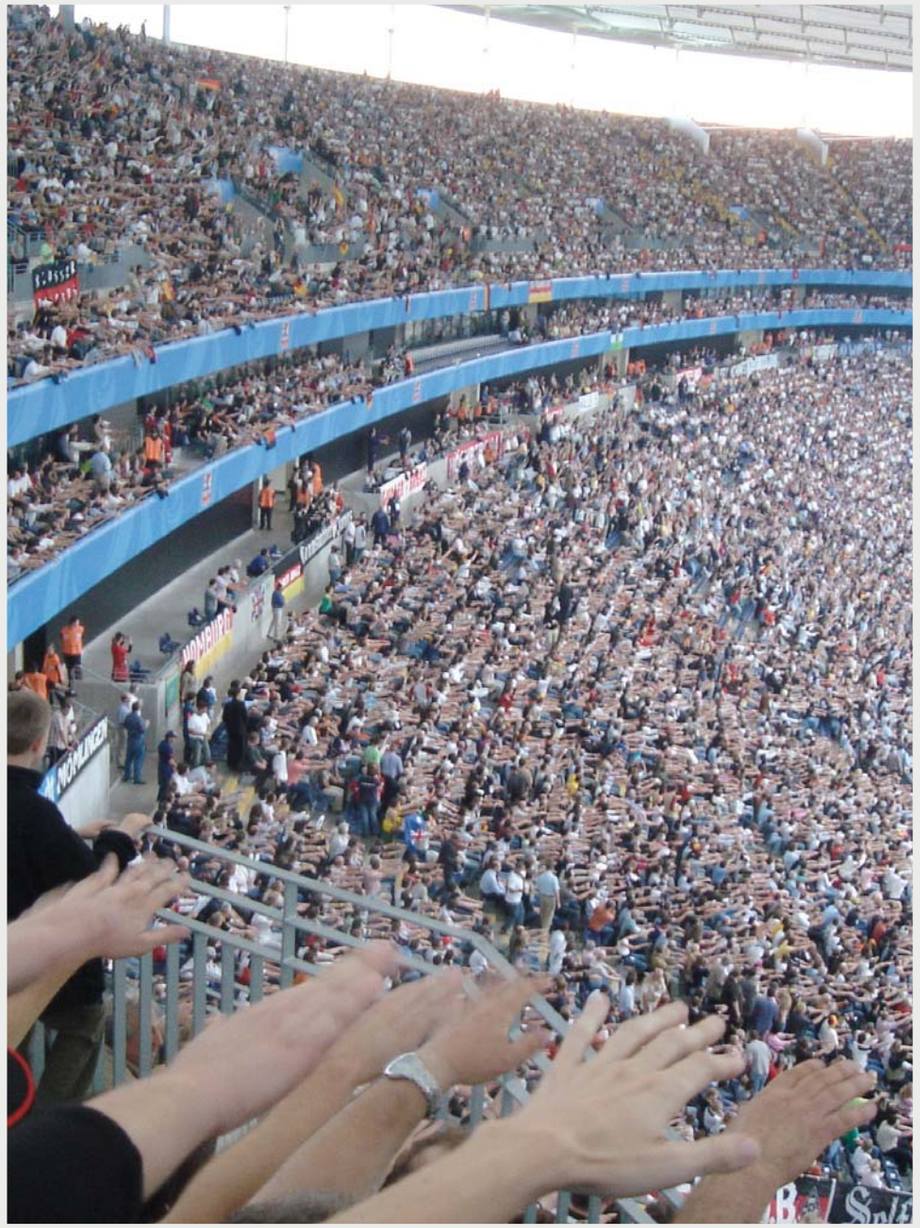



# HIGH FREQUENCY SOLUTIONS

# LOW FREQUENCY WAVES

- Not-high-frequency is worth considering.
- Smoothing does have some benefits.
- Can we take advantage of this for better modelling and/or migration?



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CREWES

PIMS

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POTSI PSEUDODIFFERENTIAL OPERATOR THEORY & SEISMIC IMAGING

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