Color correction for Gabor deconvolution and Nonstationary Phase Rotations

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CREWES

Outline

- Gabor deconvolution
- Color correction methods
- Nonstationary phase rotation
- Conclusions

Gabor deconvolution

- Gabor Transform (GT)

➢ Extend FT to the nonstationary realm

$$S_G(\tau, f) = \int_{-\infty}^{\infty} s(t) g(t-\tau) e^{-i2\pi ft} dt$$

$g(t)$: Gabor analysis window

τ : window center

Gabor deconvolution

- Nonstationary model of seismic traces

$$\hat{s}(f) = \hat{w}(f) \int_{-\infty}^{\infty} \alpha_Q(\tau, f) r(\tau) e^{-i 2\pi f \tau} d\tau$$

Constant-Q attenuation:

$$\alpha_Q(\tau, f) = e^{-\frac{\pi f \tau}{Q} + iH\left(\frac{\pi f \tau}{Q}\right)}$$

Gabor deconvolution

- GT of attenuated seismic trace

$$S_G(\tau, f) \approx \hat{w}(f)\alpha(\tau, f)R_G(\tau, f)$$

- Estimated propagating wavelet

$$\hat{w}(f)\alpha_Q(\tau, f) \approx \overline{|S_G(\tau, f)|} e^{i\varphi(\tau, f)}$$

$$\varphi(\tau, f) = \int_{-\infty}^{\infty} \frac{\ln(\overline{|S_G(\tau, f')|})}{f - f'} df'$$

Assumption of white reflectivity: $|R_G(\tau, f)| \approx 1$

Gabor deconvolution

- Estimated reflectivity

$$R_G(\tau, f)_{est} = \frac{S_G(\tau, f)}{\left|\overline{S_G(\tau, f)}\right| + \mu A_{\max}} e^{-i\varphi(\tau, f)}$$

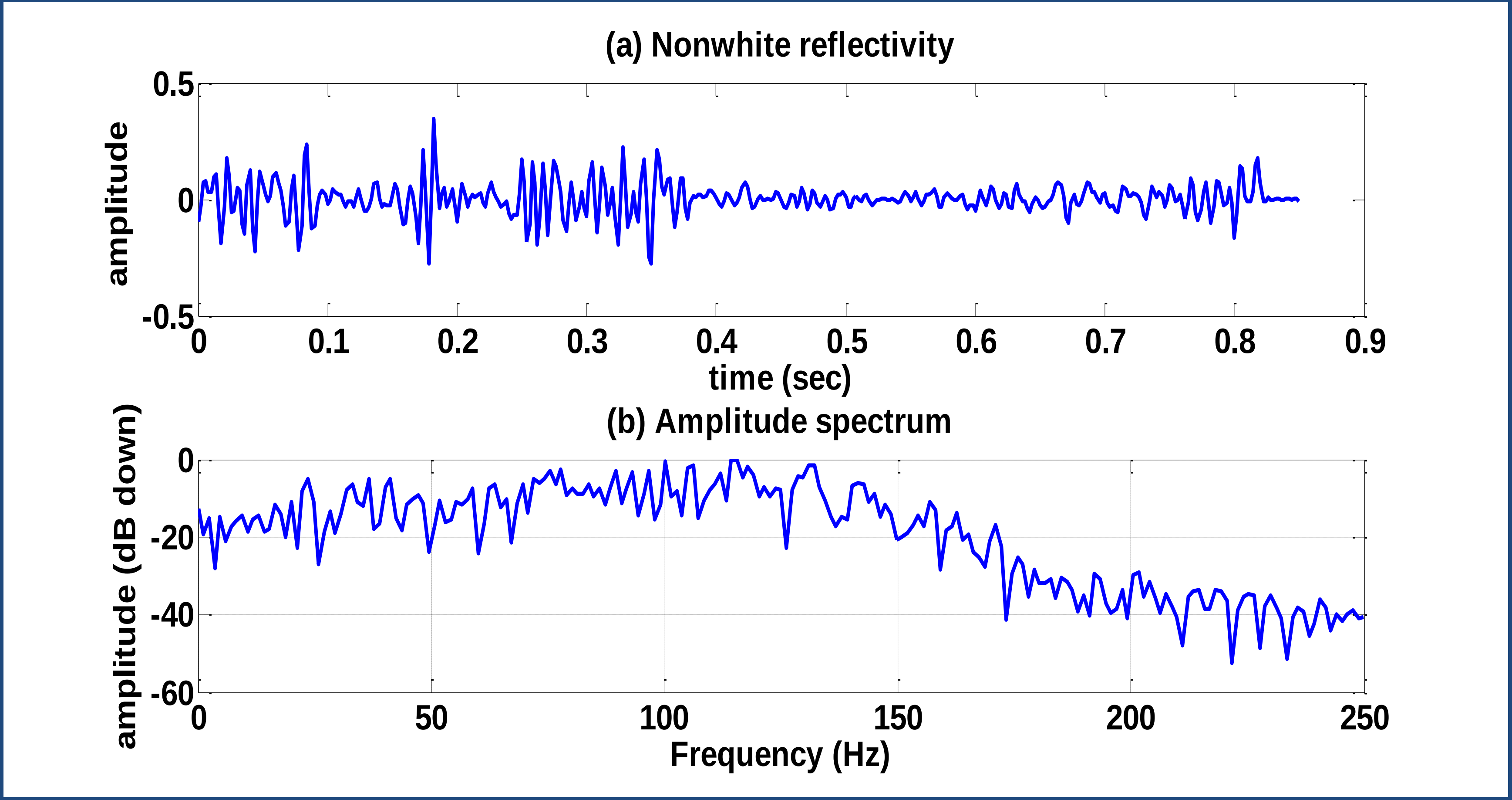
with assumption of white reflectivity

Color correction

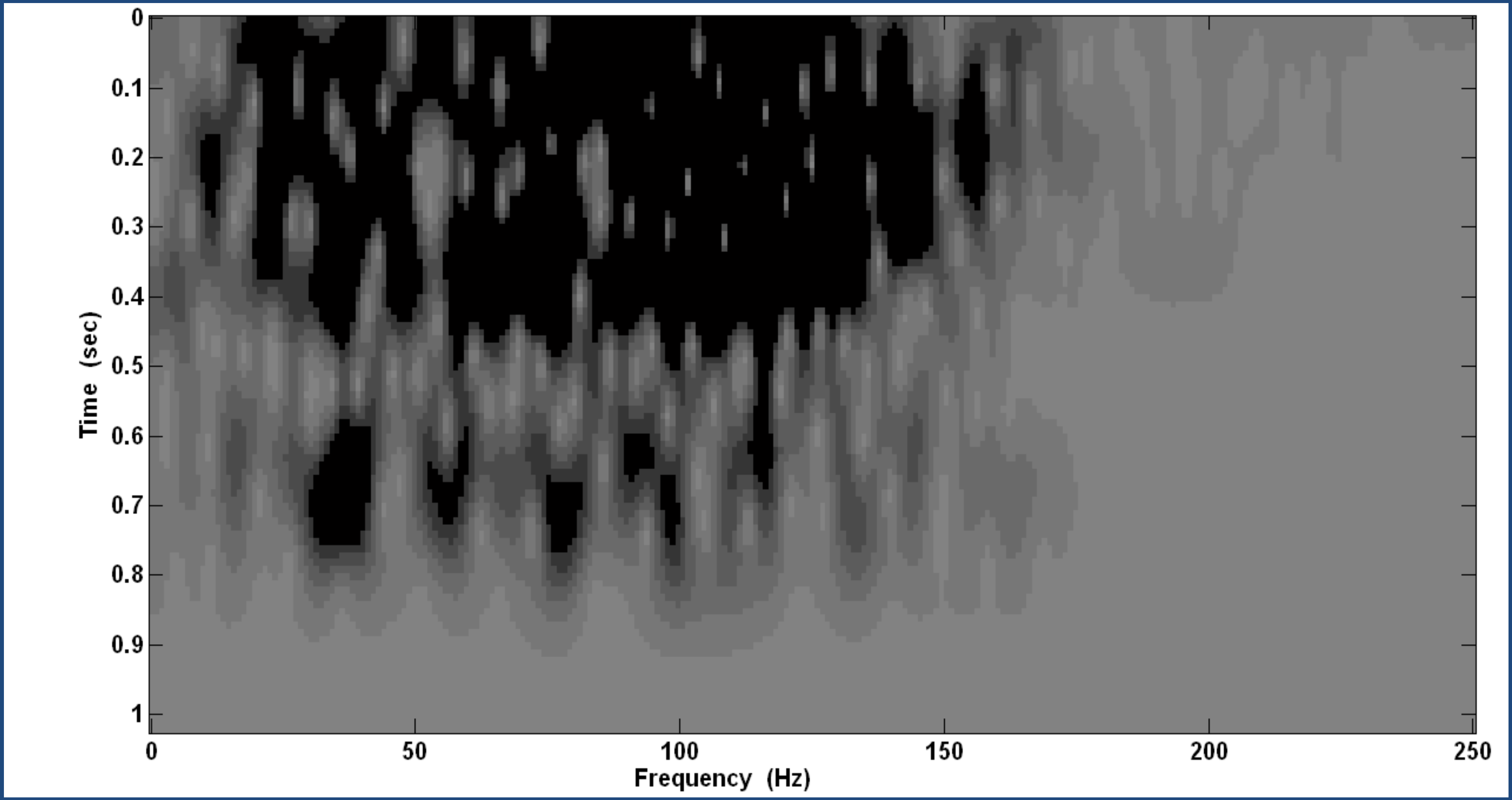
- Nonwhite reflectivity in practice

$$r_c(t) \rightarrow | R'_G(\tau, f) | \neq 1$$

Nonwhite reflectivity



Nonwhite reflectivity



Color correction

- Condition:

  $\overline{\left|R'_G(\tau, f)\right|}$ is available from well log

- Estimation of nonwhite reflectivity

$$R'_G(\tau, f)_{est} = \frac{S_G(\tau, f)\overline{\left|R'_G(\tau, f)\right|}}{\overline{\left|S_G(\tau, f)\right|} + \mu A_{\max}} e^{i\varphi_c(\tau, f)}$$

$$\varphi_c(\tau, f) = H\left(\ln\left|\frac{\overline{R'_G(\tau, f)}}{\overline{S_G(\tau, f)} + \mu A_{\max}}\right|\right)$$

Color correction

- Effect of color correction

$$R'_G(\tau, f)_{est} = \frac{S_G(\tau, f)\overline{|R'_G(\tau, f)|}}{\overline{|S_G(\tau, f)|} + \mu A_{\max}} e^{i\varphi_c(\tau, f)}$$

How much does $\overline{|R'_G(\tau, f)|}$ depart from unity?

How reliable is $\overline{|R'_G(\tau, f)|}$?

Influential factors:

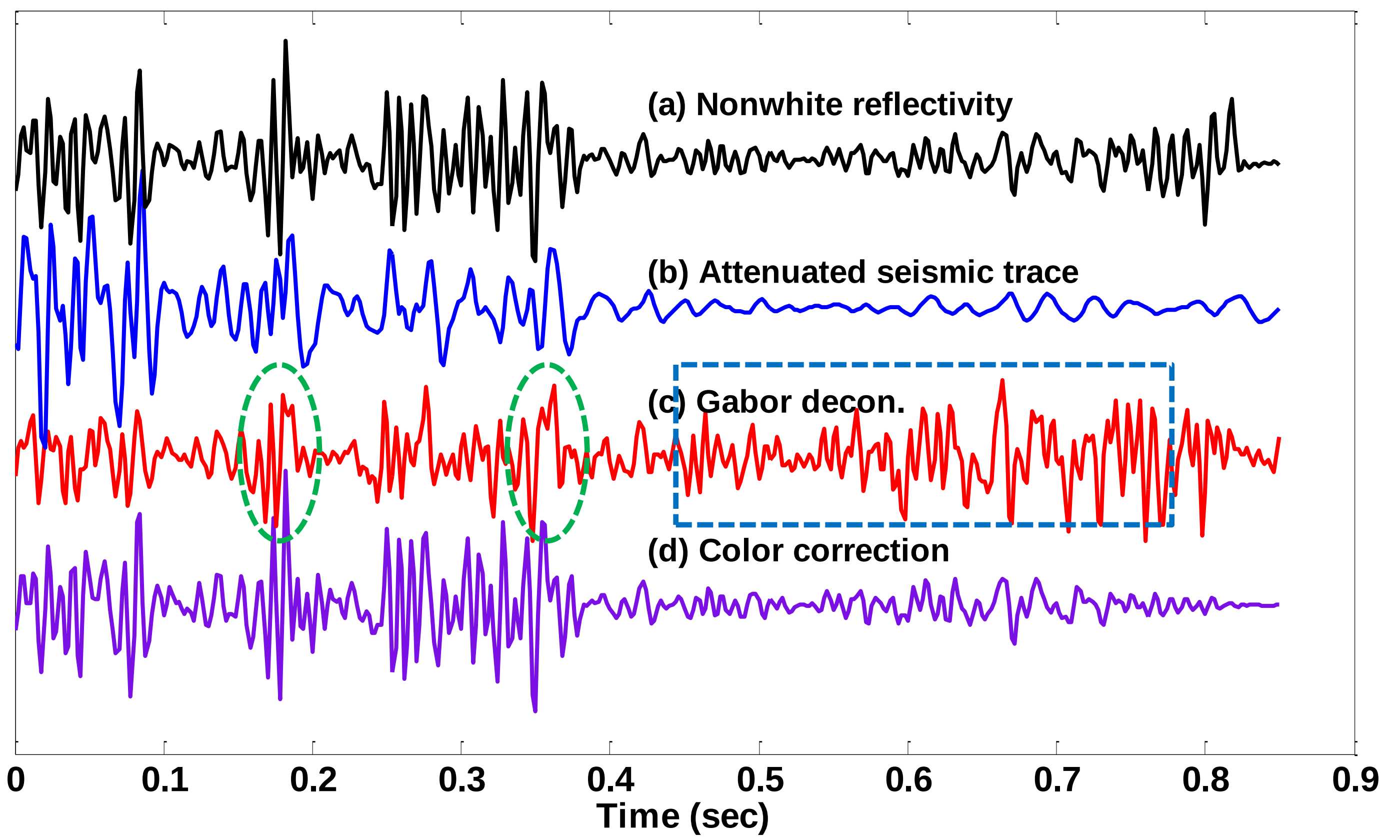
- available frequency band
- completeness of well log

An ideal case of color correction

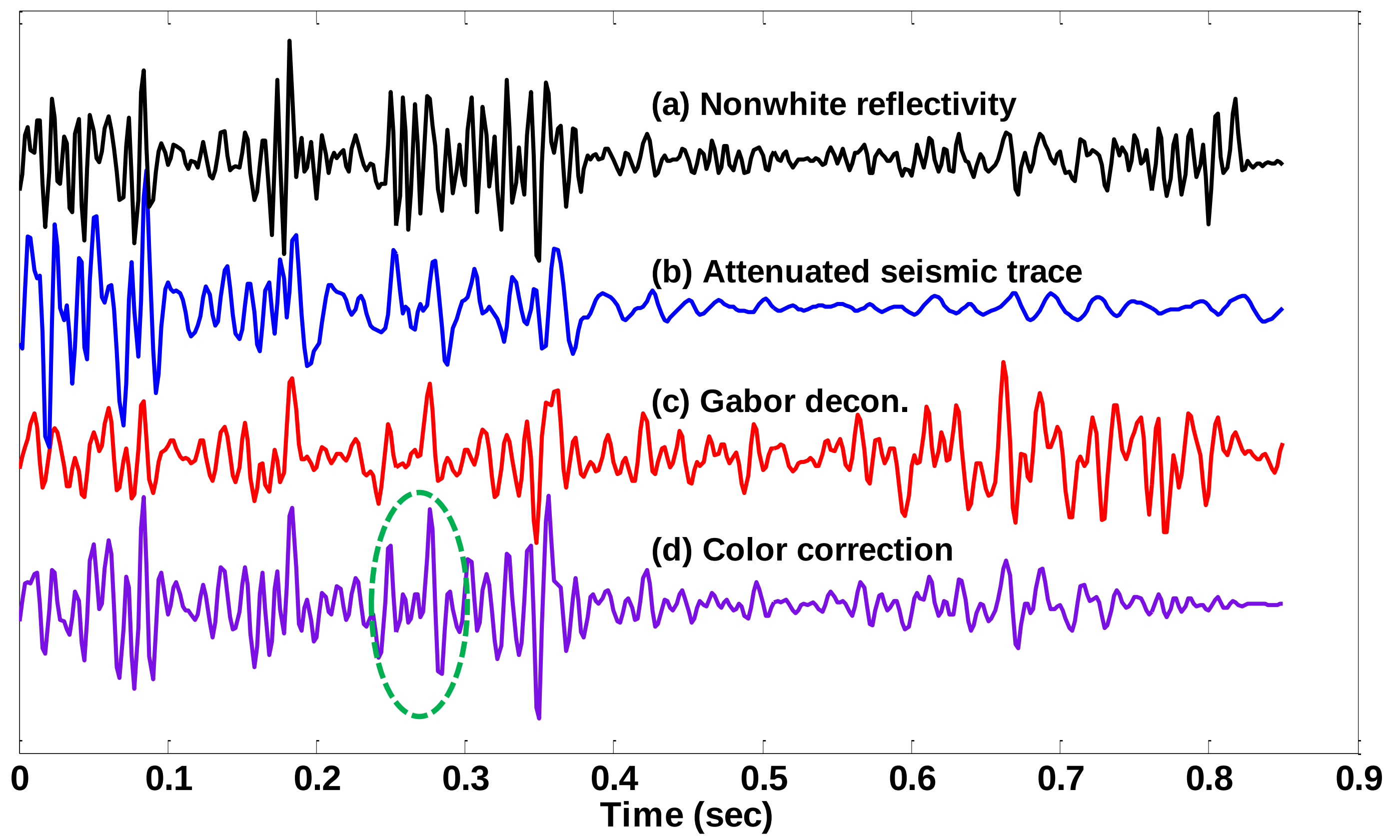
- Complete well log

- Broad frequency band for deconvolution

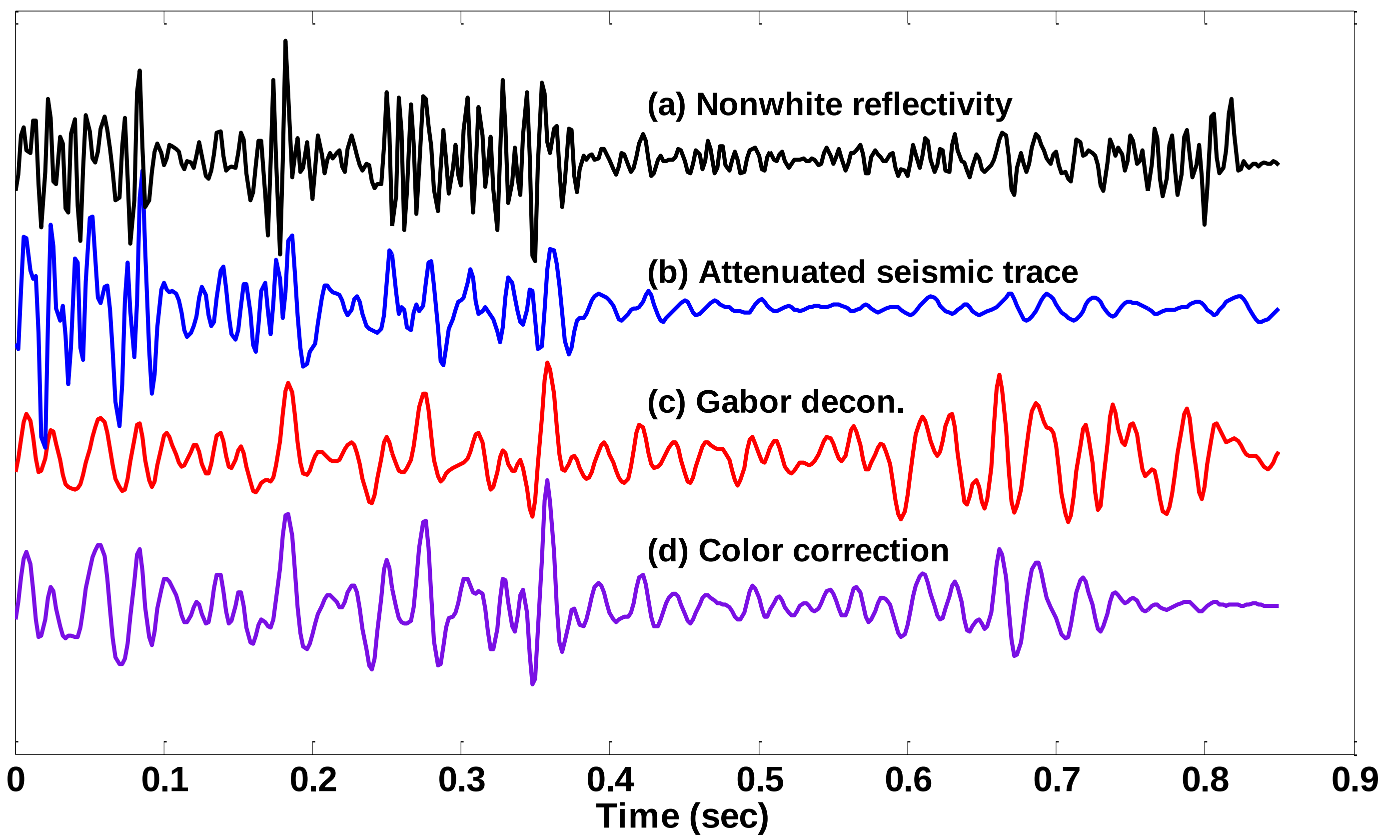
An ideal case of color correction (10-150Hz)



Effect of limited frequency band (10-100Hz)



Effect of limited frequency band (10-60Hz)



Practical color correction

- Incomplete well log

seismic trace: $s(t) \quad t \in (0, t_{\max})$

well log: $\tilde{r}_c(t) \quad t \in (t_1, t_2) \quad 0 < t_1 < t_2 < t_{\max}$

$$\tilde{r}_c(t) \rightarrow \overline{\left| R'_G(\tau, f) \right|}?$$

Practical color correction

- Creation of correction pattern

Method 1:

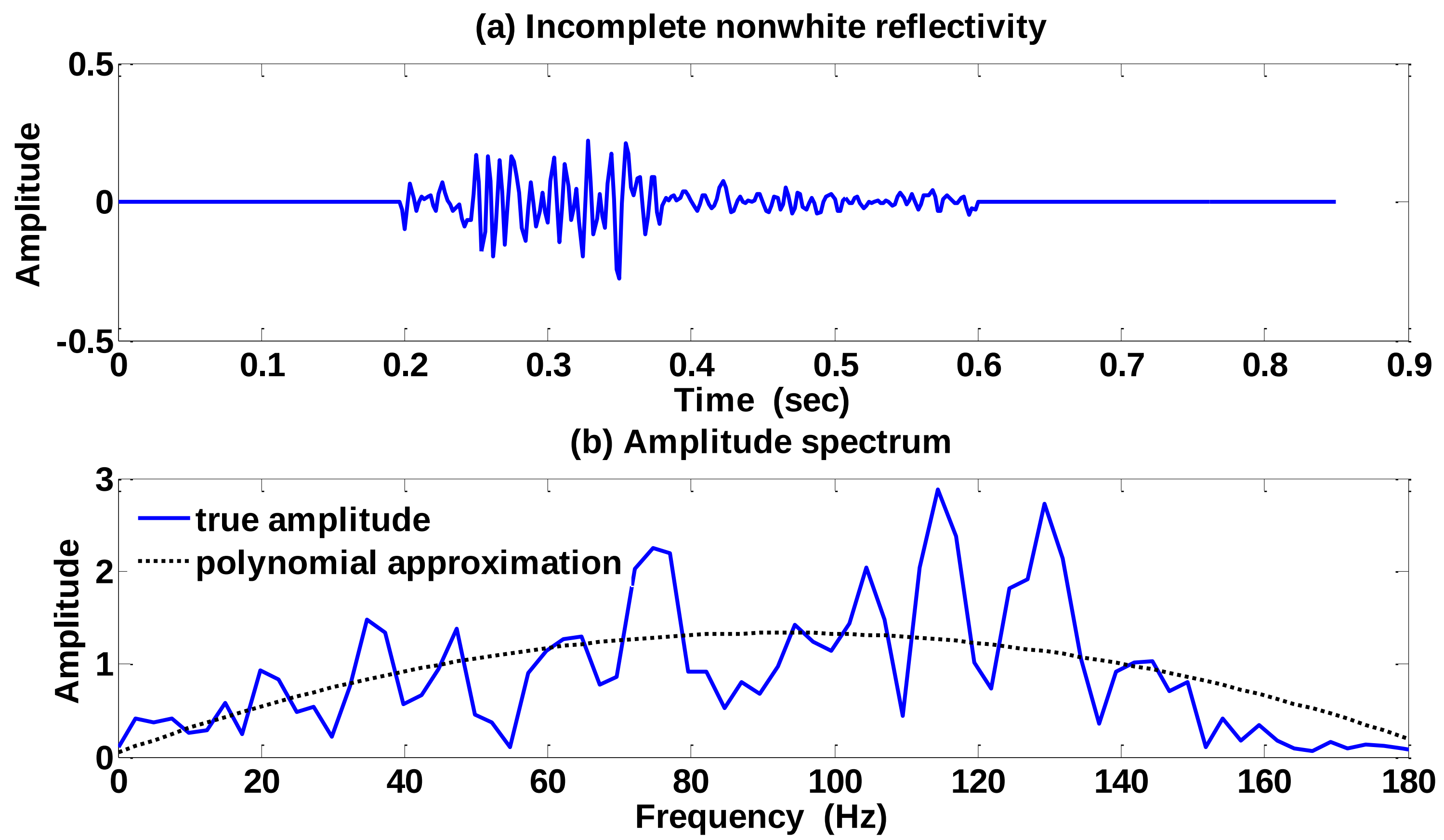
$$\tilde{r}_c(t) \rightarrow \tilde{R}_c(f)$$

$$|\tilde{R}_c(f)| \approx a_0 + a_1 f + a_2 f^2$$

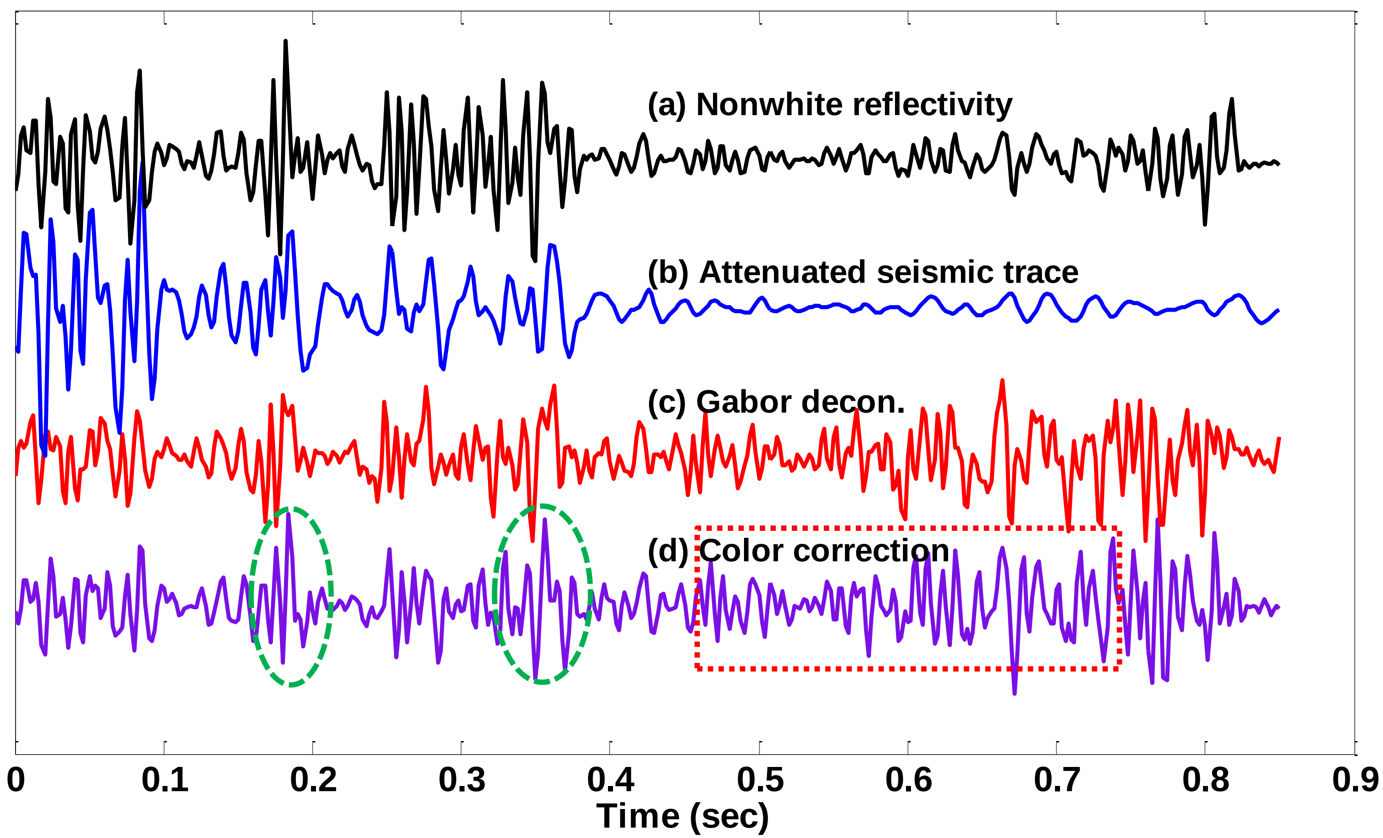
$$\overline{|R'_G(\tau, f)|} = a_0 + a_1 f + a_2 f^2$$

Color feature is temporally stationary

Practical color correction (method 1)



Practical color correction (method 1)



Practical color correction

- Method 2: color feature is time-variant

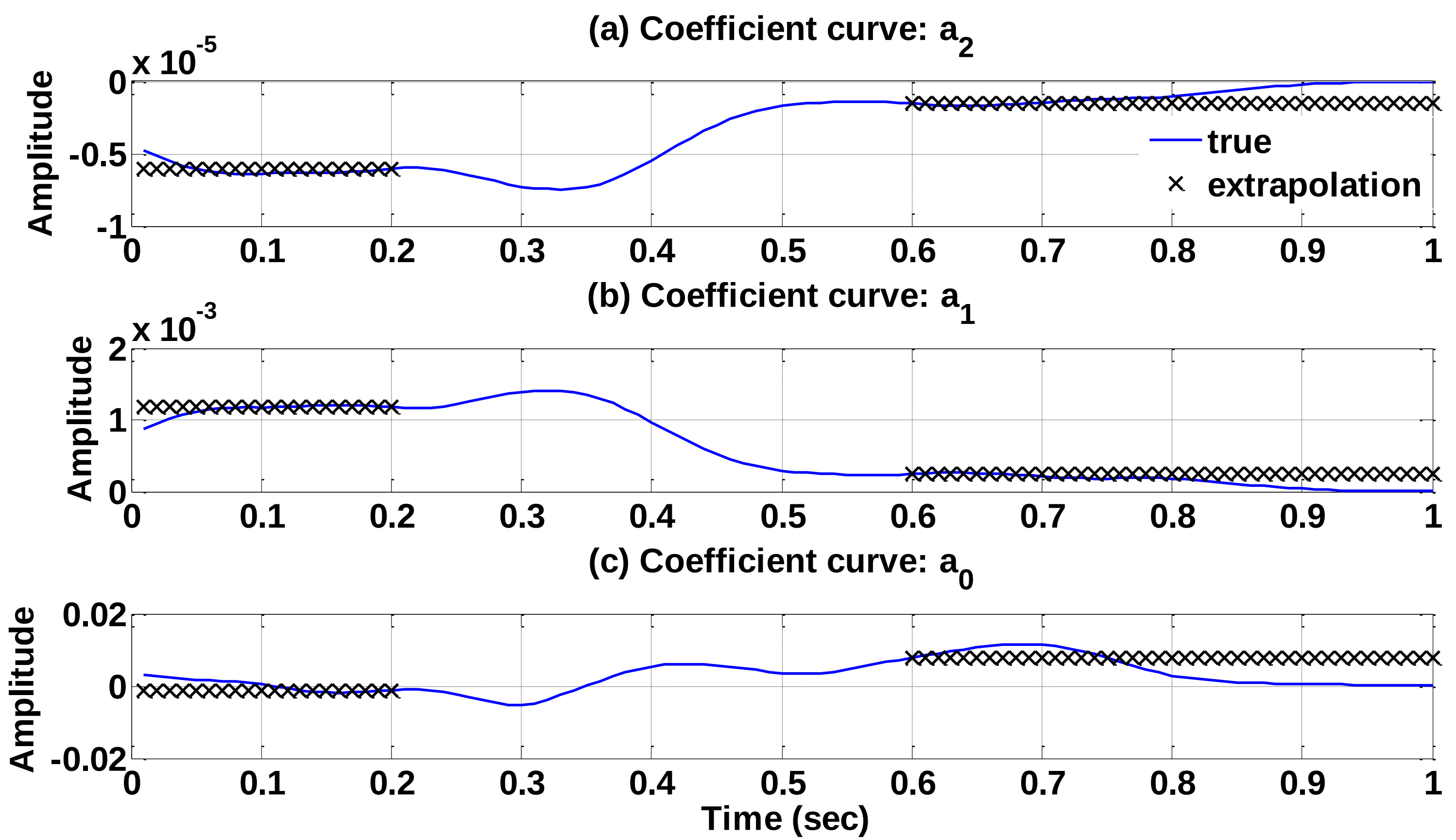
$$\tilde{r}_c(t) \rightarrow \tilde{R}_G(\tau, f)$$

$$|\tilde{R}_G(\tau, f)| \approx a_0'(\tau) + a_1'(\tau) f + a_2'(\tau) f^2 \qquad \tau \in [t_1, t_2]$$

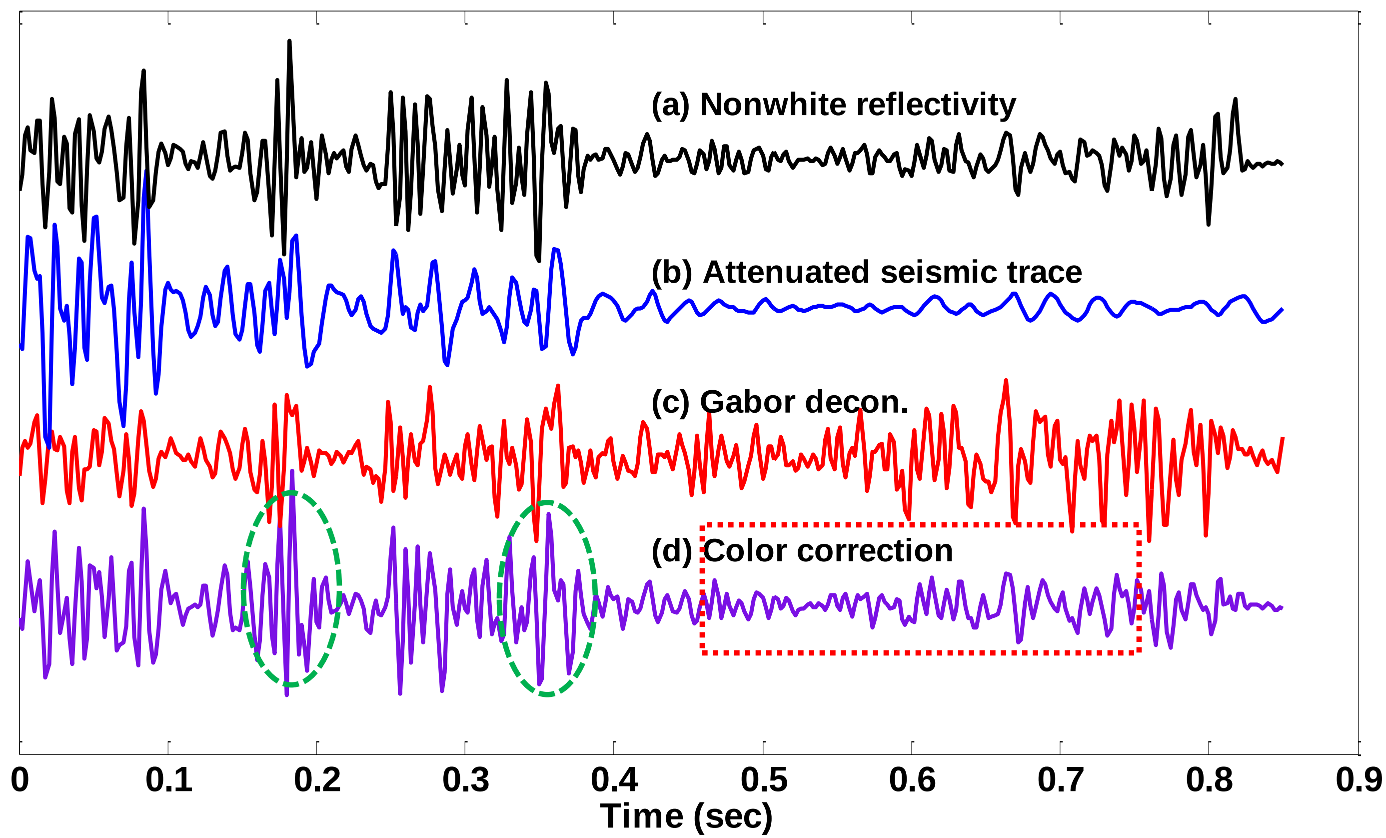
$$\overline{|R_G'(\tau, f)|} = a_0(\tau) + a_1(\tau) f + a_2(\tau) f^2 \qquad \tau \in [0, t_{max}]$$

$$a_i(\tau) = \begin{cases} a_i'(t_1), 0 \le \tau \le t_1 \\ a_i'(\tau), t_1 < \tau < t_2 \quad i = 1,2,3 \\ a_i'(t_2), t_2 \le \tau \le t_{max} \end{cases}$$

Practical color correction (method 2)



Practical color correction (method 2)

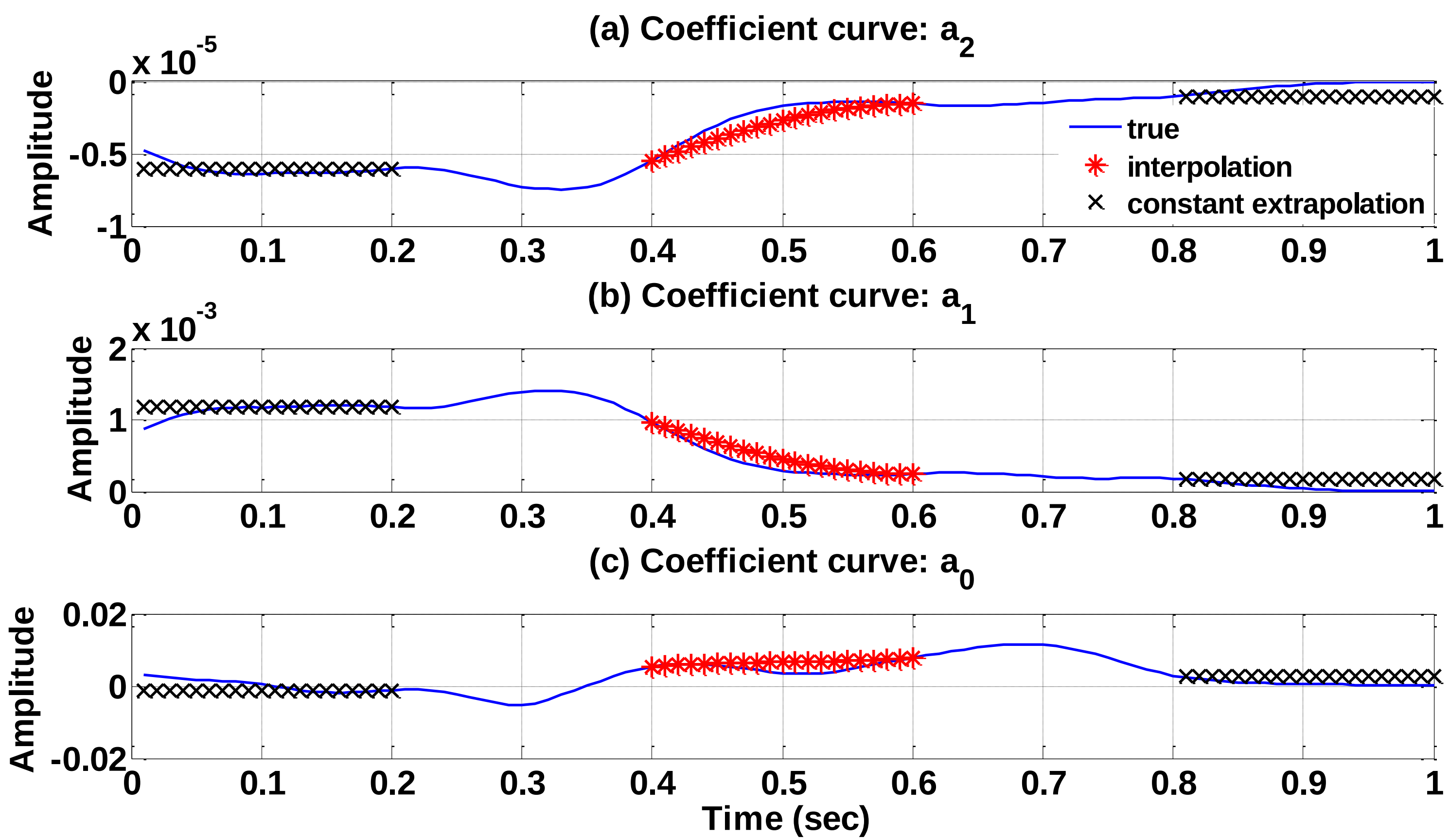


Practical color correction

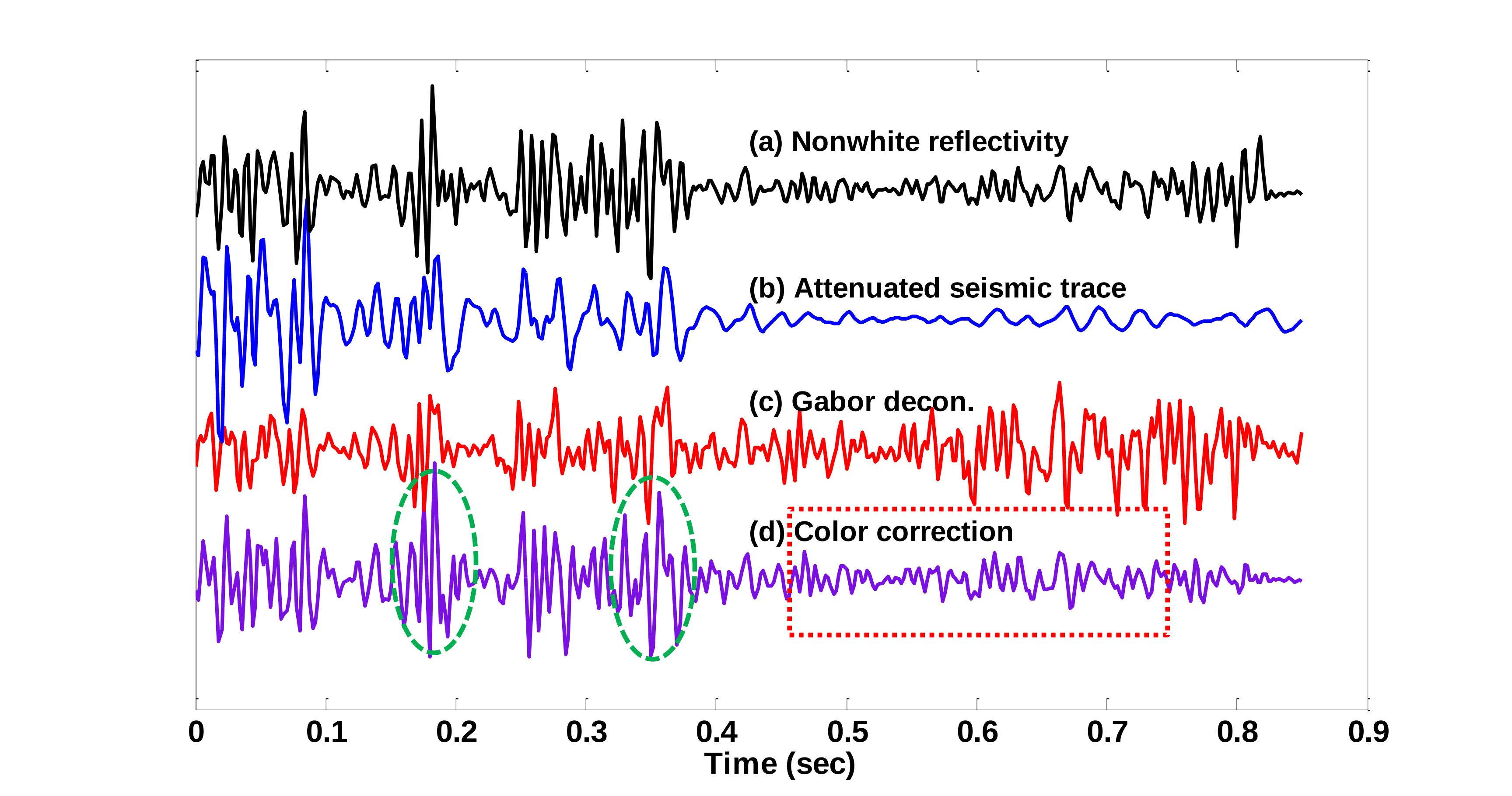
- Method 3: extension of method 2

  $a_i(\tau)$ can be obtained through interpolation when multiple well logs are available.

Practical color correction (method 3)



Practical color correction (method 3)



Nonstationary phase rotation

- Constant phase rotation

$$s_\theta(t) = s(t)\cos\theta + s_{\pi/2}(t)\sin\theta$$

- Nonstationary phase rotation

$$s'_\theta(t) = s(t)\cos\theta(t) + s_{\pi/2}(t)\sin\theta(t) \quad ?$$

Removal of nonstationary phase rotation

- Method 1

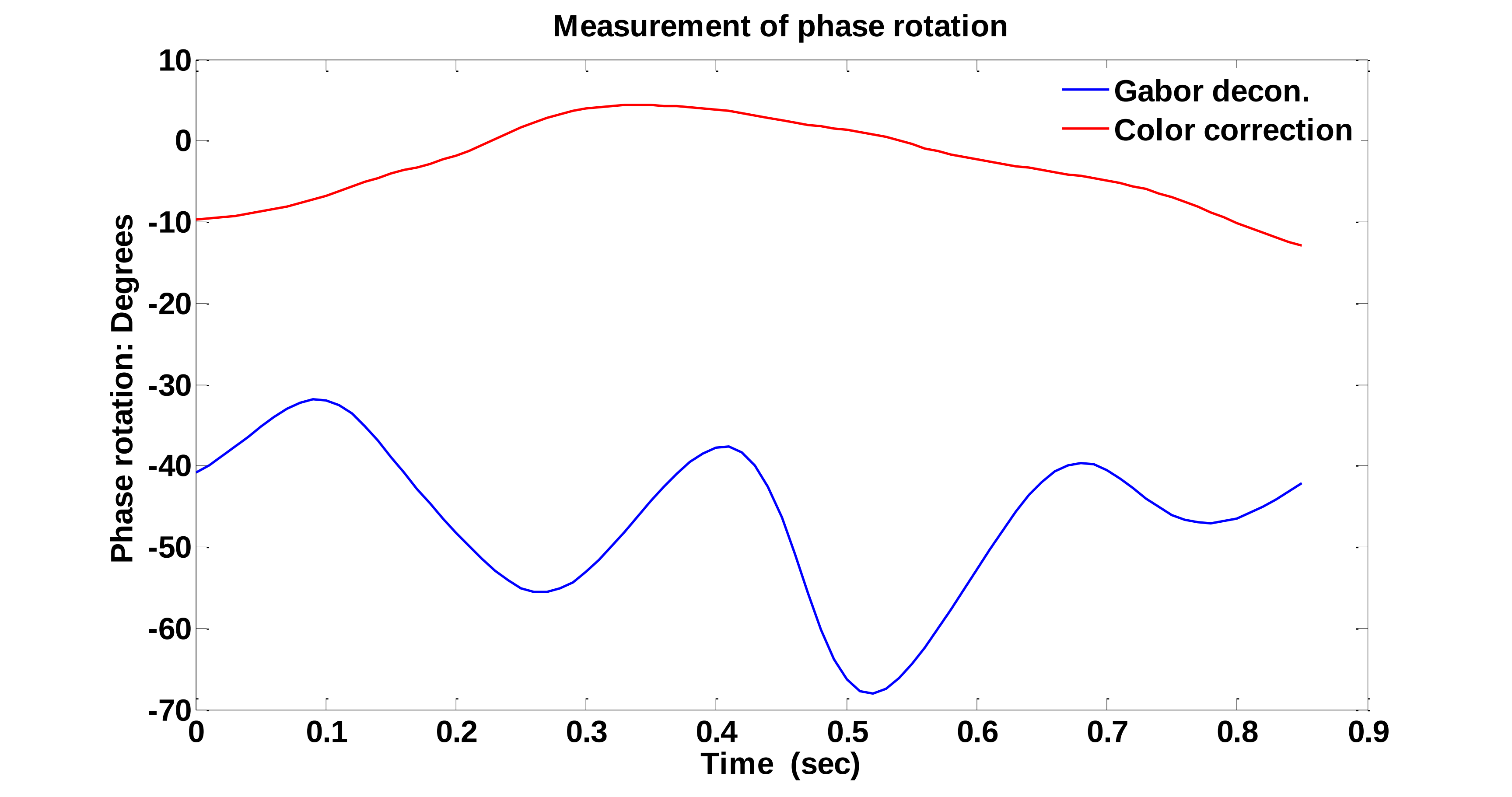
remove the phase rotation in Gabor windows

- Method 2

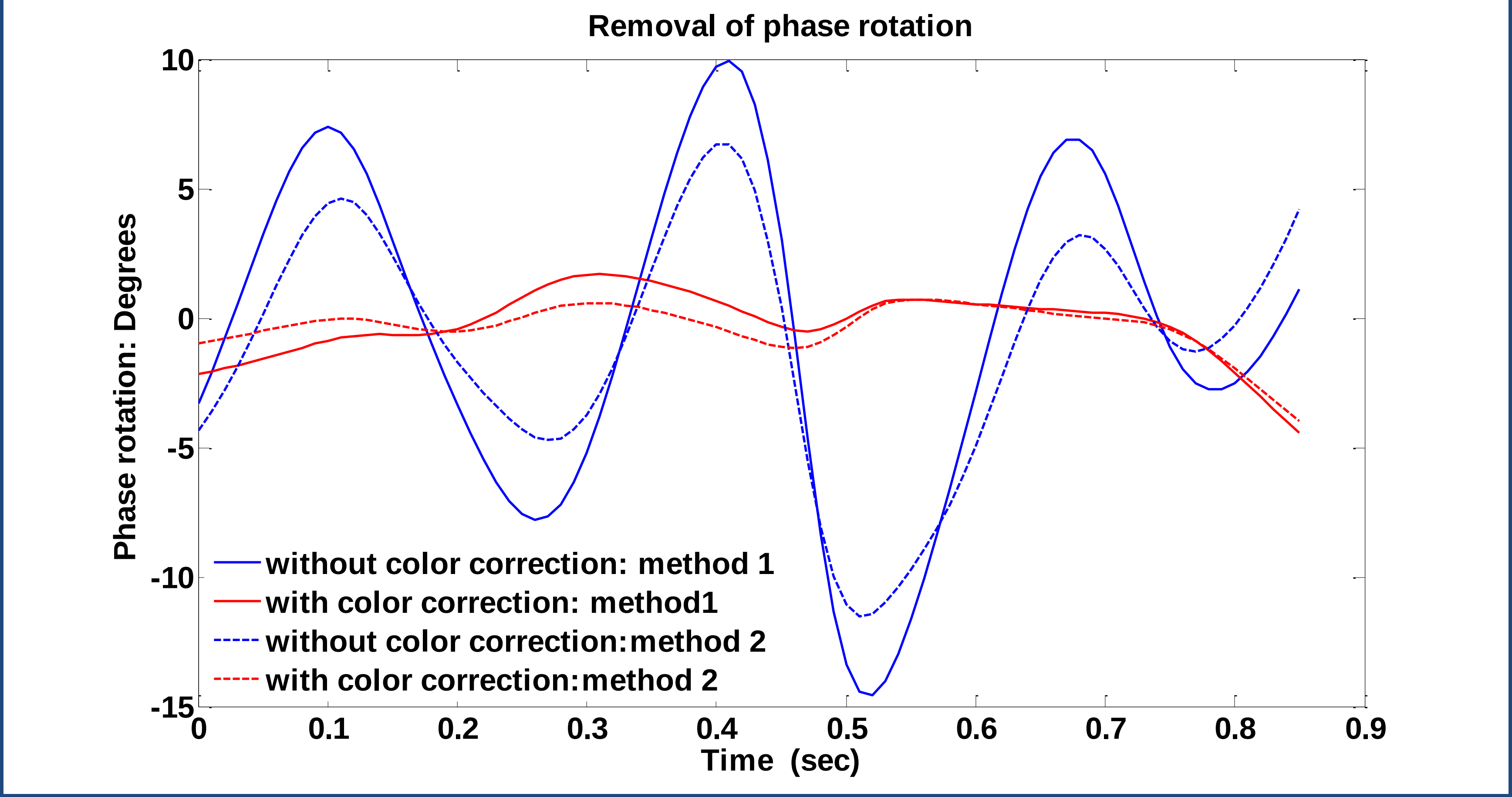
$$g(t) = s'_\theta(t)$$

$$s'(t) = g(t)\cos\theta(t) - g_{\pi/2}(t)\sin\theta(t) \quad ?$$

Measured T.V. phase rotation after G.D.



Measured T.V. phase rotation after phase rotation removal



Conclusions

- Real reflectivity is not white and its color feature is time-variant.
- White assumption causes distortion in conventional Gabor decon.
- Color correction is the most important if frequency band is broad
- Three effective methods for practical color correction were shown
- Practical way to define and remove nonstationary phase rotation was investigated

Acknowledgement

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Thank you!