Microseismic Monitoring:

Insights from Moment Tensor Inversion

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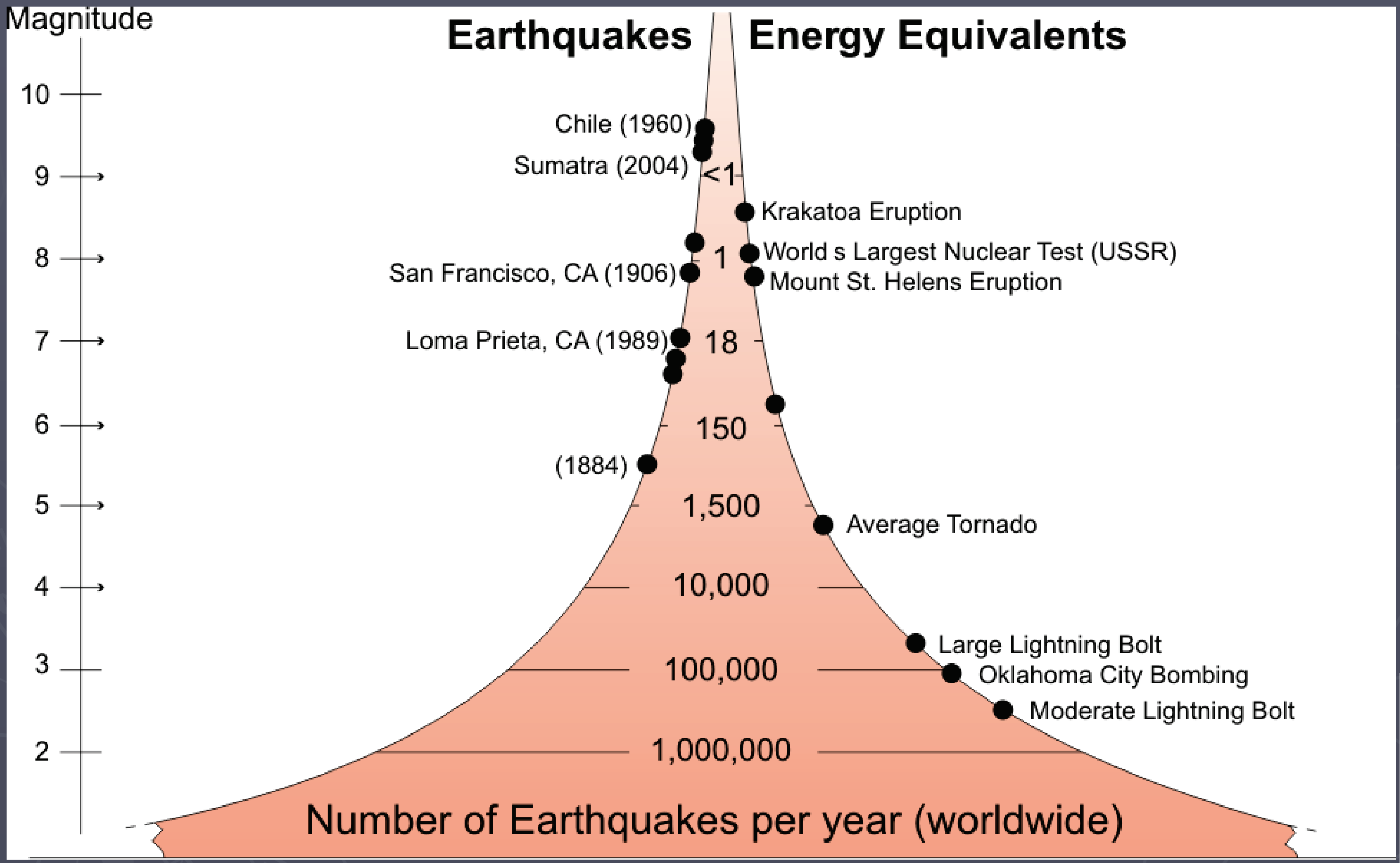
Outline

- Moment magnitude and scalar
- Moment tensor & equivalent body forces
- Moment tensor (MT)
- Seismic sources
- Tensile earthquake and faulting
- Decomposition of moment tensor
- Synthetic modeling results
- Conclusion

Magnitude of Microseismic Events



Typical range for micro-seismic events

Seismic Moment Scalar & Magnitude

- Seismologists report the size of quakes in terms of moment magnitude and seismic moment scalar.
- Seismic moment scalar: Measure of an earthquake rupture size related to the leverage of forces across the area where the fault slips.

$$M_0 = \mu D A$$

- *μ:* shear modulus
- *D:* average slip
- *A:* fault area.

Magnitude & Moment Scalar

General form:

$$M = \log\left(A/T^n\right) + Q(h,\mathrm{D})$$

where

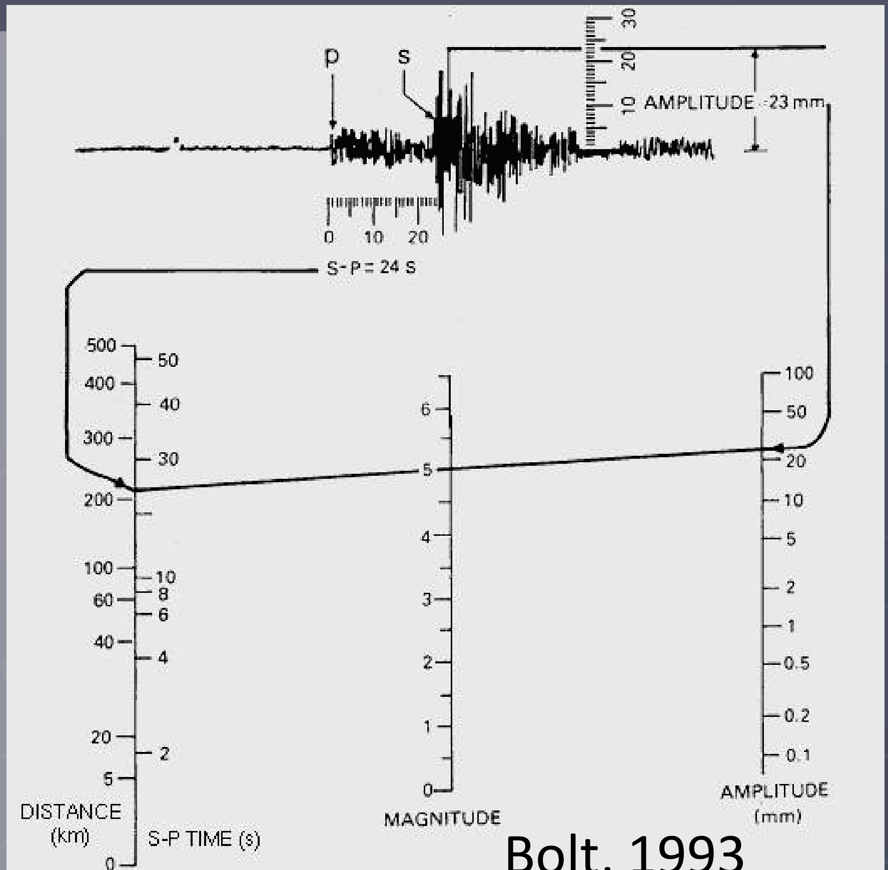
A: signal amplitude

T : dominant period

h: focal depth

D: distance

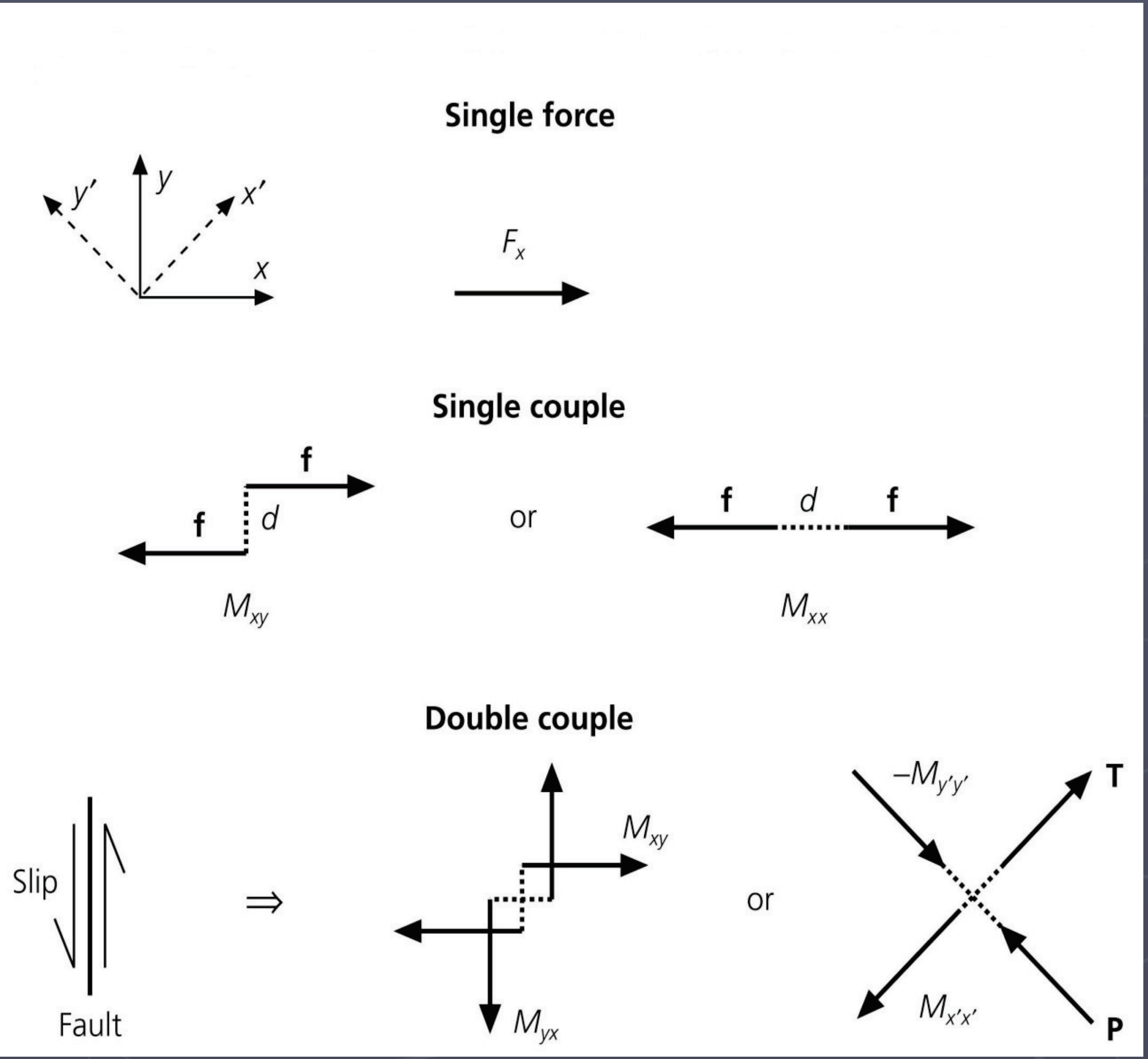
$$Mw = \frac{2}{3}\log_{10}\left(M_0\right) - 10.7$$



Bolt, 1993

Seismic Moment – Equivalent Body Forces

- Equivalent Body Forces
- General description capable of representing various seismic sources

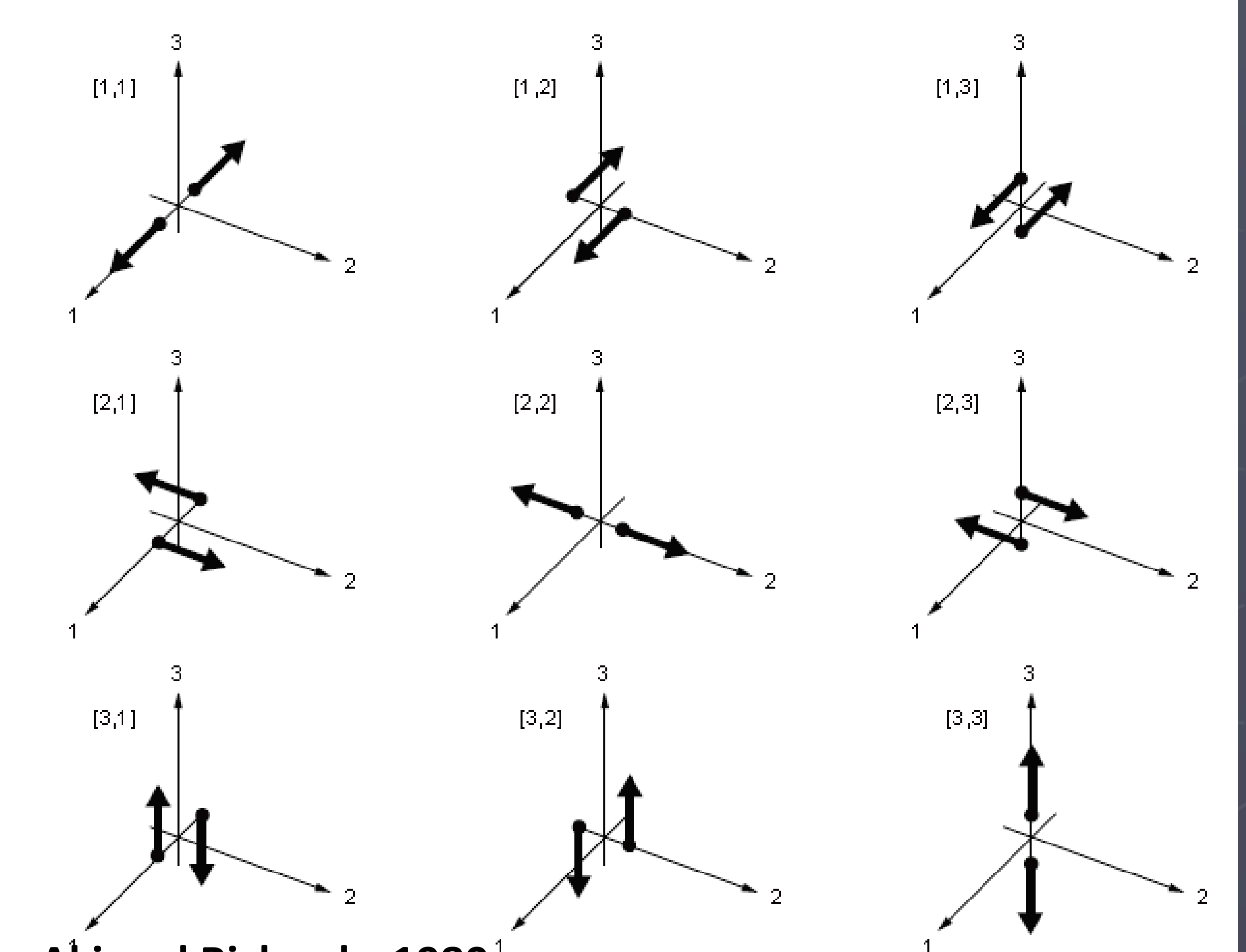


Seismic Moment Tensor

- **Seismic moment tensor**: Mathematical representation of seismic source.

The nine generalized couples of the seismic moment tensor.

$$M = M_0 \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$



Aki and Richards, 1980

Why Seismic Moment Tensor?

- Mathematical description of the source mechanism
- Computational properties of MT:
  - Linear mathematical expressions for displacement field.
  - Easier forward modeling and inversion analyses

$$d_n(x,t) = M_{kj}[G_{nk,j} * s(\bar{t})]$$

s(t) **is the source time function**

d_n displacement

Moment Tensor Inversion

$$d = G\overline{m}$$

$$\begin{bmatrix} d1 \\ d2 \\ \vdots \\ dn \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{n1} & G_{n2} & G_{n3} & G_{n4} & G_{n5} & G_{n6} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_6 \end{bmatrix}$$

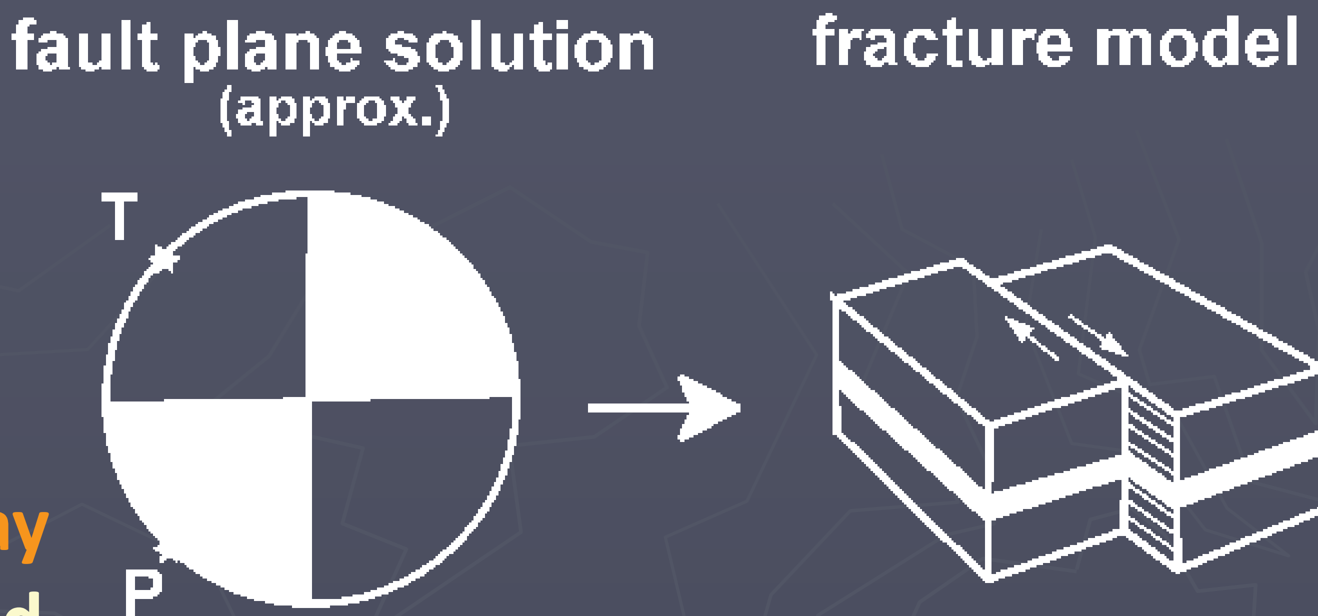
$$m = \left(G^T G\right)^{-1} G^T u$$

(Jost & Herrmann, 1989)

General Seismic Point Sources

- Pure double-couple (pure shear)
- ISO + DC + CLVD (Tensile Earthquakes)
- ISO + VSS + VDS + 45°DS (special case)
- Isotropic (ISO) + 3 double-couples (DC)
- ISO + 3 vector-dipoles (VD)
- ISO + 3 compensated linear vector dipoles (CLVDs)
- ISO + major couple + minor couple

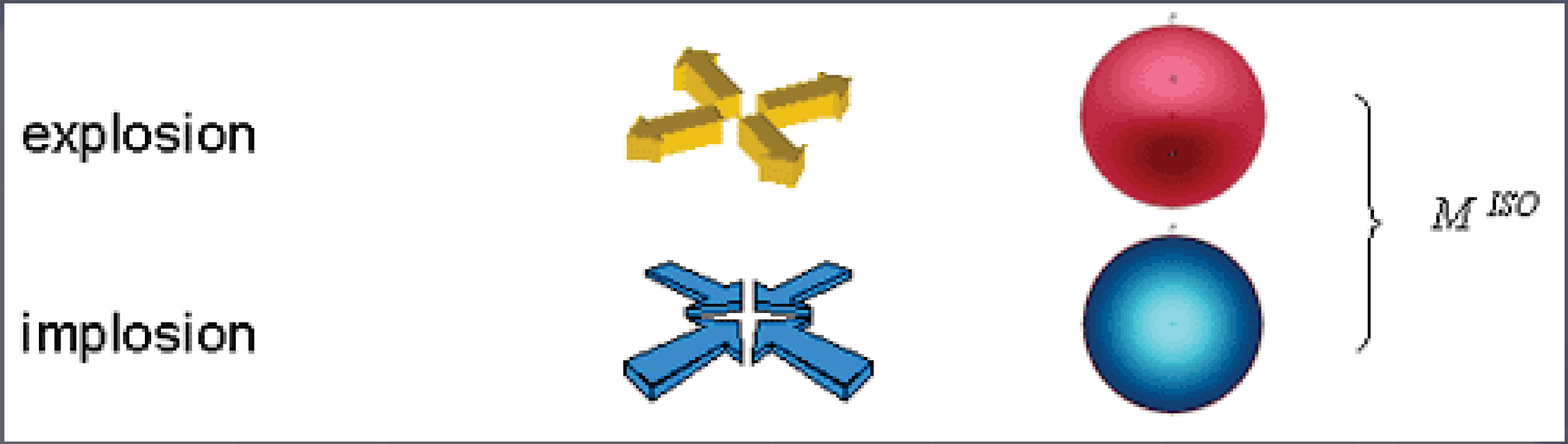
Seismic Source: Double-Couple(DC)



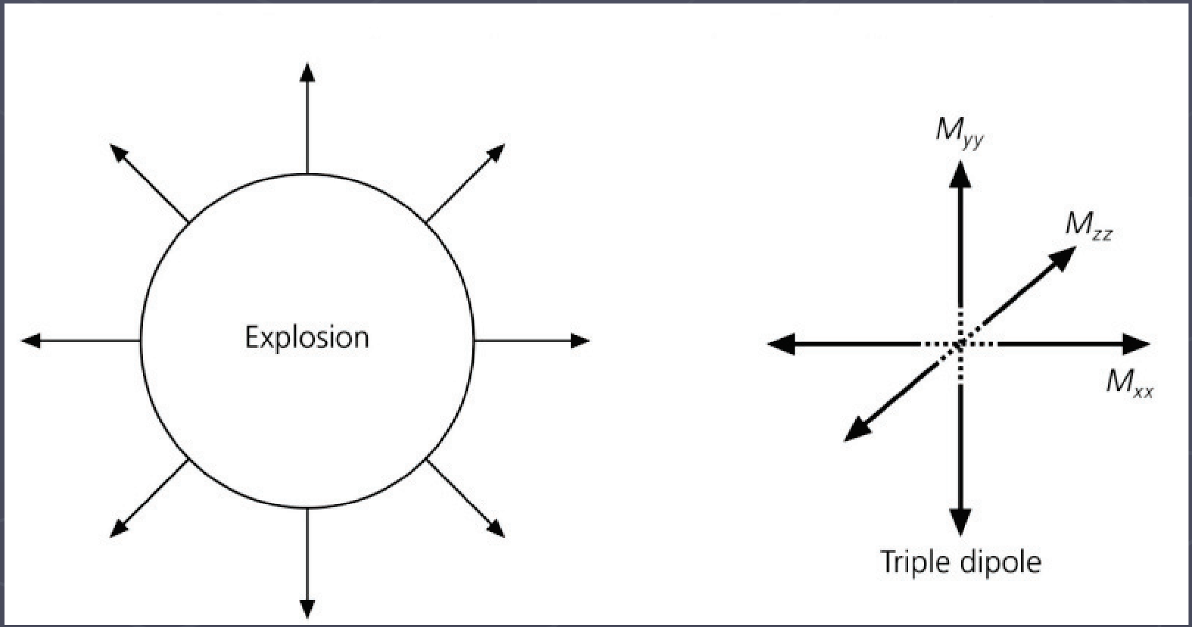
- Most earthquakes may be approximated by a double-couple
- eigenvectors of M yield principal stress axes (P, T)

$$\begin{bmatrix} 0 & M_0 & 0 \\ M_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Seismic Source : Non-DC (Isotropic)



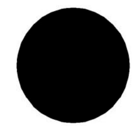
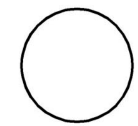
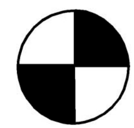
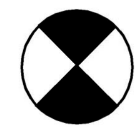
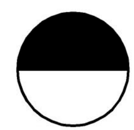
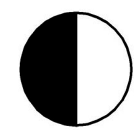
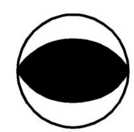
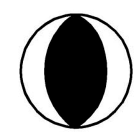
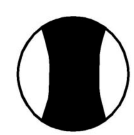
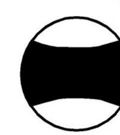
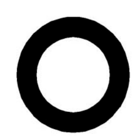
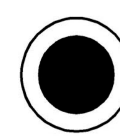
$$M_{iso} = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{bmatrix}$$



Seismic Source: Linear Vector Dipole

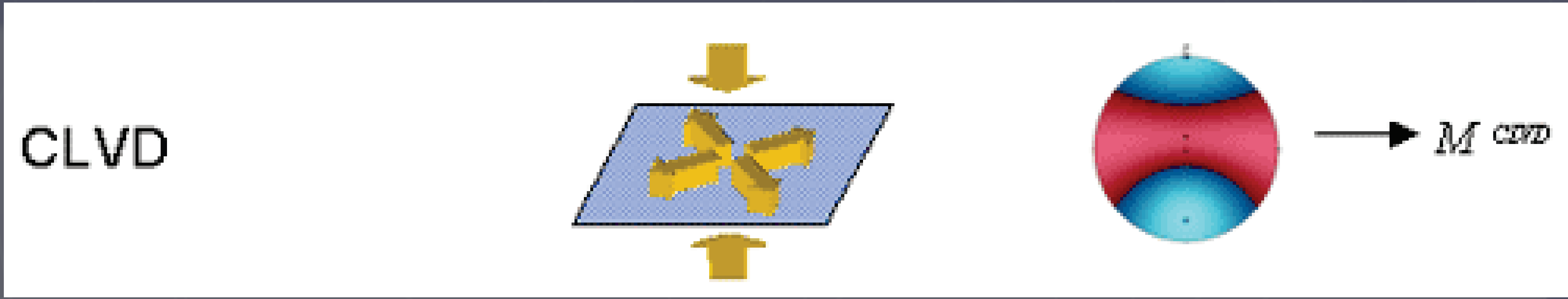
- Sudden change in **shear modulus** in presence of axial strain.
- **CLVD**: a dipole corrected for the effect of volume change

Figure 4.4-6: Selected moment tensors and their associated focal mechanisms.

Moment tensor	Beachball	Moment tensor	Beachball
$\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$		$-\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	
$-\frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&-1\\0&0&0\\-1&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&0&-1\\0&-1&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}-1&0&0\\0&0&0\\0&0&1\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&-1&0\\0&0&1\end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&-2&0\\0&0&1\end{pmatrix}$		$\frac{1}{\sqrt{6}}\begin{pmatrix}-2&0&0\\0&1&0\\0&0&1\end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$		$-\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$	

Seismic Sources : Non-DC (CLVD)

- **What is CLVD?**
  - **Represents crack opening under tension.**

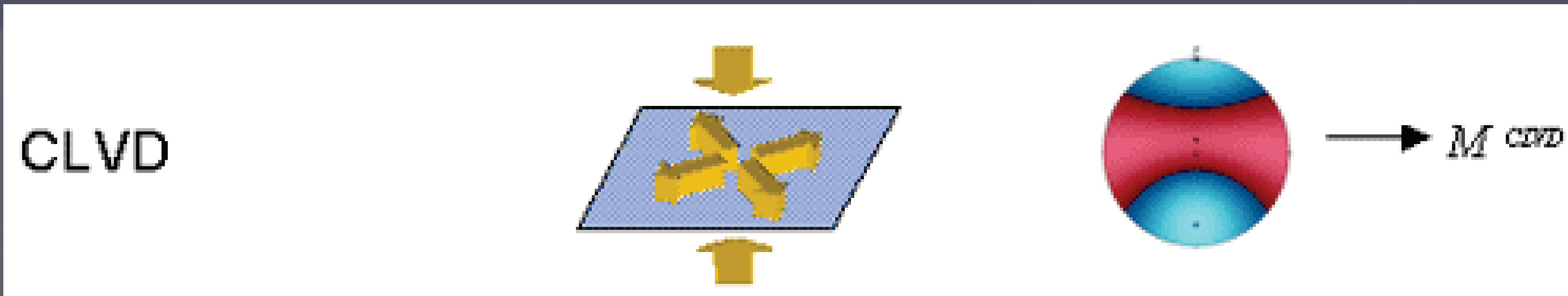


$$M_{clvd} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda + 2\mu \end{bmatrix}$$

$$M_{clvd} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda + 2\mu \end{bmatrix} = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{bmatrix} + \begin{bmatrix} -2/3\mu & 0 & 0 \\ 0 & -2/3\mu & 0 \\ 0 & 0 & 4/3\mu \end{bmatrix}$$

Seismic Sources : Non-Double-Couple

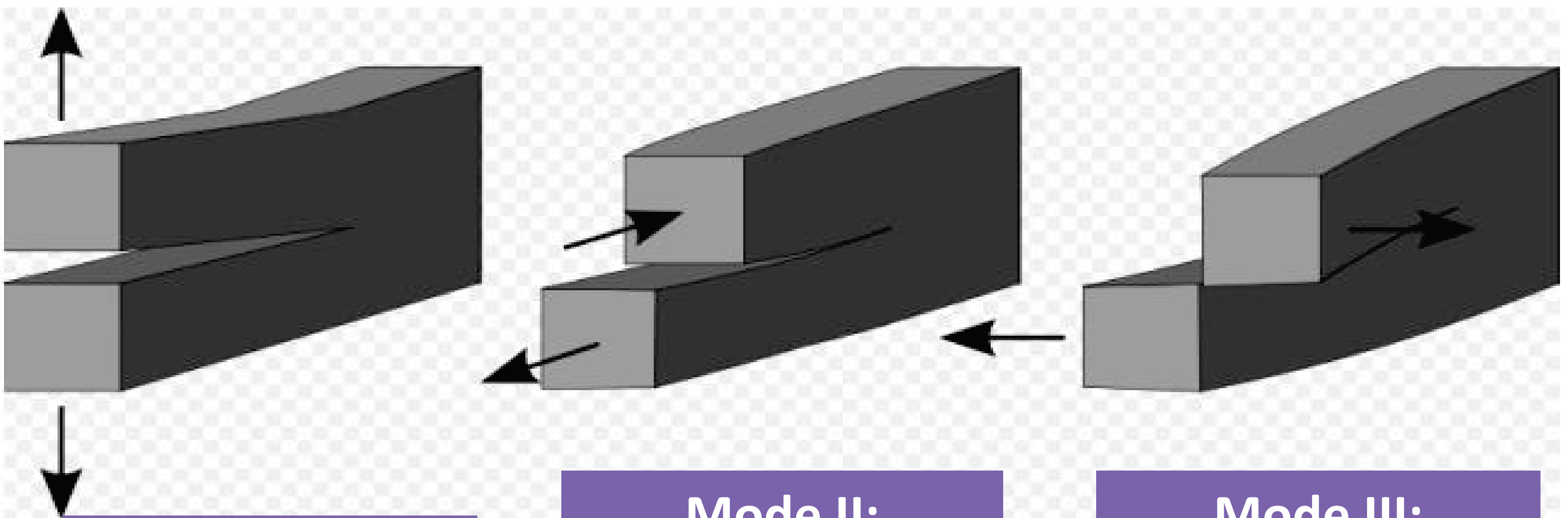
- **What is CLVD?**
  - **Represents near-simultaneous earthquakes on close faults of different geometries!**



$$\begin{bmatrix} M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -M_0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2M_0 & 0 \\ 0 & 0 & 2M_0 \end{bmatrix} = \begin{bmatrix} M_0 & 0 & 0 \\ 0 & -2M_0 & 0 \\ 0 & 0 & M_0 \end{bmatrix}$$

Types of Fracture (Propagation Modes)

- Definition for ideal flat perfectly sharp crack of zero thickness (Gibowicz & Kijko, 1994)



Mode I: Tensile Opening

Crack wall displacement normal to crack

Mode II: In-plane Shear

Displacement in the plane of crack wall and normal to crack edge

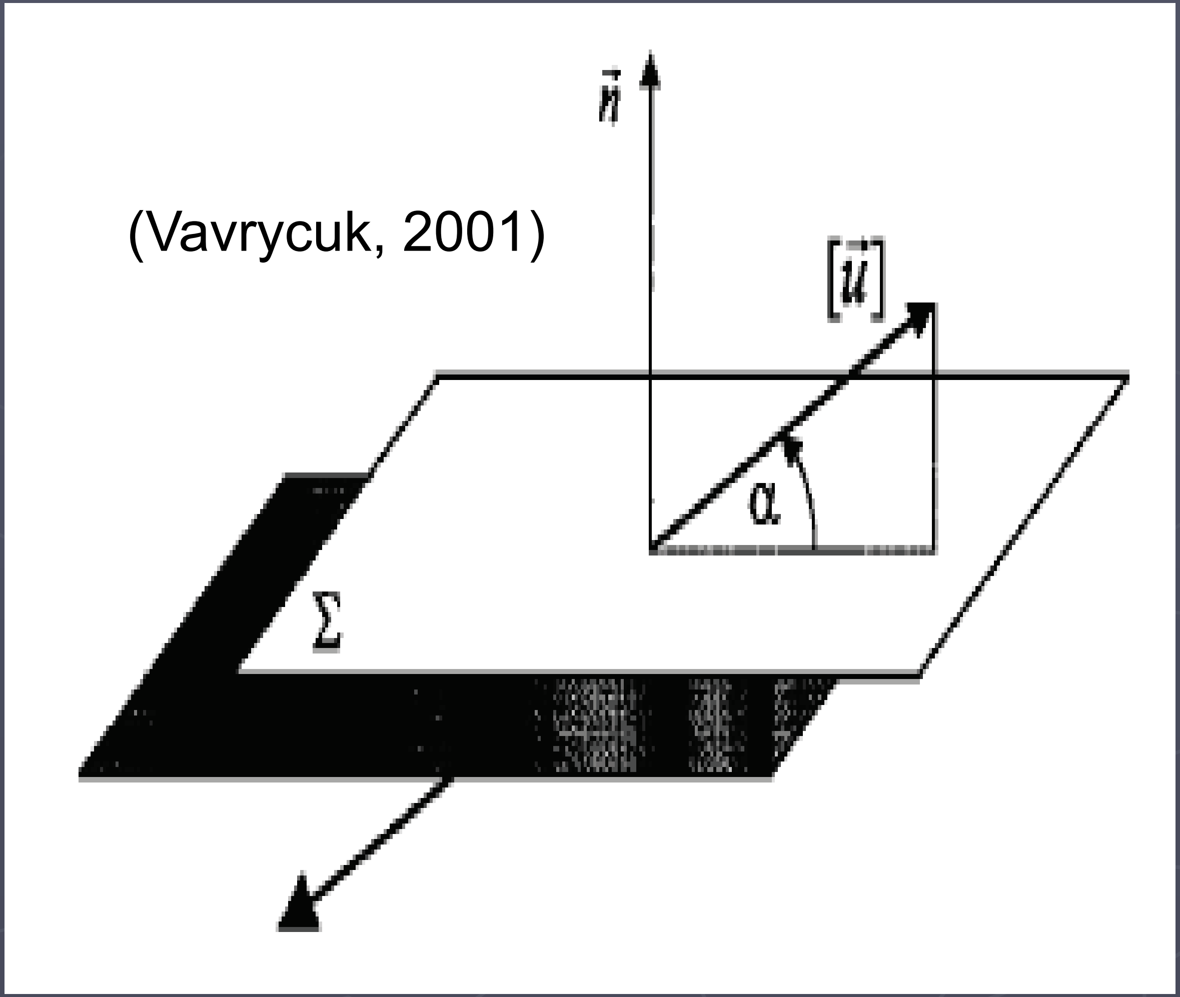
Mode III: Anti-plane Shear

Displacement in the plane of crack wall and parallel to crack edge

Tensile Earthquakes

- Slip vector deviates from the fault and causes opening or closing

- Inferred from moment tensor inversion, ISO + CLVD show the deviation of source mechanism from pure DC or shear.



Moment Tensor of Tensile Faulting

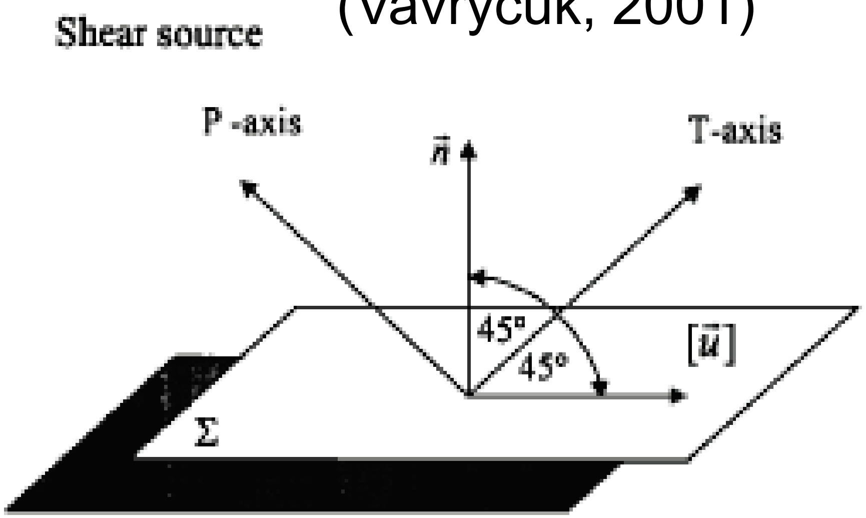
$$\mathbf{M} = u \begin{bmatrix} \lambda \sin\alpha & 0 & \mu\cos\alpha \\ 0 & \lambda\sin\alpha & 0 \\ \mu\cos\alpha & 0 & (\lambda + 2\mu)\sin\alpha \end{bmatrix}$$

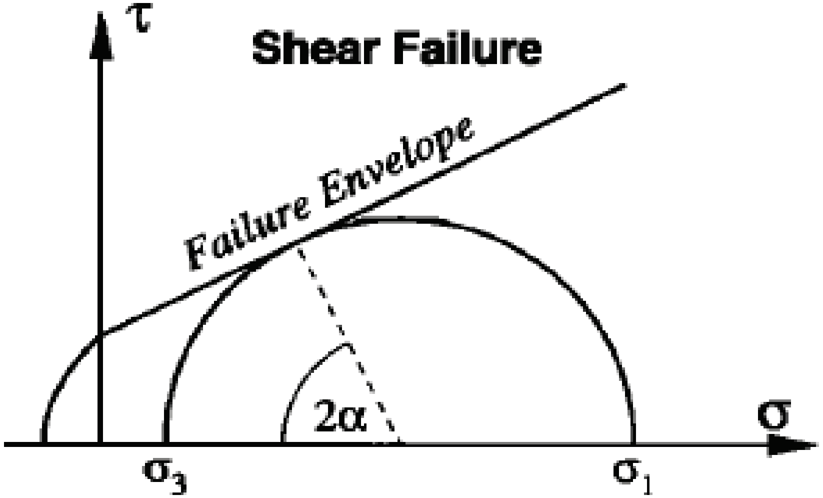
$$\mathbf{M} = \mathbf{M}^{\mathrm{ISO}} + \mathbf{M}^{\mathrm{CLVD}} + \mathbf{M}^{\mathrm{DC}}$$

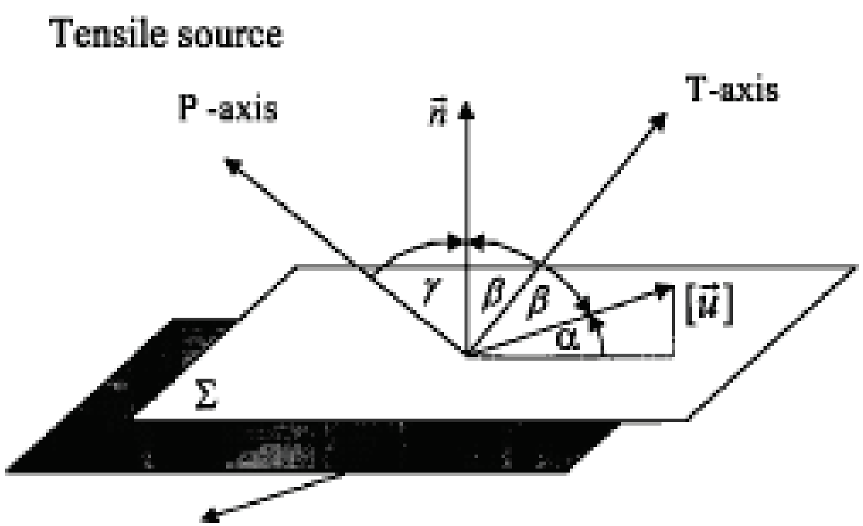
$$M = u \begin{bmatrix} \lambda\sin\alpha - \mu(1 - \sin\alpha) & 0 & 0 \\ 0 & \lambda\sin\alpha & 0 \\ 0 & 0 & \lambda\sin\alpha + \mu(1 + \sin\alpha) \end{bmatrix}$$

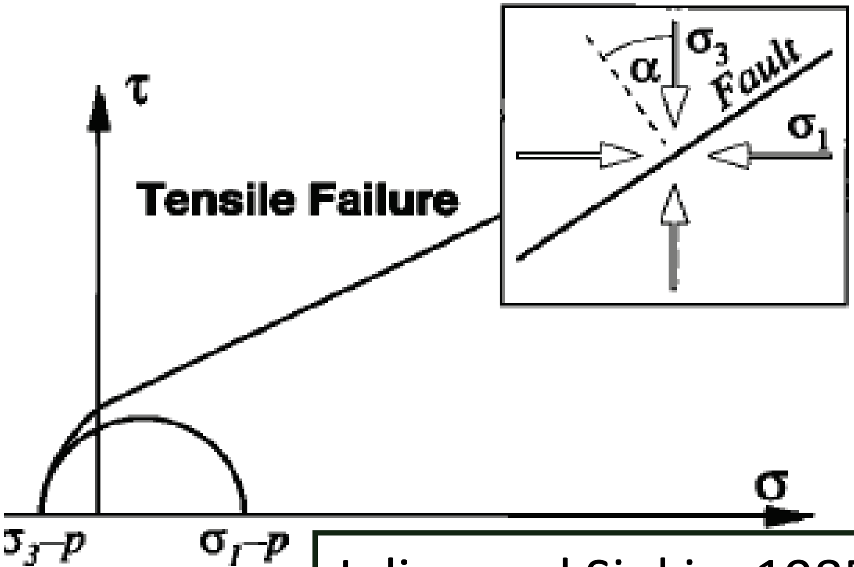
Shear & Tensile Failure

(Vavrycuk, 2001)









Julian and Sipkin, 1985

Decomposition of Moment Tensor

- M_{min} & M_{max} : Absolute values of the eigenvalues of deviatoric moment ***M****
- **ε**: measure of the size of CLVD relative to DC component.
- For a pure CLVD source, ε = ±0.5 and for a pure DC source, ε = 0

$$\boldsymbol{M} = \boldsymbol{M}^{ISO} + \boldsymbol{M}^{CLVD} + \boldsymbol{M}^{DC}$$

$$M^{ISO} = \frac{1}{3} tr(M) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$M^{CLVD} = |\varepsilon| M^*_{|Max|} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

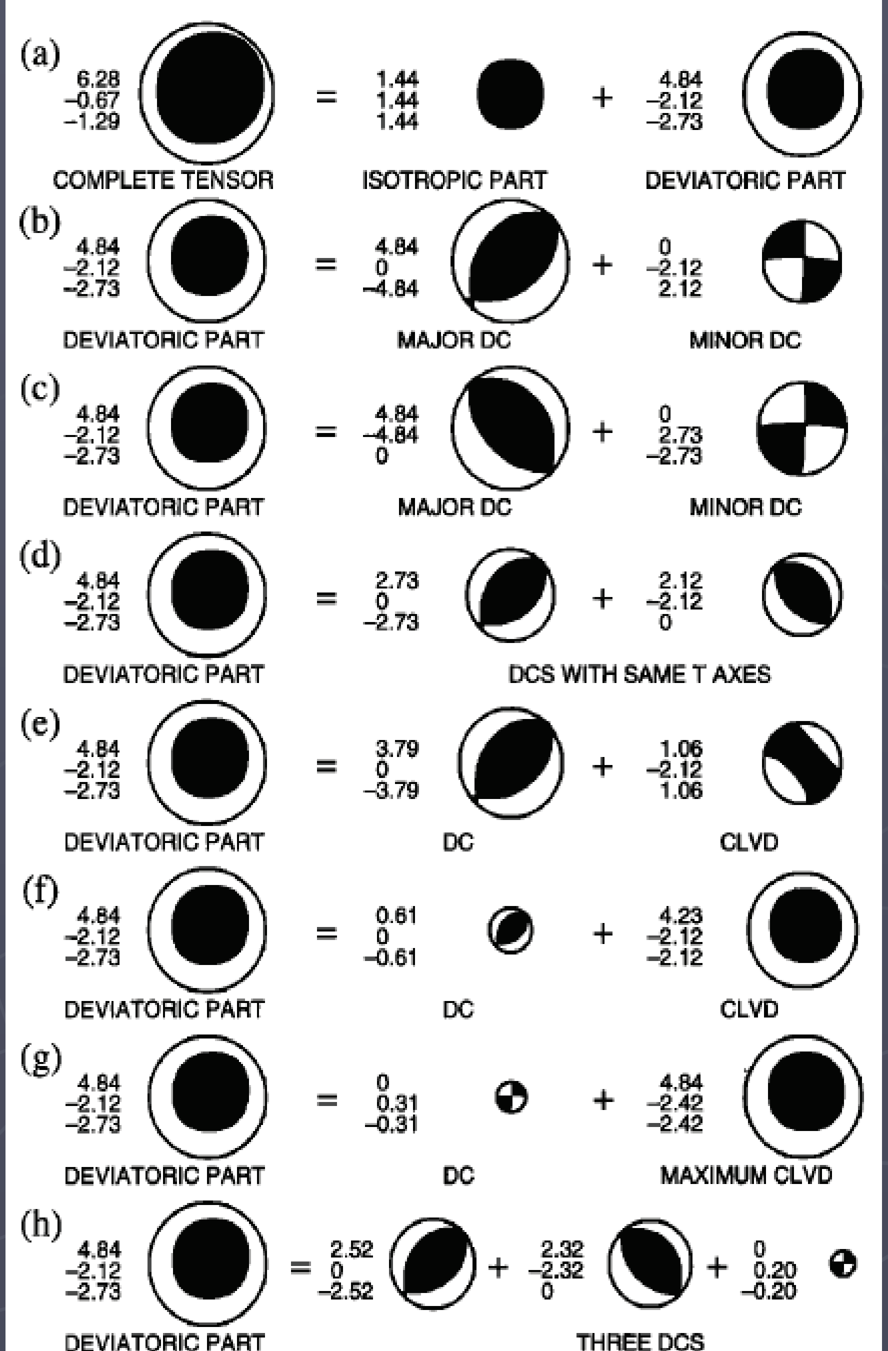
$$M^{DC} = (1 - 2|\varepsilon|) M^*_{|\max|} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\varepsilon = -\frac{M^*_{|\min|}}{\left| M^*_{|Max|} \right|}$$

(Vavrycuk, 2001)

MT Decomposition

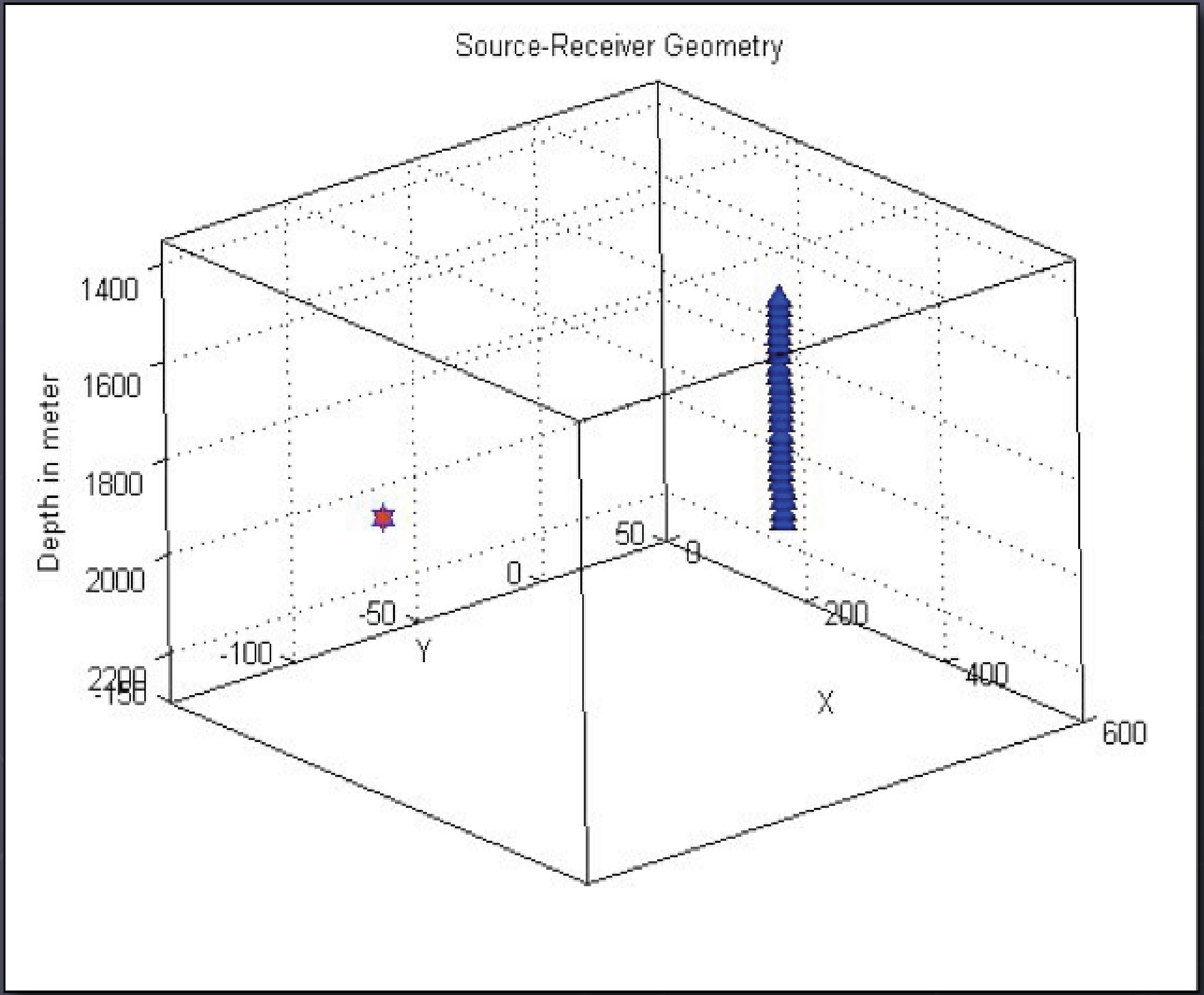
- Decomposition is not unique!!
- Various decomposition may lead to different interpretation.



Julian and Sipkin, 1985

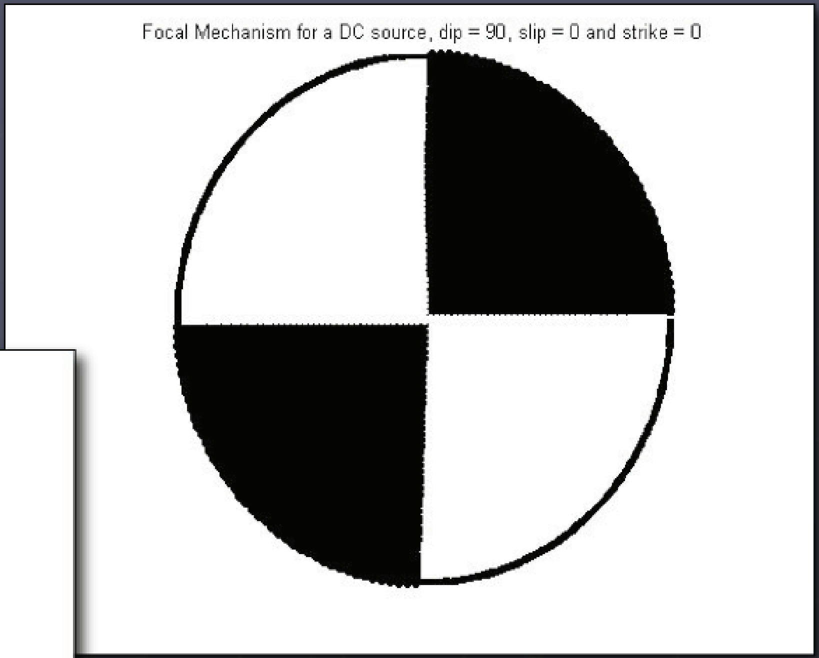
Synthetic Modeling

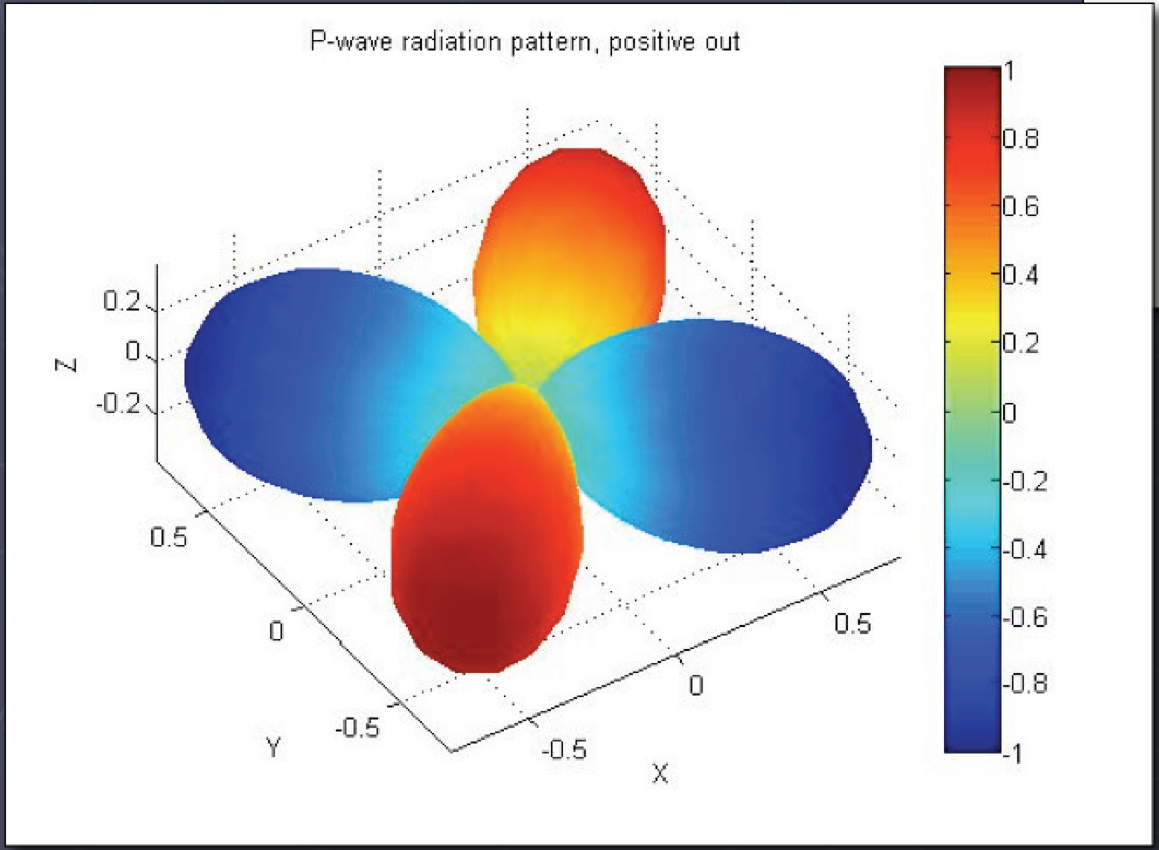
- Double-couple source
- Source with 20% isotropic, 50% double-couple and 30% CLVD components



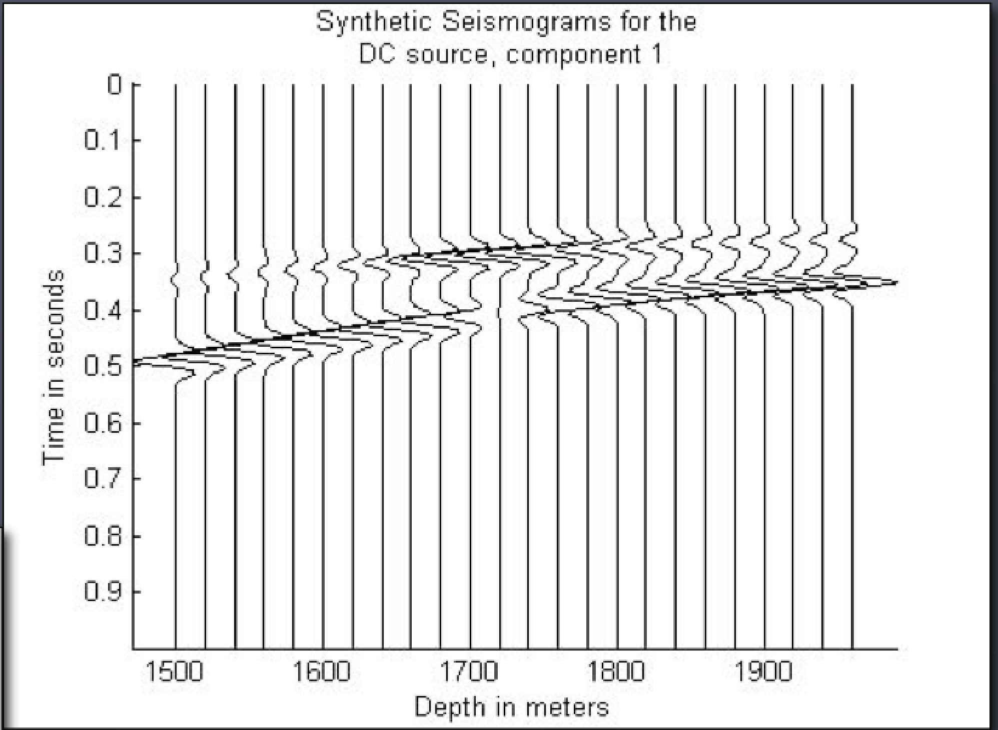
Focal Mechanism & Radiation Pattern

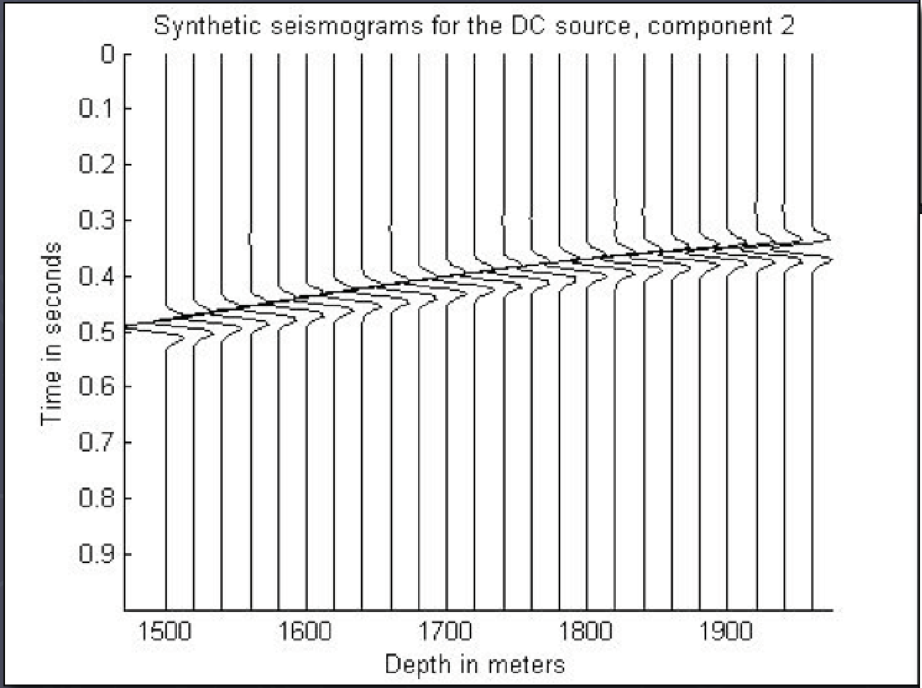
$$M_{dc} = 1e+21 \times \begin{bmatrix} 0 & 3.5481 & 0 \\ 3.5481 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

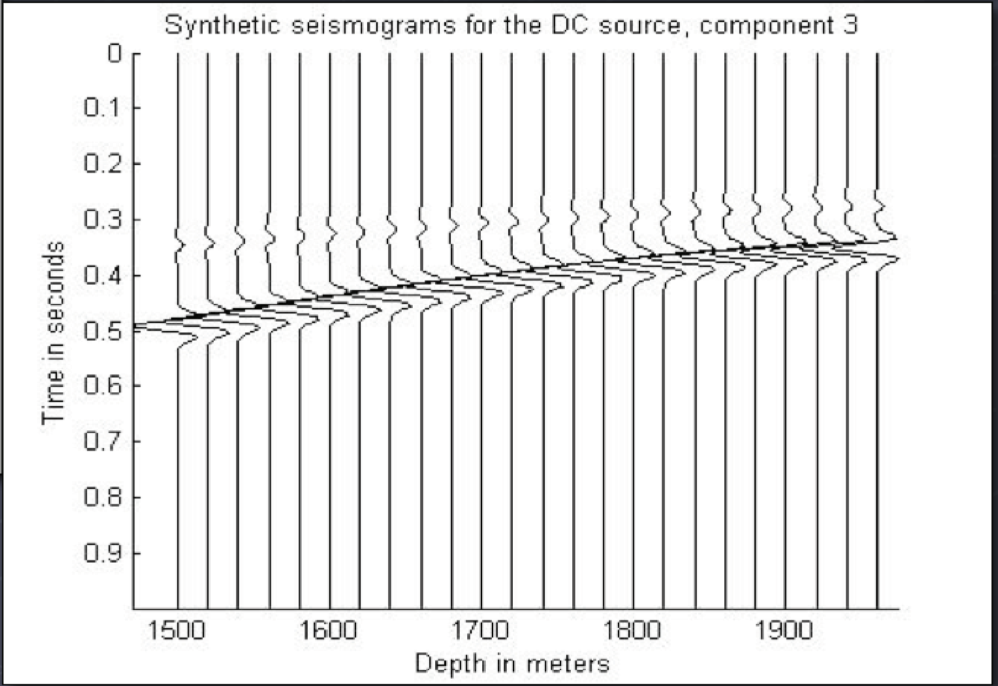




Forward Modeling Double-couple source

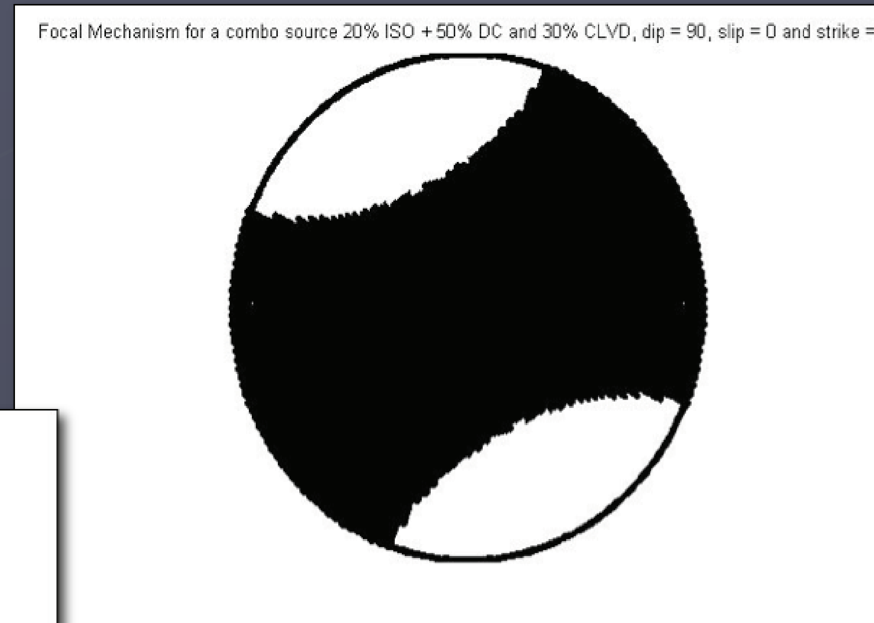


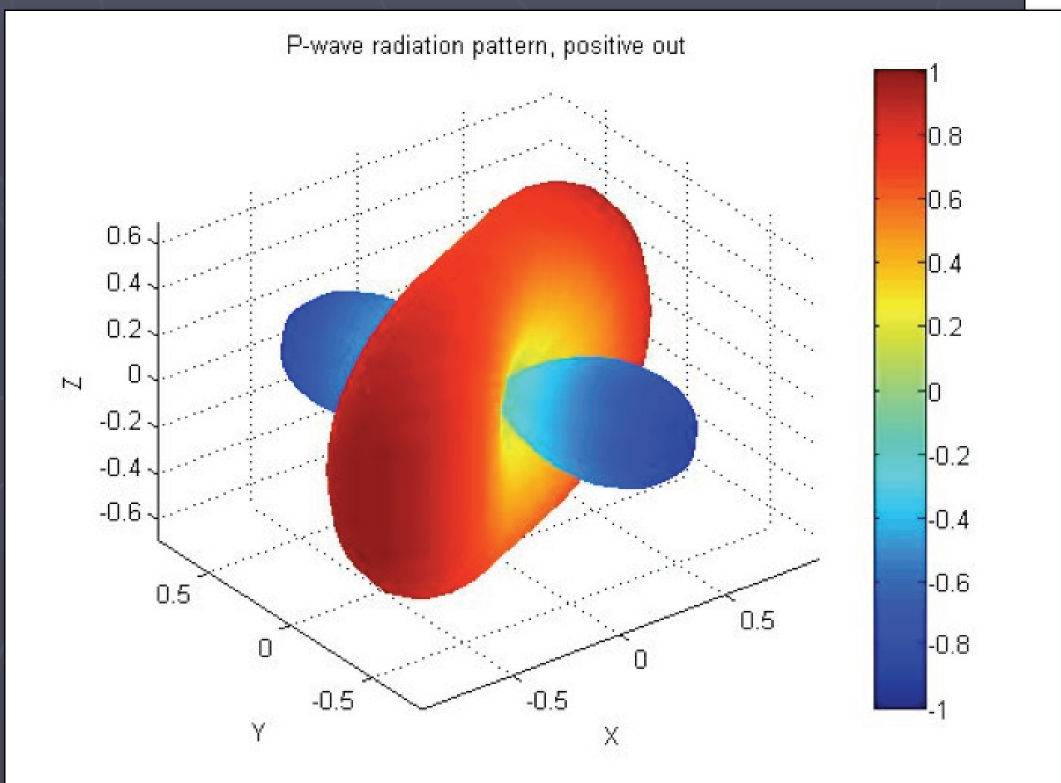




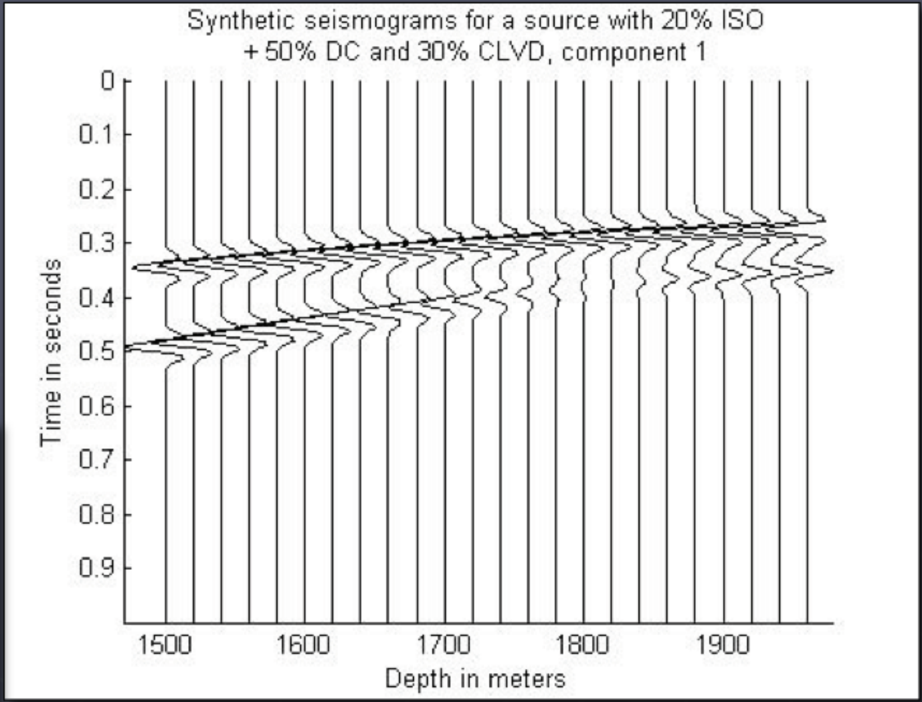
Focal Mechanism & Radiation Pattern

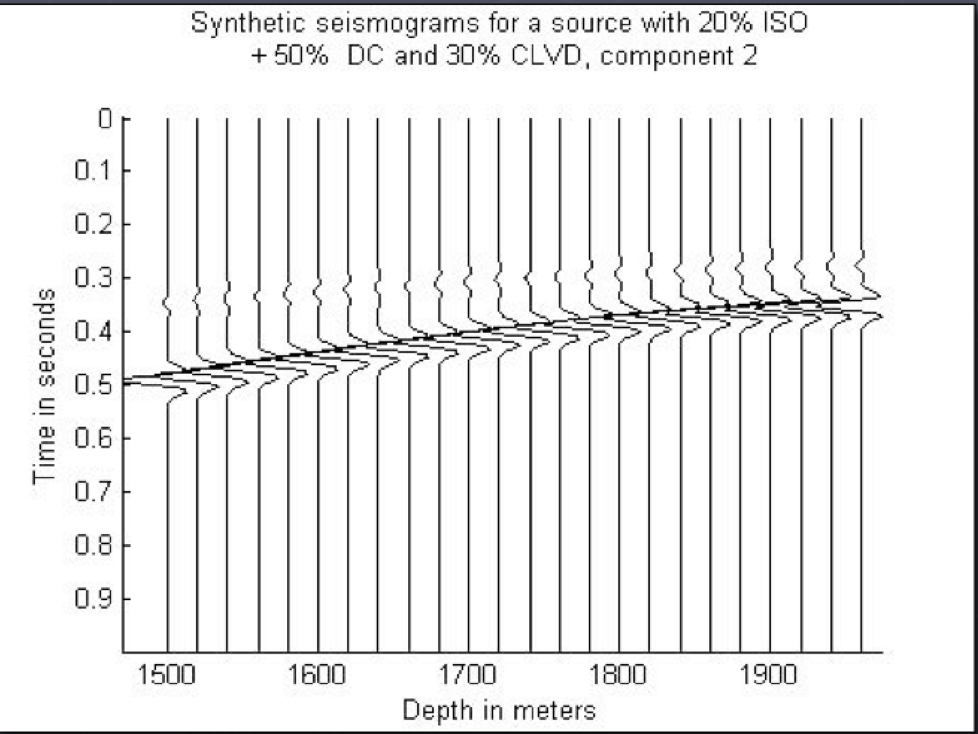
$$M_{combo} = 1e+21\times\begin{bmatrix}1.7741 & 1.7741 & 0\\ 1.7741 & -1.4193 & 0\\ 0 & 0 & 1.7741\end{bmatrix}$$

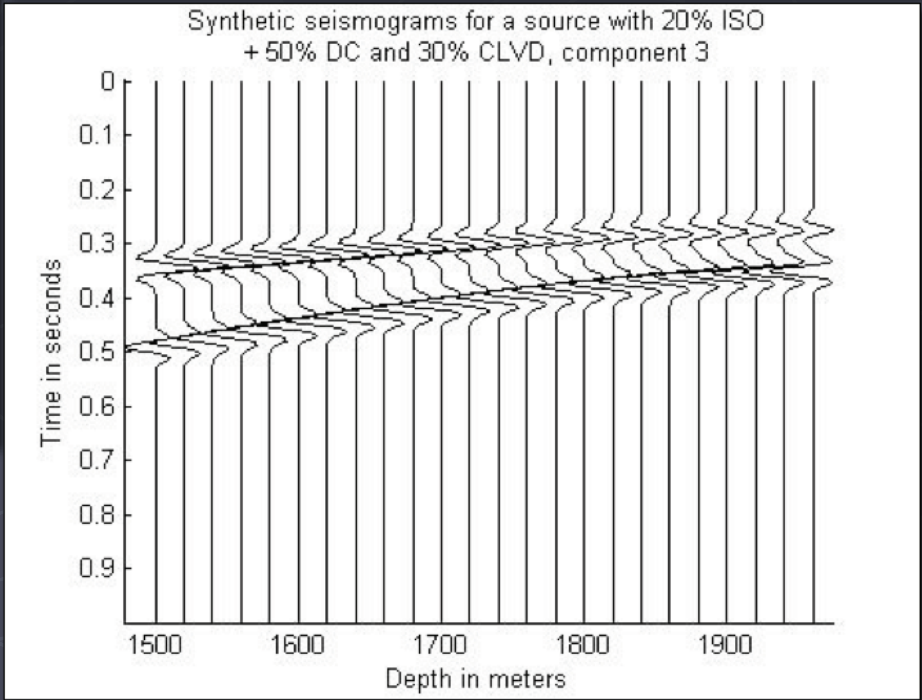




Forward Modeling Combo Source

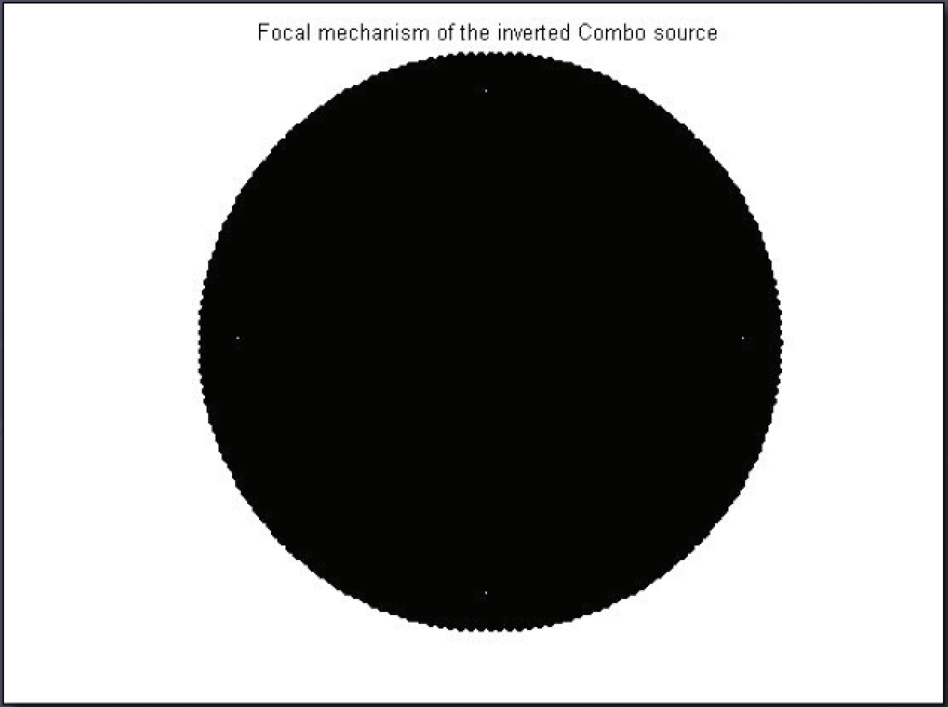






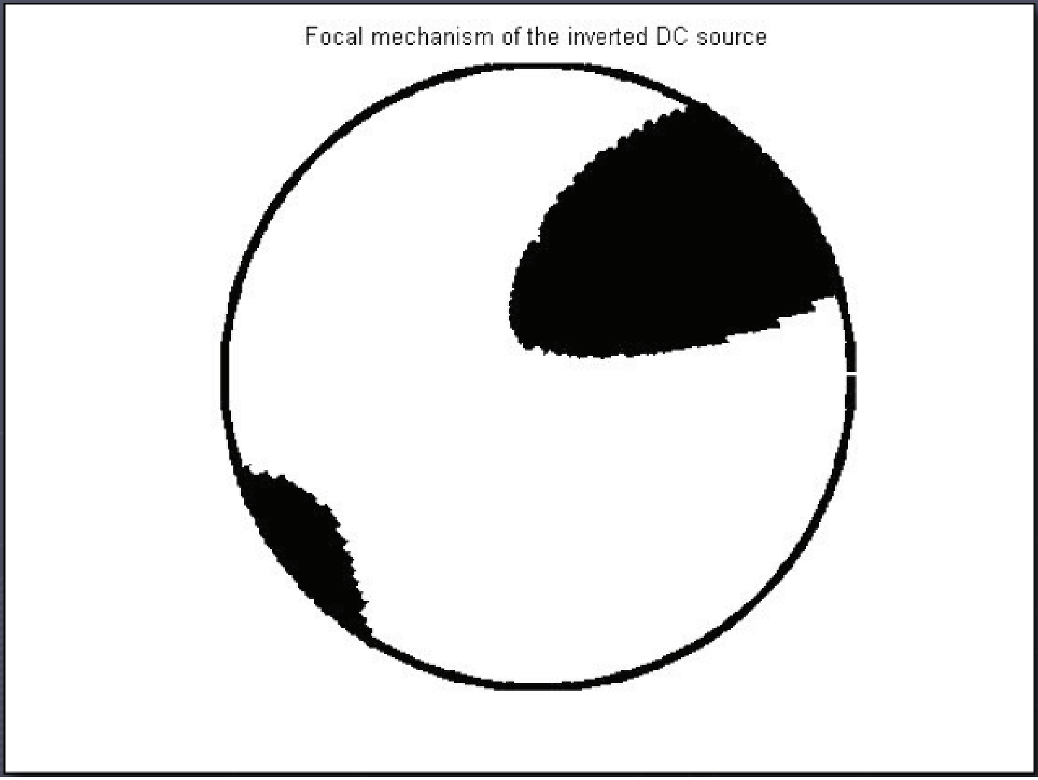
Inversion Results – Combo Source
Single Observation Well

Components	Combo FWD Modeling	Combo Inverted
ISO %	20	3.7
CLVD %	30	27.6
DC %	50	68.7



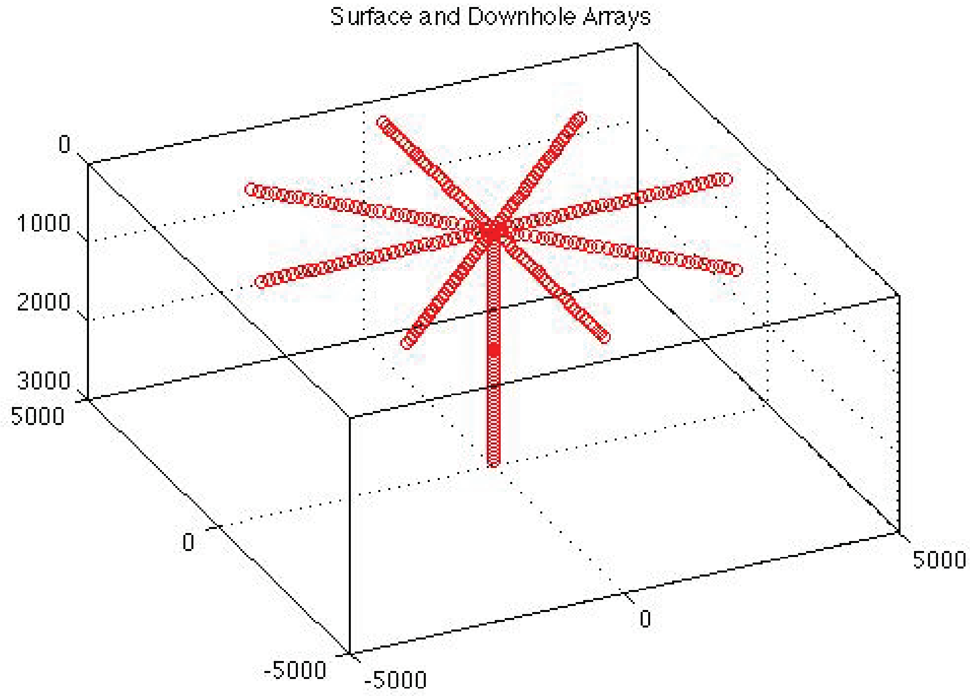
$$M_{Inv-combo} = 1e+21 \times \begin{bmatrix} 2.2835 & -0.6231 & 0.9191 \\ -0.6231 & 5.7821 & -0.0014 \\ 0.9191 & -0.0014 & 1.5267 \end{bmatrix}$$

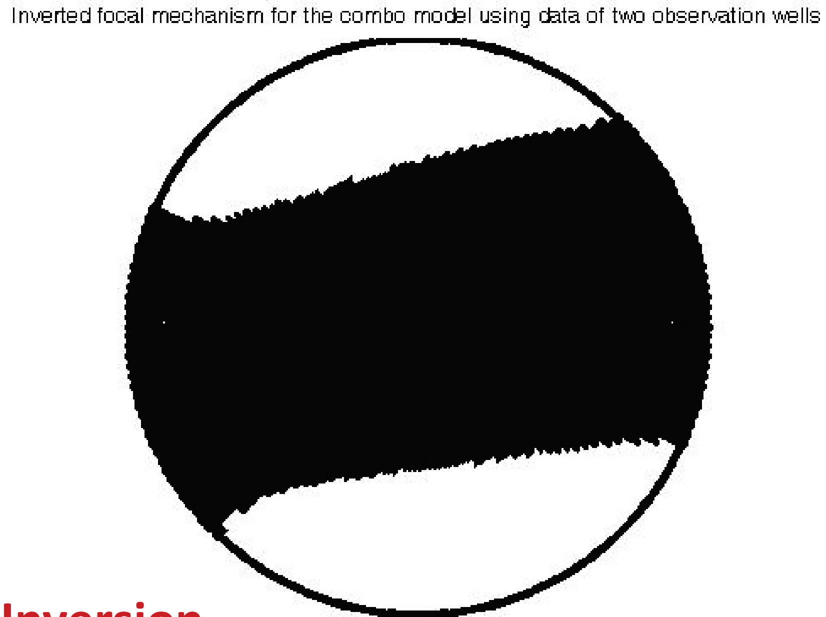
Inversion Results – DC Source Single Observation Well



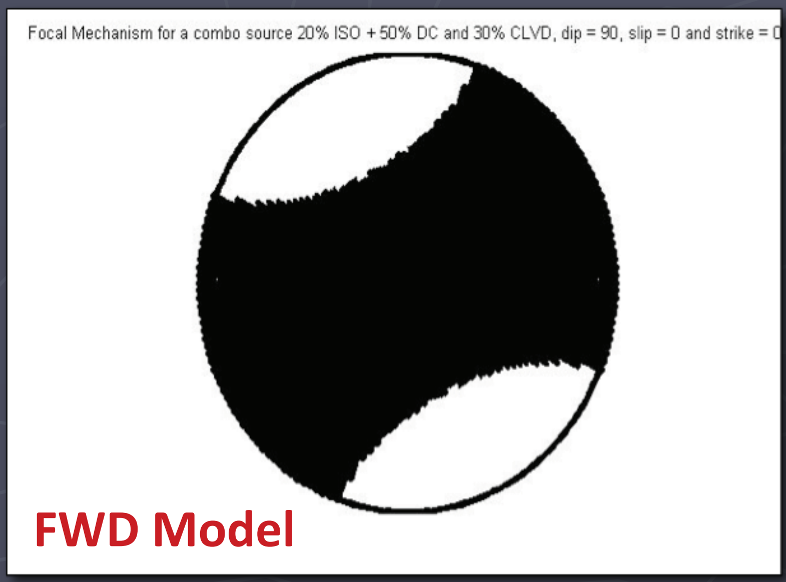
$$M_{Inv-dc} = 1e+21 \times \begin{bmatrix} -4.7251 & 9.4066 & 0.6223 \\ 9.4066 & -9.5789 & 4.1920 \\ 0.6223 & 4.1920 & -0.8151 \end{bmatrix}$$

Inversion Using Surface and Downhole Data





Inversion



Conclusion

- Moment tensor: a powerful tool in delineating the source parameters mathematically.
- Linear computational properties of MT
- With a single observation borehole, it is impossible to solve for the full moment tensor elements
- The isotropic component is the least well-resolved parameter obtained from one single observation well.
- Decomposition of MT is non-unique and various decompositions may lead to different interpretations.

Thank You!