Ray Tracing in Anisotropic Media: Exact, Approximate and Linearized Formulations

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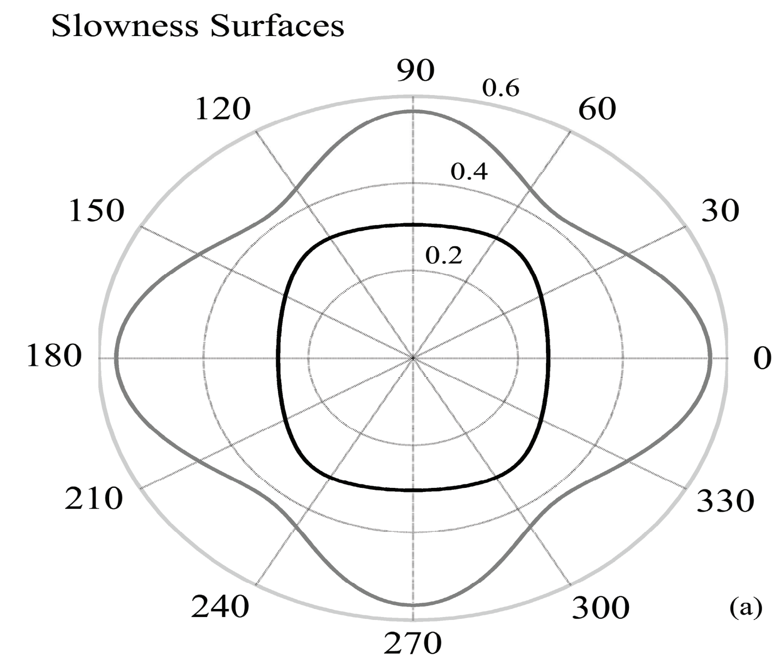
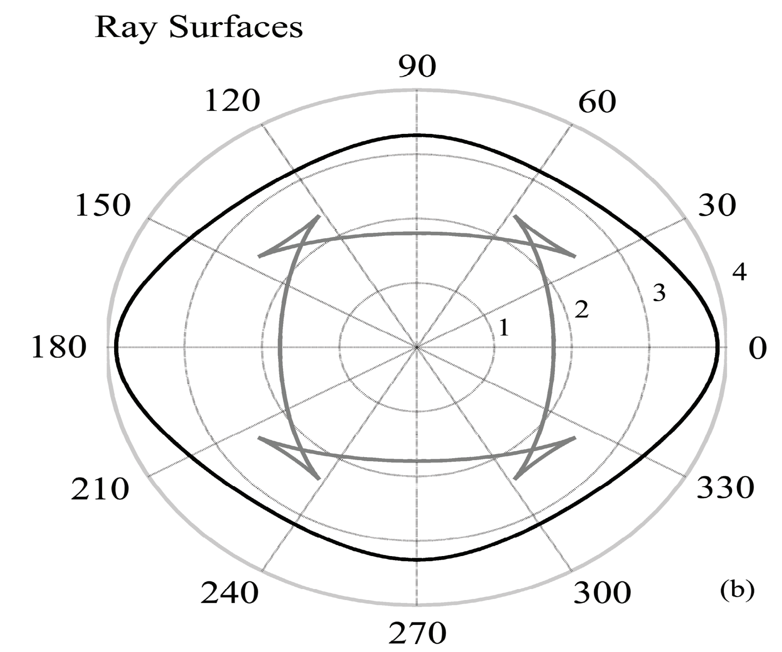
Olivine: slowness and ray surfaces (qP and qSv)

Use convex hull of qSv slowness surface.





Group and Phase Velocity in spatial coordinates and slowness space for anisotropic materials (from Shearer, *Introduction to Seismology*, 1999, p.220)

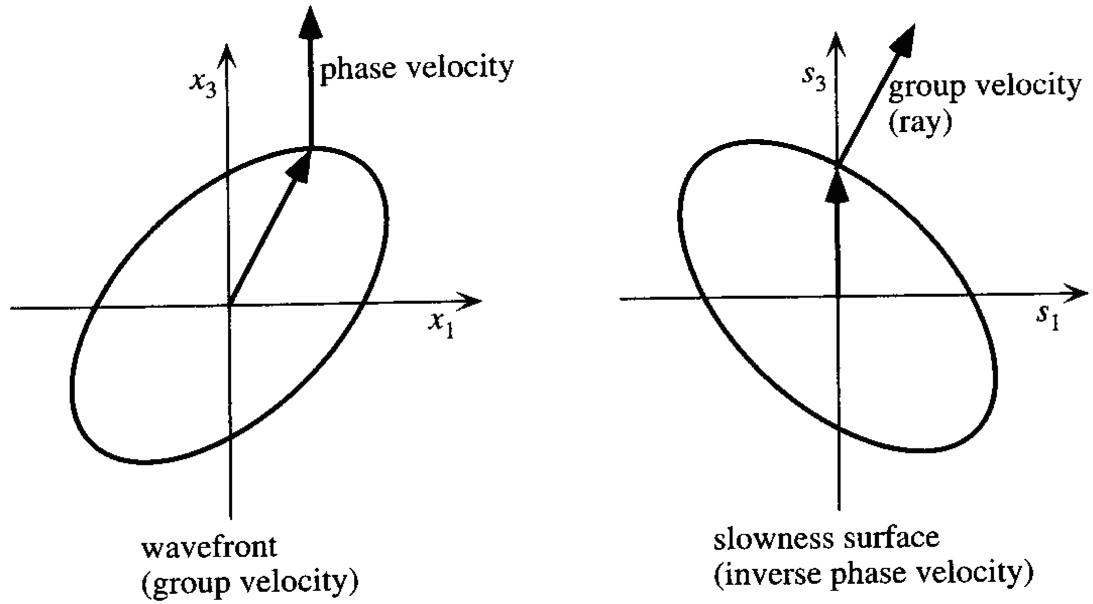
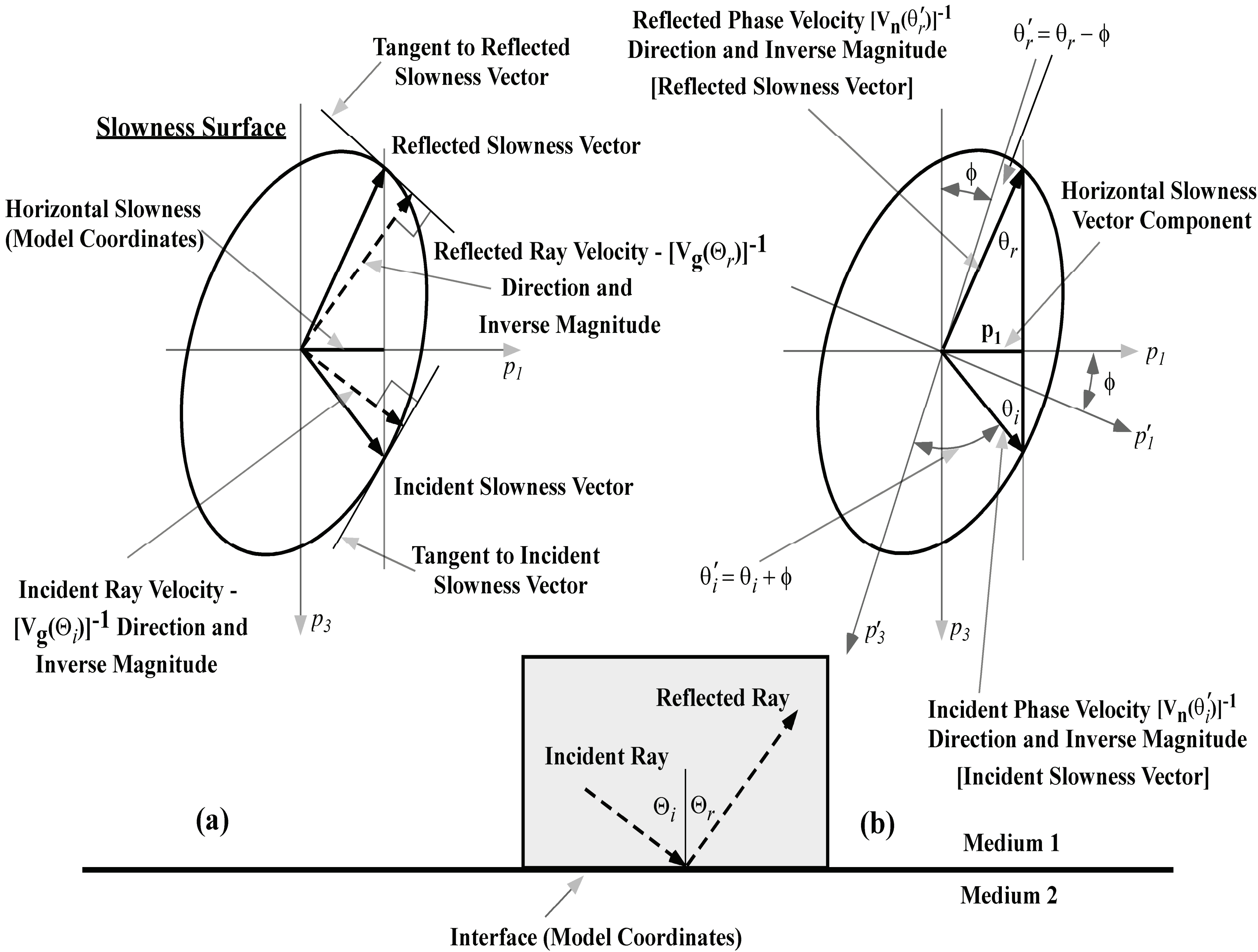
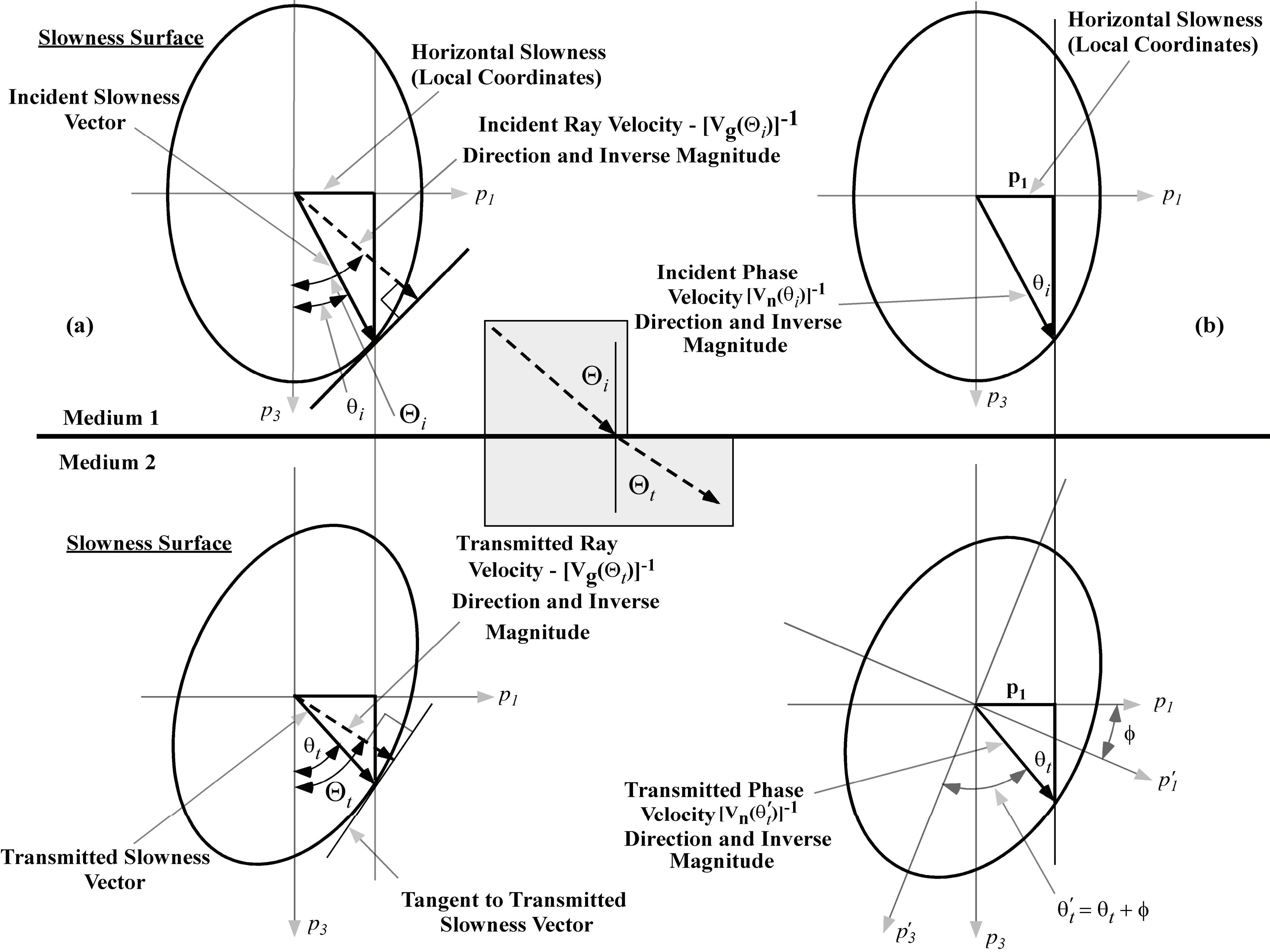


Fig. 11.10. The relationship between the wavefront and slowness surface for anisotropic material.



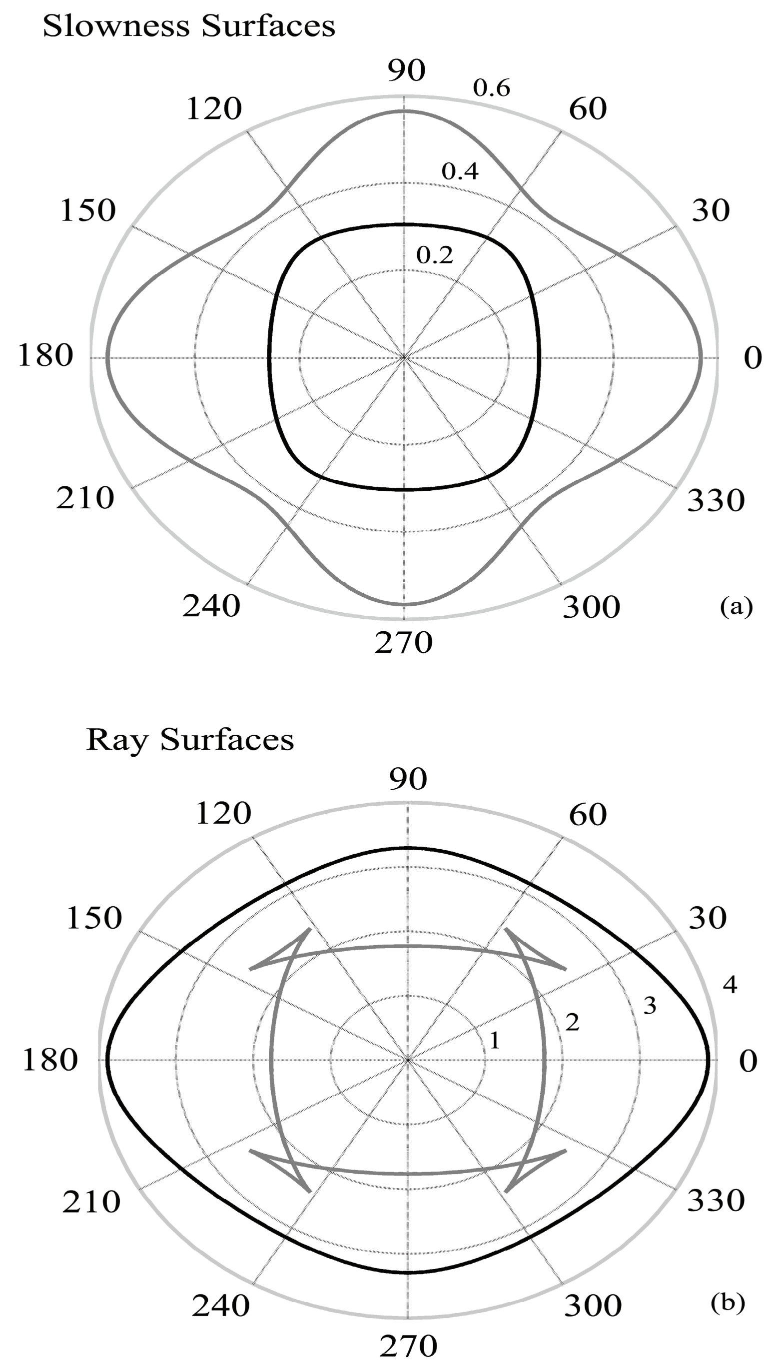
Slowness Surface
Tangent to Reflected Slowness Vector
Reflected Slowness Vector
Horizontal Slowness (Model Coordinates)
Reflected Ray Velocity - $[V_g(\Theta_r)]^{-1}$ Direction and Inverse Magnitude
p_1
Incident Slowness Vector
Tangent to Incident Slowness Vector
Incident Ray Velocity - $[V_g(\Theta_i)]^{-1}$ Direction and Inverse Magnitude
p_3
(a)
Reflected Phase Velocity $[V_n(\theta_r')]^{-1}$ Direction and Inverse Magnitude
[Reflected Slowness Vector]
$\theta_r' = \theta_r - \phi$
ϕ
θ_r
Horizontal Slowness Vector Component
p_1
p_1
ϕ
θ_i
p_1'
$\theta_i' = \theta_i + \phi$
p_3'
p_3
Incident Phase Velocity $[V_n(\theta_i')]^{-1}$ Direction and Inverse Magnitude
[Incident Slowness Vector]
(b)
Reflected Ray
Incident Ray
Θ_i
Θ_r
Medium 1
Medium 2
Interface (Model Coordinates)



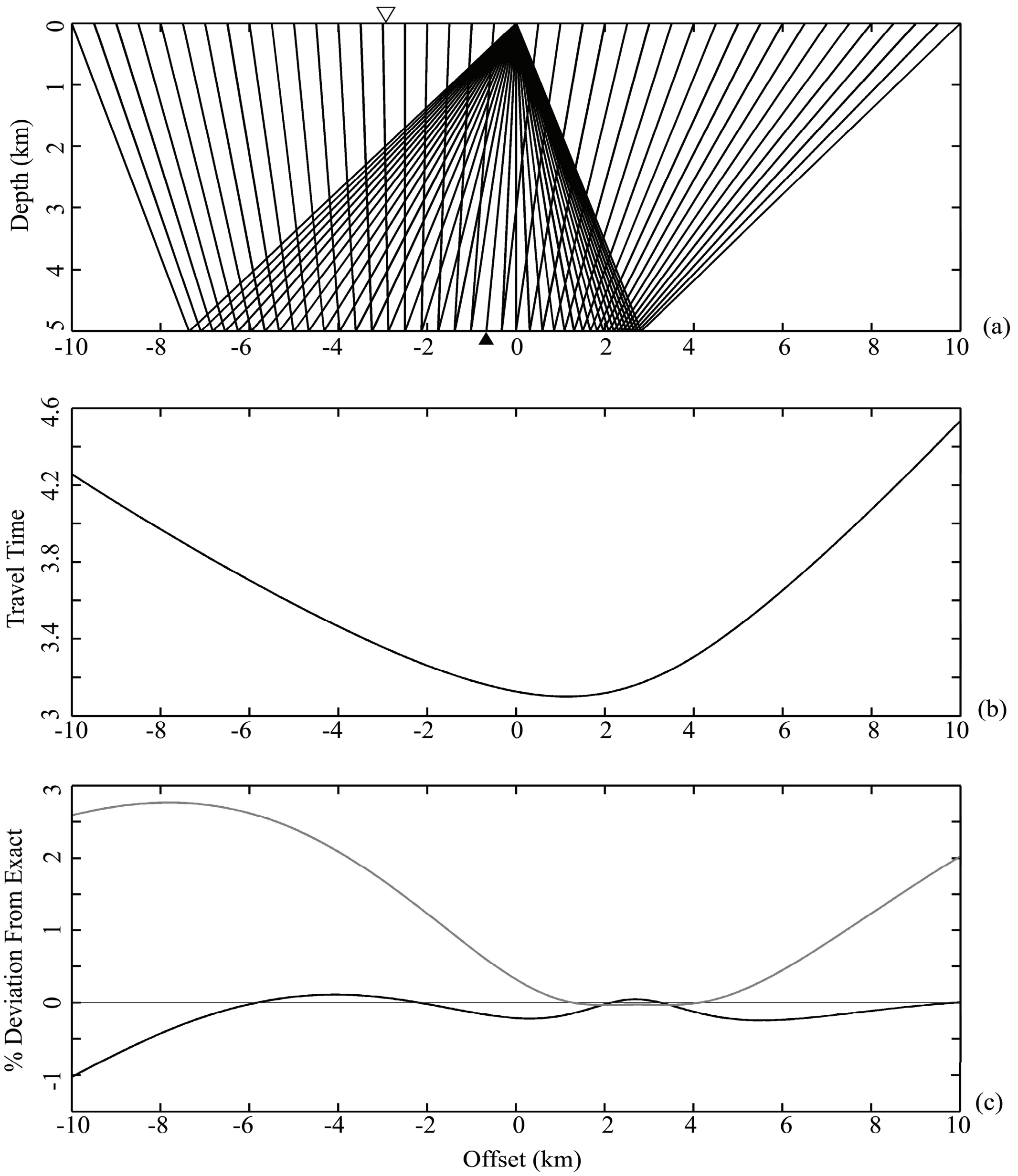
Slowness Surface
Horizontal Slowness (Local Coordinates)
Incident Slowness Vector
Incident Ray Velocity - $[V_g(\Theta_i)]^{-1}$
Direction and Inverse Magnitude
p_1
p_3
θ_i
Θ_i
(a)
(b)
Horizontal Slowness (Local Coordinates)
$\mathbf{p_1}$
Incident Phase Velocity $[V_n(\theta_i)]^{-1}$ Direction and Inverse Magnitude
Medium 1
Medium 2
Θ_t
Slowness Surface
Transmitted Ray Velocity - $[V_g(\Theta_t)]^{-1}$ Direction and Inverse Magnitude
θ_t
Transmitted Slowness Vector
Tangent to Transmitted Slowness Vector
ϕ
p_1'
p_3'
Transmitted Phase Velocity $[V_n(\theta_t')]^{-1}$ Direction and Inverse Magnitude
$\theta_t' = \theta_t + \phi$

Olivine: slowness and ray surfaces (qP and qSv)

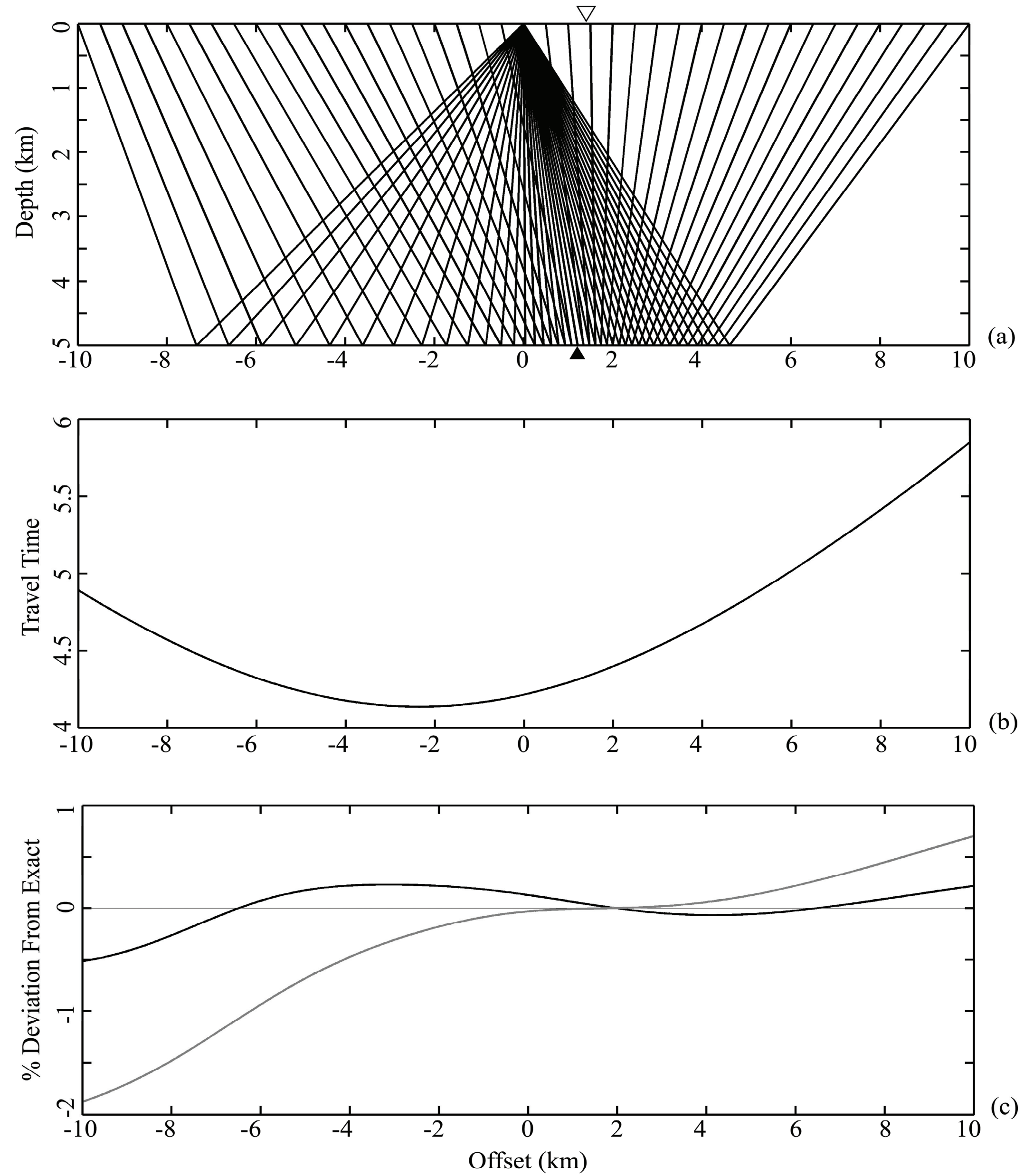
Use convex hull of qSv slowness surface.



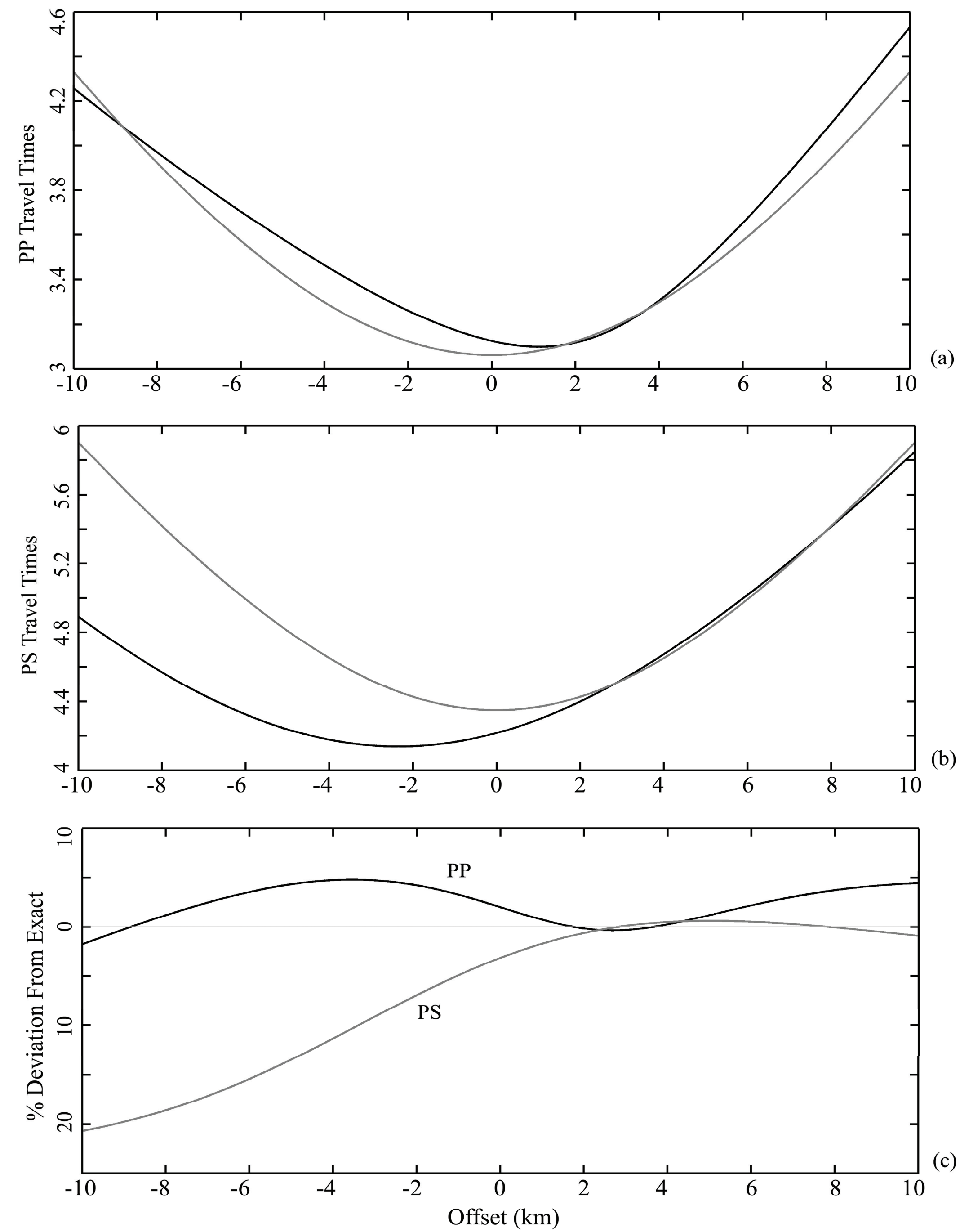
qP - qP Reflected Arrival:
30 degree rotation



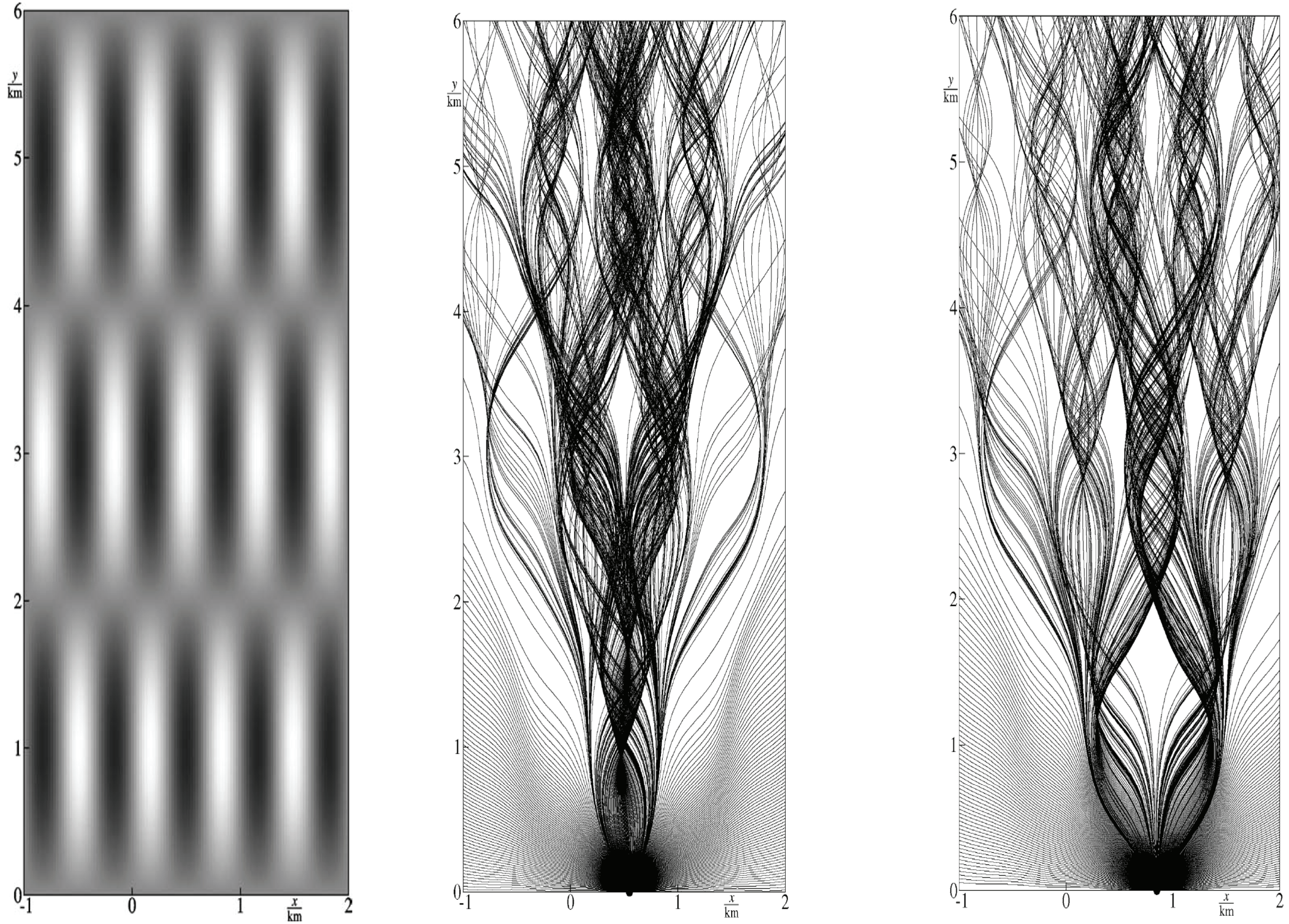
qP - qSv Reflected Arrival: 30 degree rotation



Transversely Isotropic vs. Isotropic



Ludek Klimes: 2D "Egg Crate"



Ray Tracing in Complex 2D TI Structures

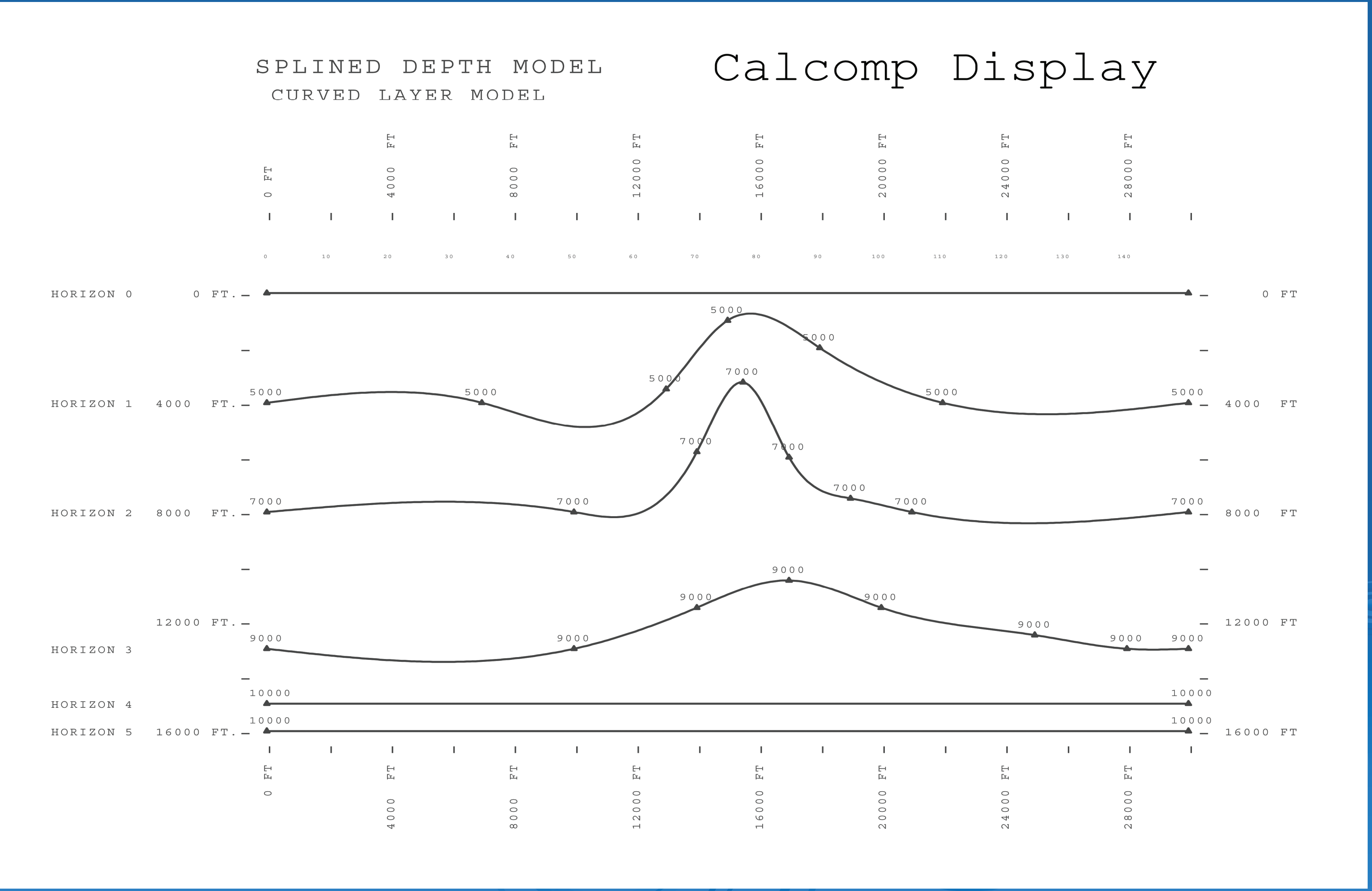
- Grid, Smooth, Trace, Display → Bad Words
- Finer Grid, More Smoothing, Trace, Display → Again
- Real Bad Words

Another Option:

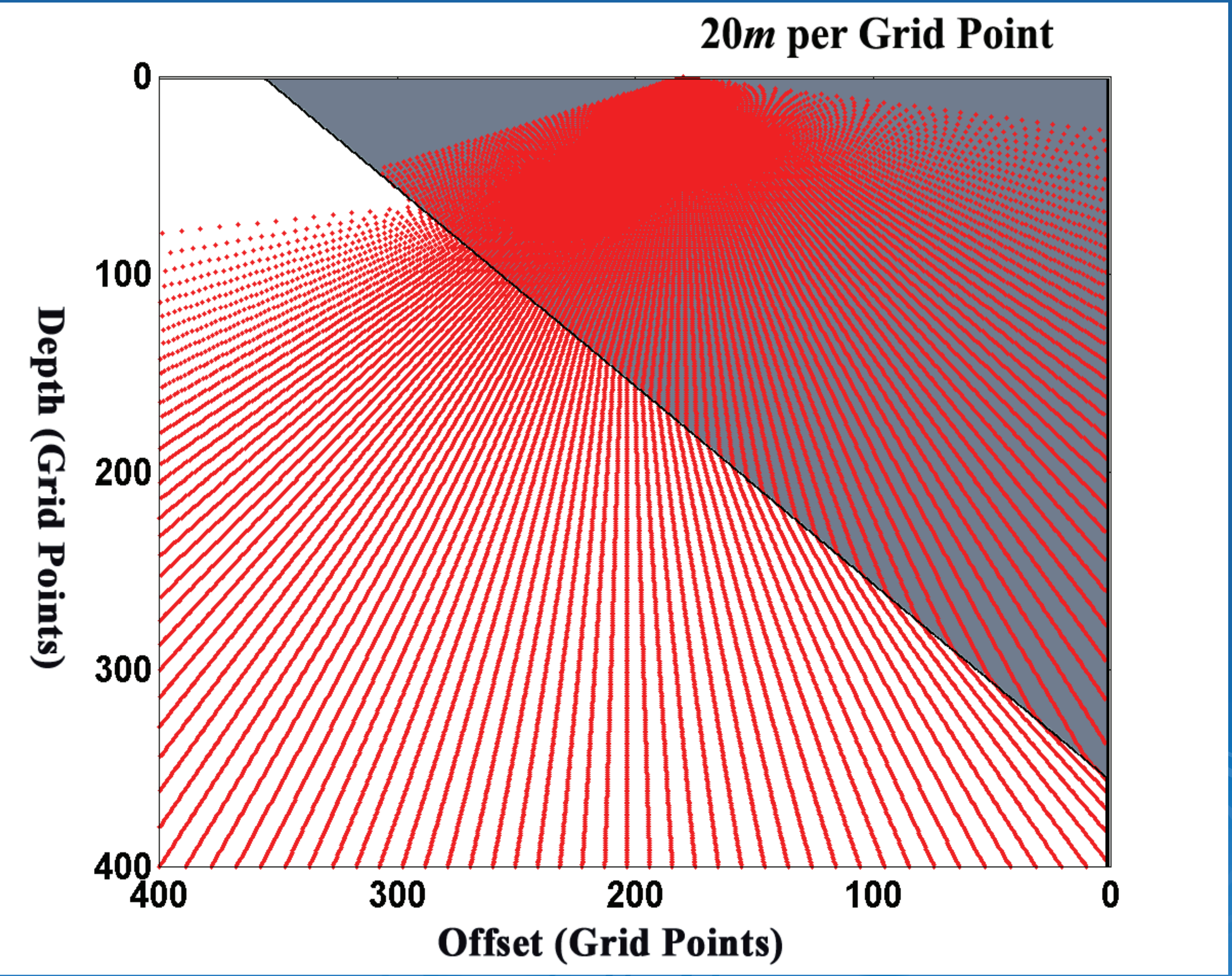
(1). Treat Anisotropic Parameter Discontinuities as Interfaces and Trace Rays Appropriately.

(2). This Type of Approach Will Be Required, at Least in the Near Term, For 3D Structures.

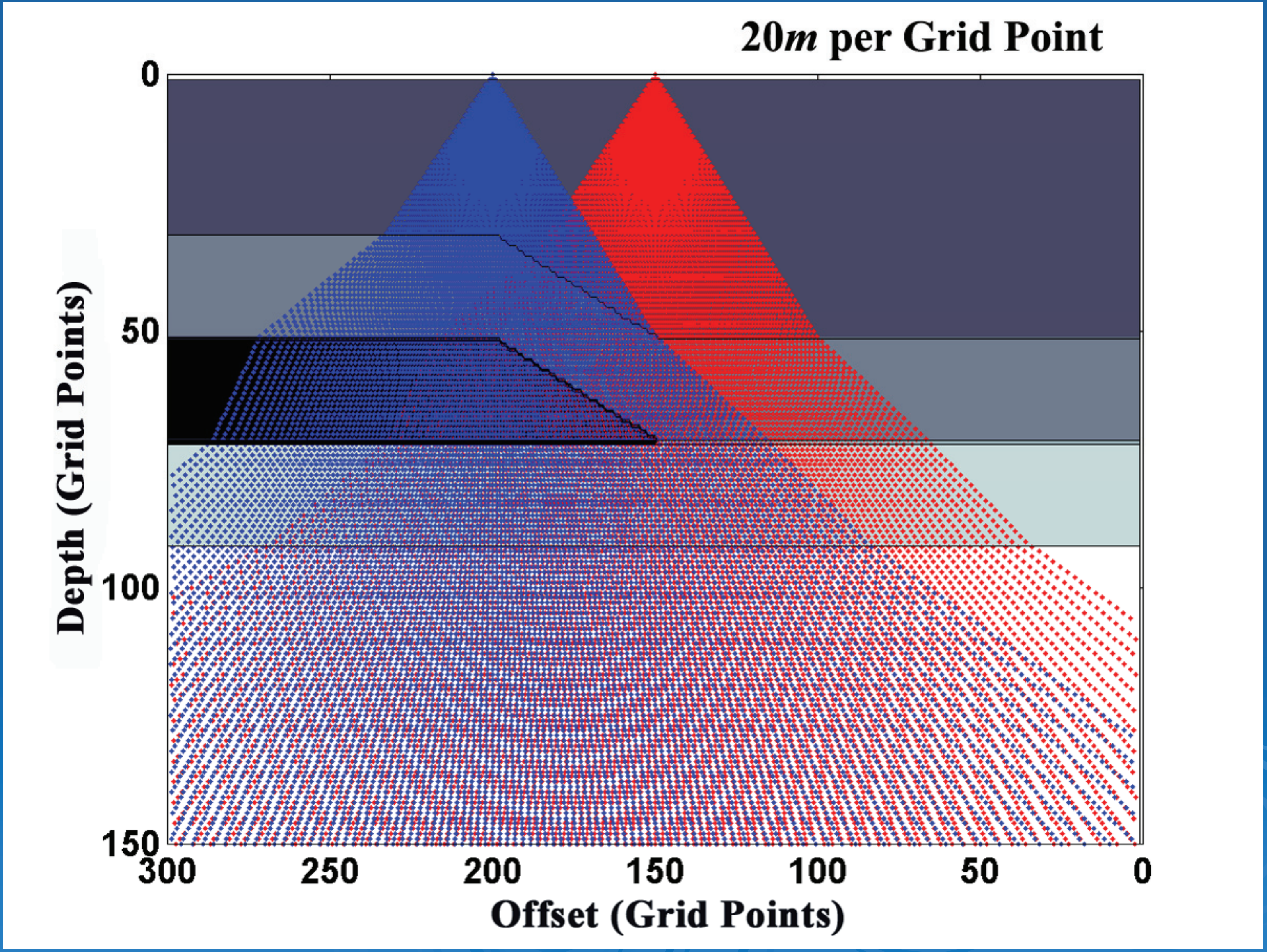
What Was Available (2D) in the 1960s



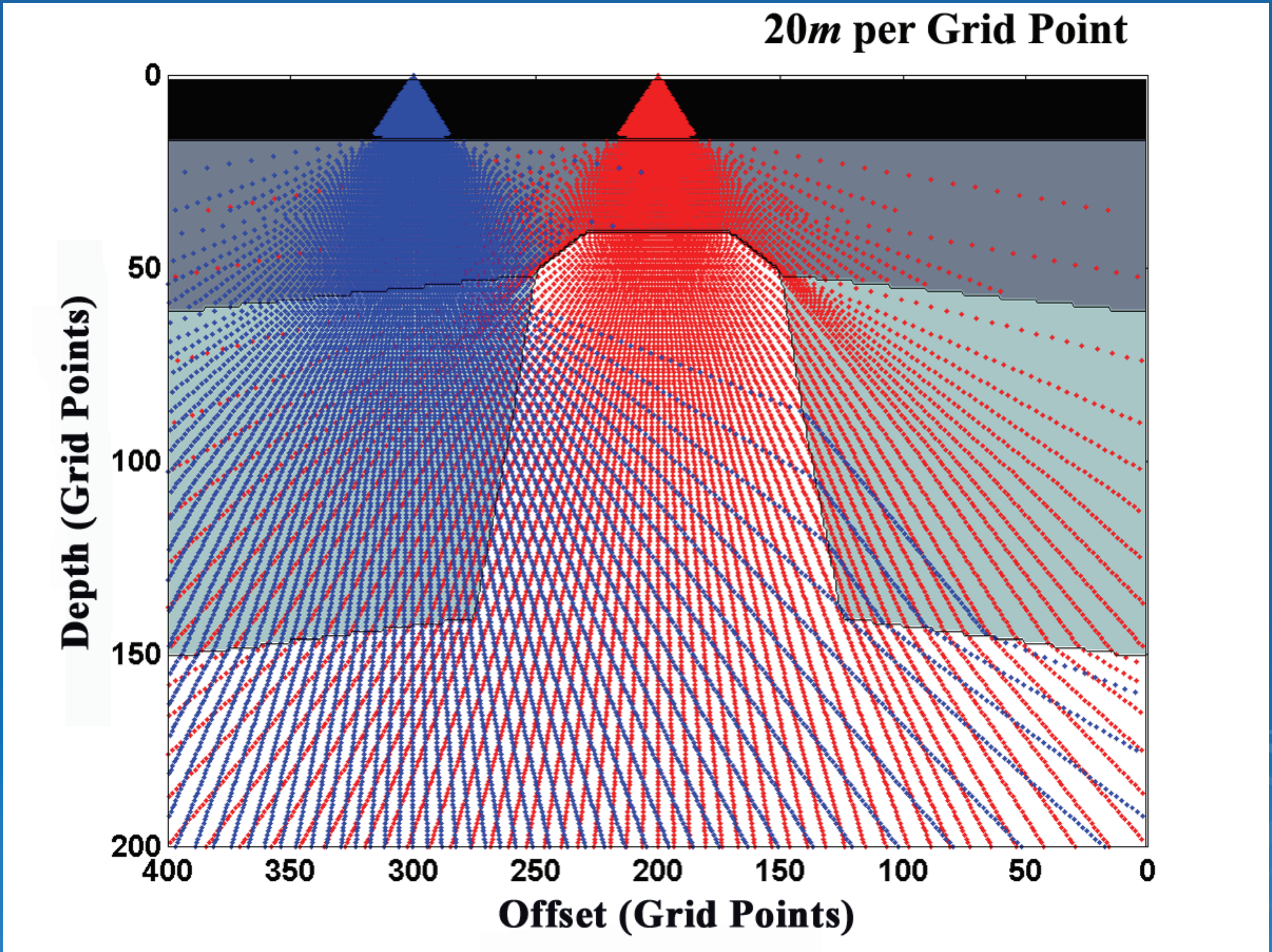
Arkoma (Amoco) Model: Angled T.I. Corner



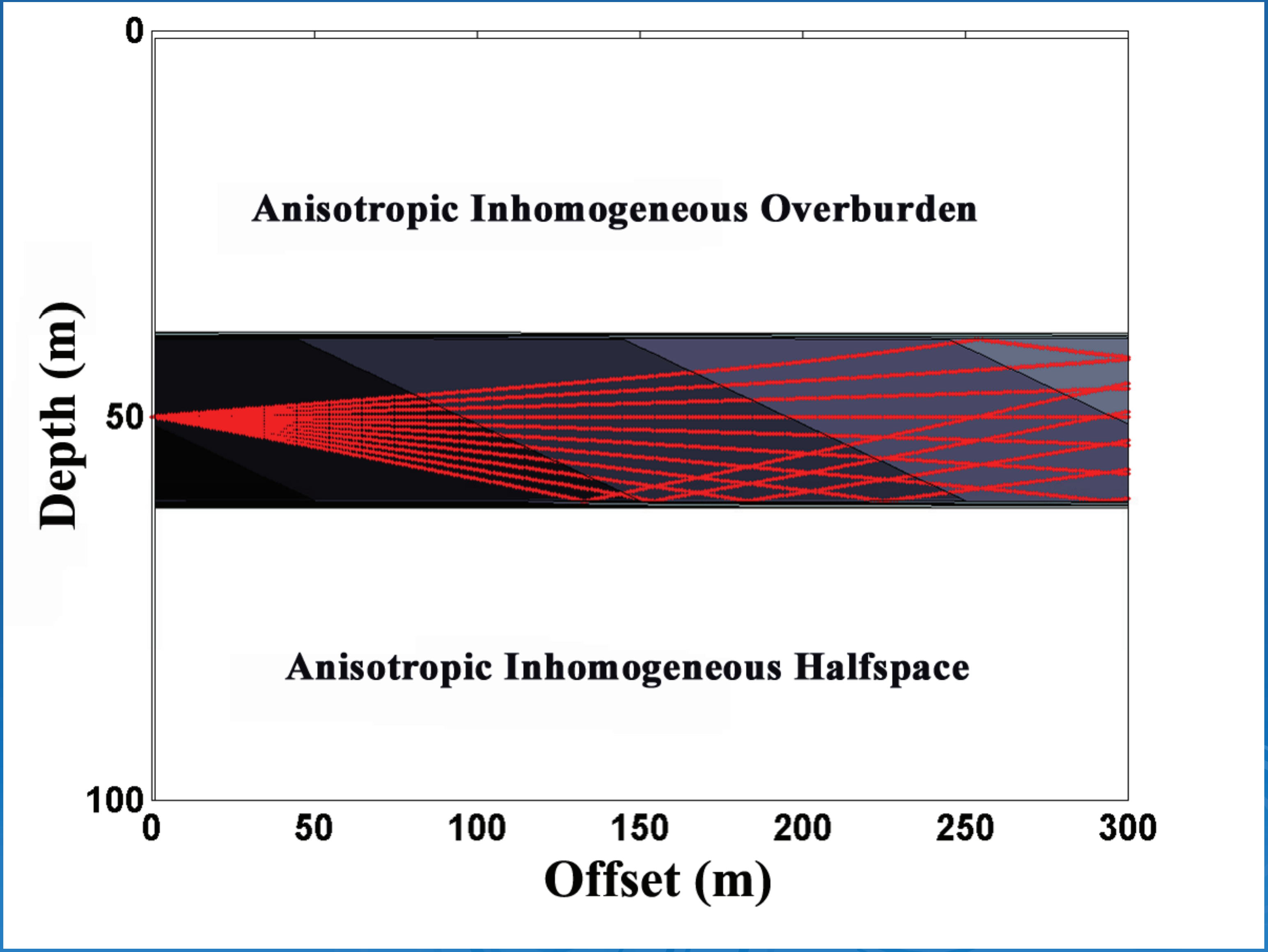
Wedge Model (Perez)



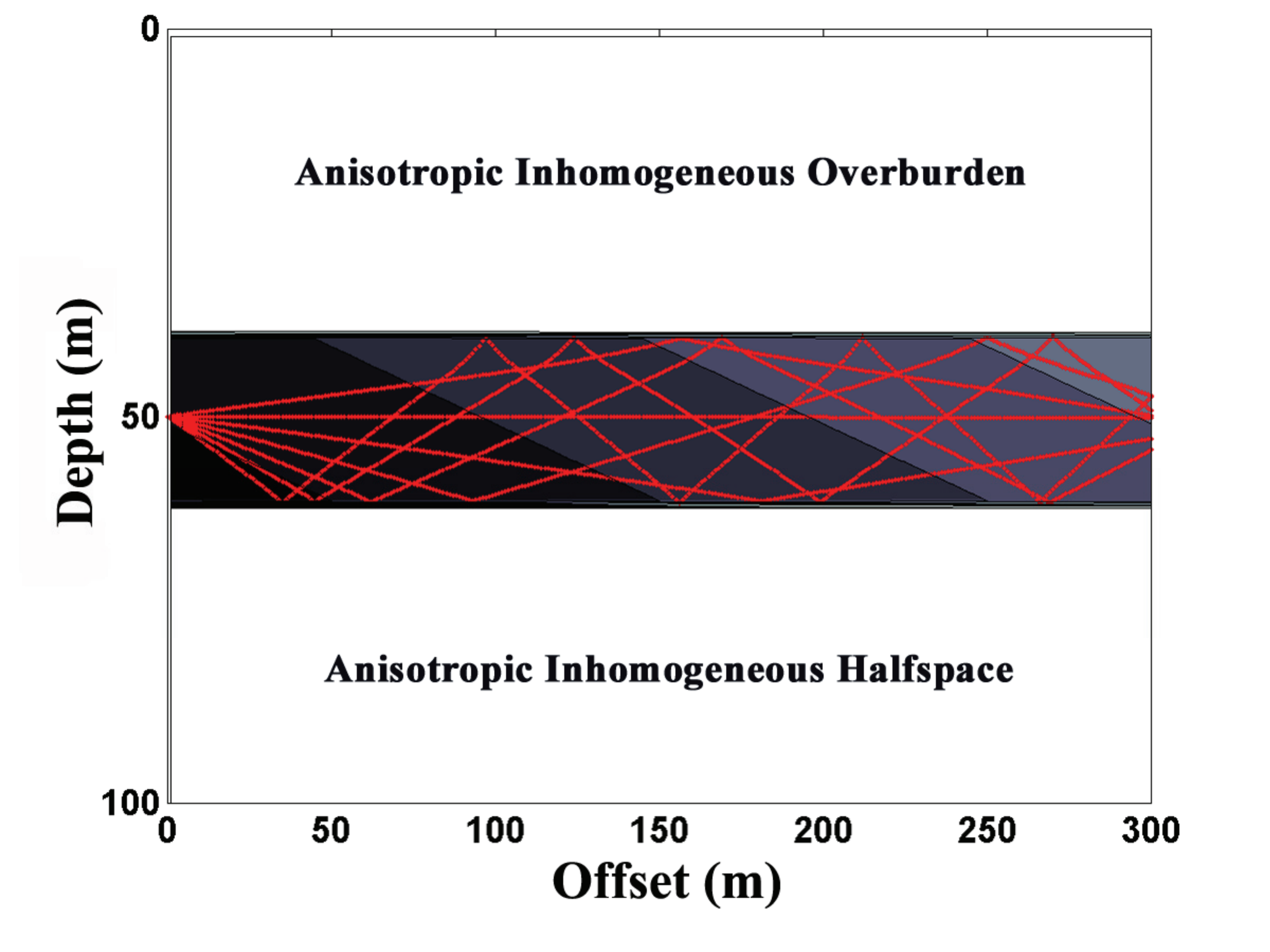
Salt Dome Model (Perez)



Coal Seam – Cross Hole I



Coal Seam – Cross Hole II



Possibly the Earth is 3D:

- 3D Ray Tracing.
- Fairly Complex (Comparatively) Anisotropic Media: Orthorhombic to 21 Parameter.
- Shear Waves Become Problematic.

Linearized Group Velocity for qP Rays in an Orthorhombic Medium.

$$\frac{1}{V_{Group}^2} = \frac{N_1^2}{A_{11}} + \frac{N_2^2}{A_{22}} + \frac{N_3^2}{A_{33}} - \frac{E_{12}N_1^2N_2^2}{A_{11}A_{22}} - \frac{E_{13}N_1^2N_3^2}{A_{11}A_{33}} - \frac{E_{23}N_2^2N_3^2}{A_{22}A_{33}}$$

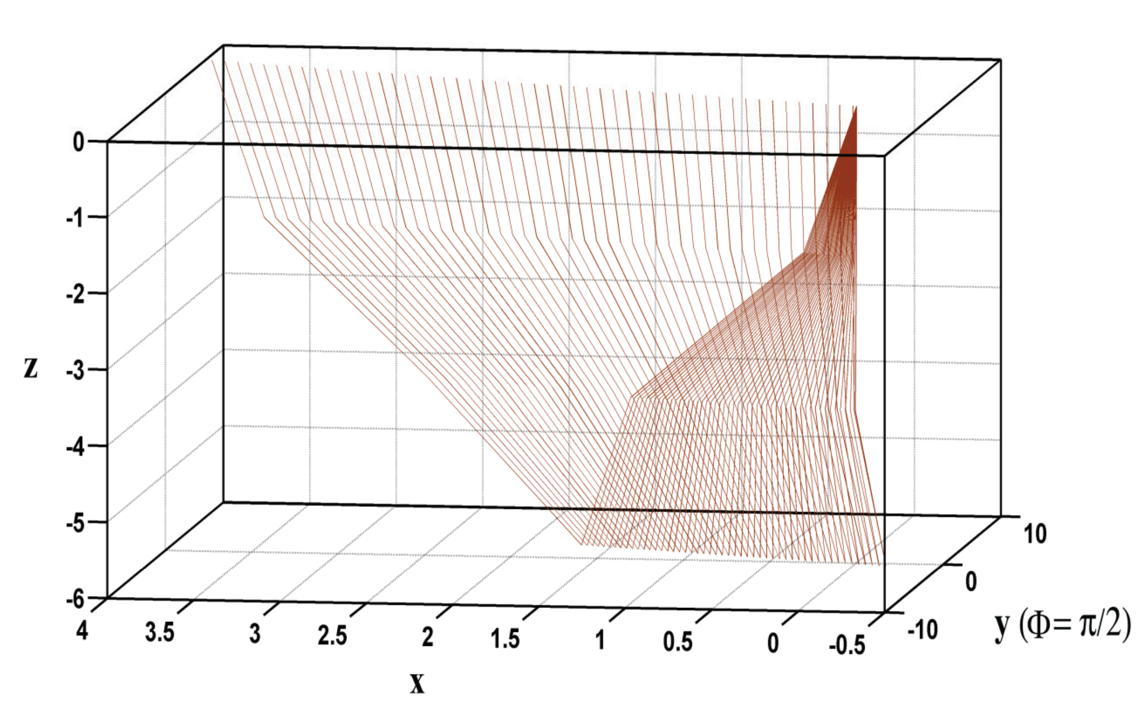
$$E_{12} = 2\left(A_{12} + 2A_{66}\right) - \left(A_{11} + A_{22}\right)$$

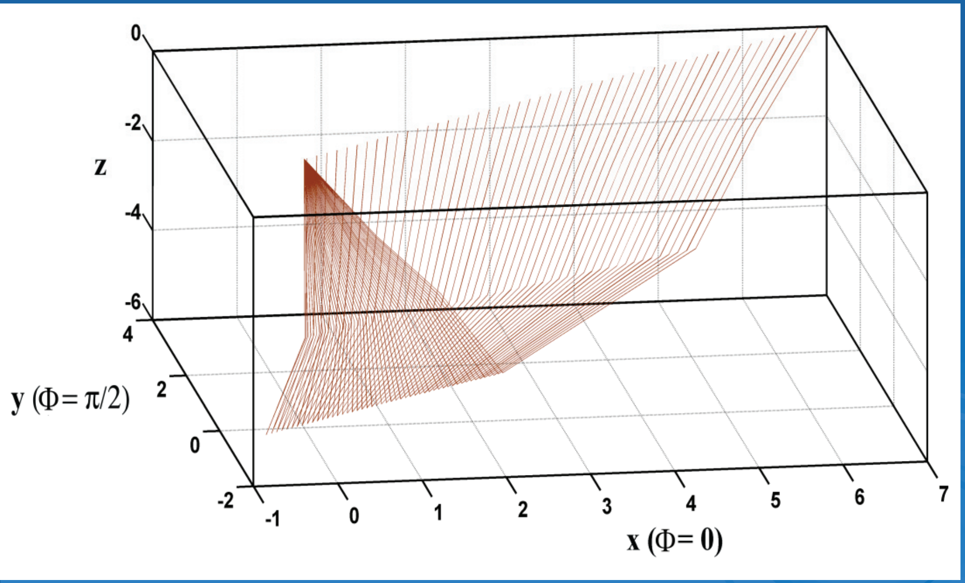
$$E_{13} = 2\left(A_{13} + 2A_{55}\right) - \left(A_{11} + A_{33}\right)$$

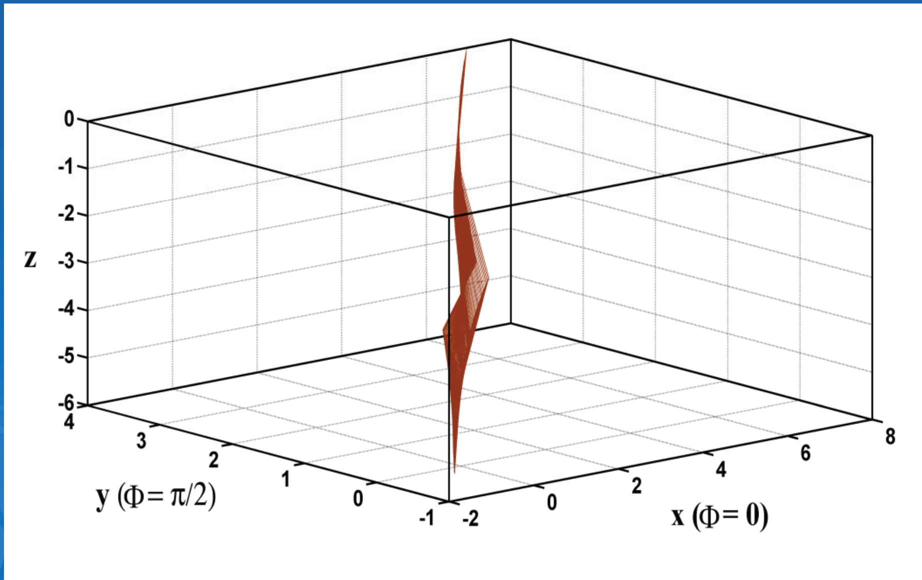
$$E_{23} = 2\left(A_{23} + 2A_{44}\right) - \left(A_{22} + A_{33}\right)$$

$$N = \left(\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta\right)$$

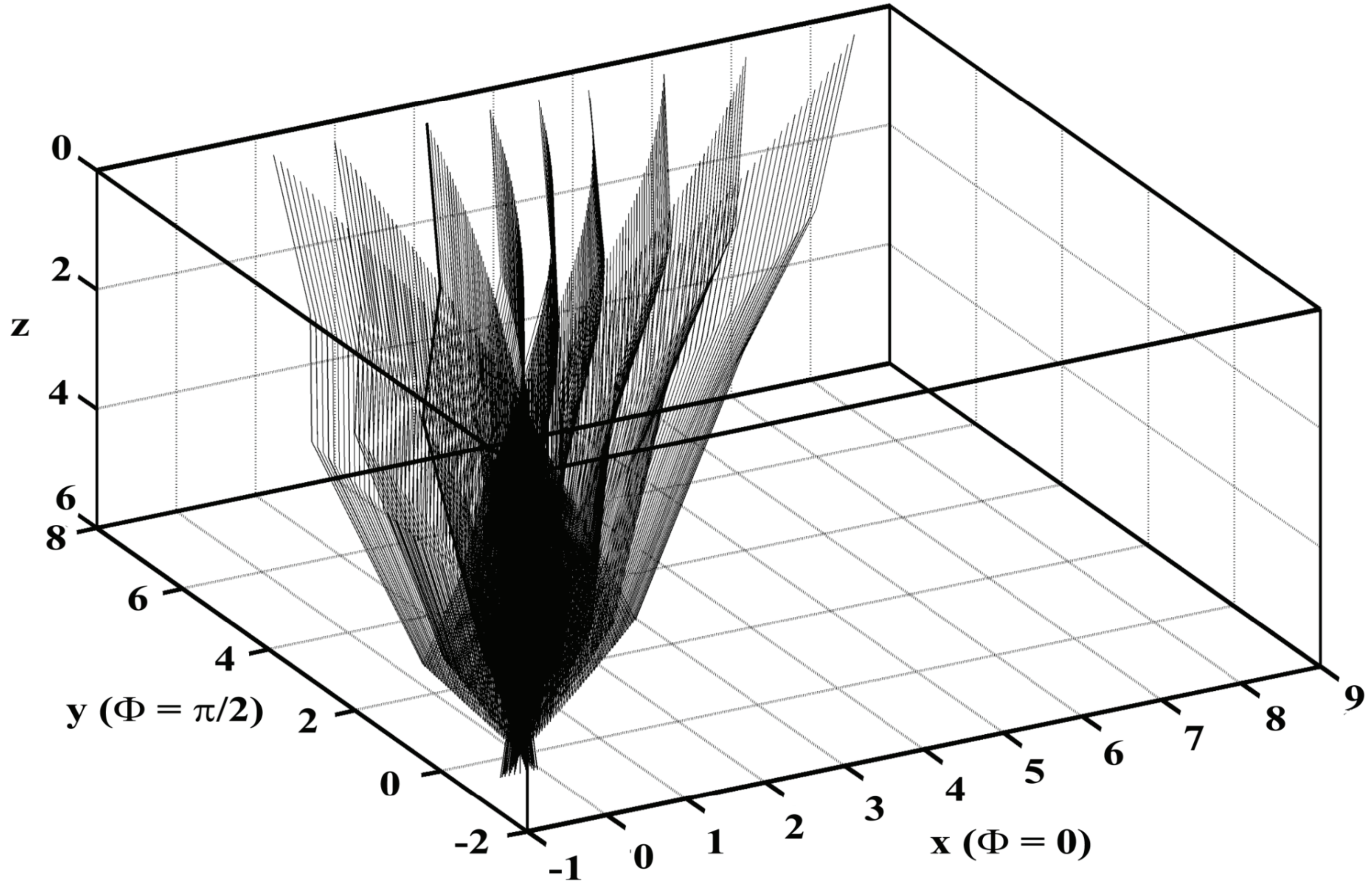
3 Layered Orthorhombic Medium: Linearized qP Reflected Rays





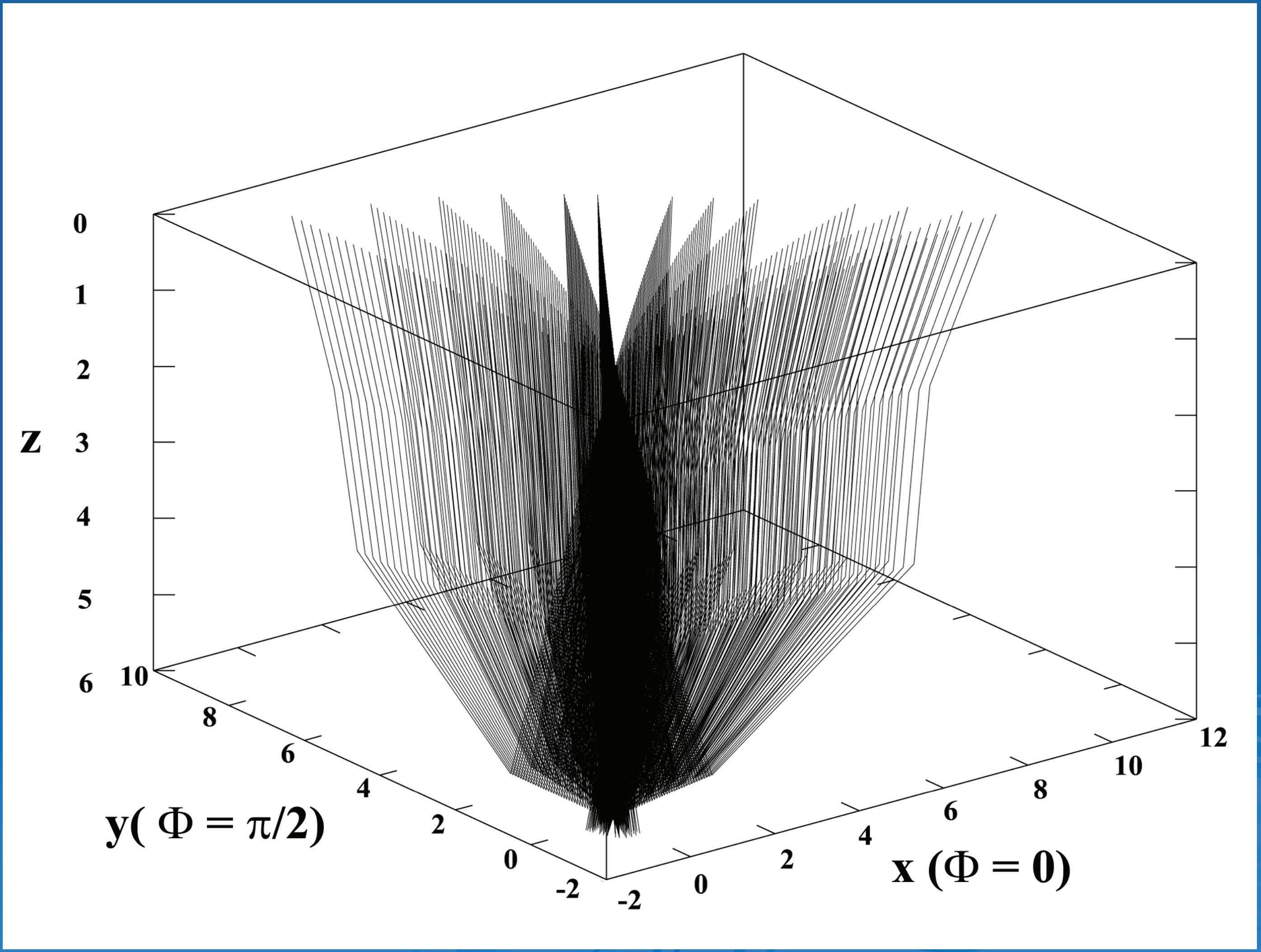


Spread of Azimuths: Surface Receivers

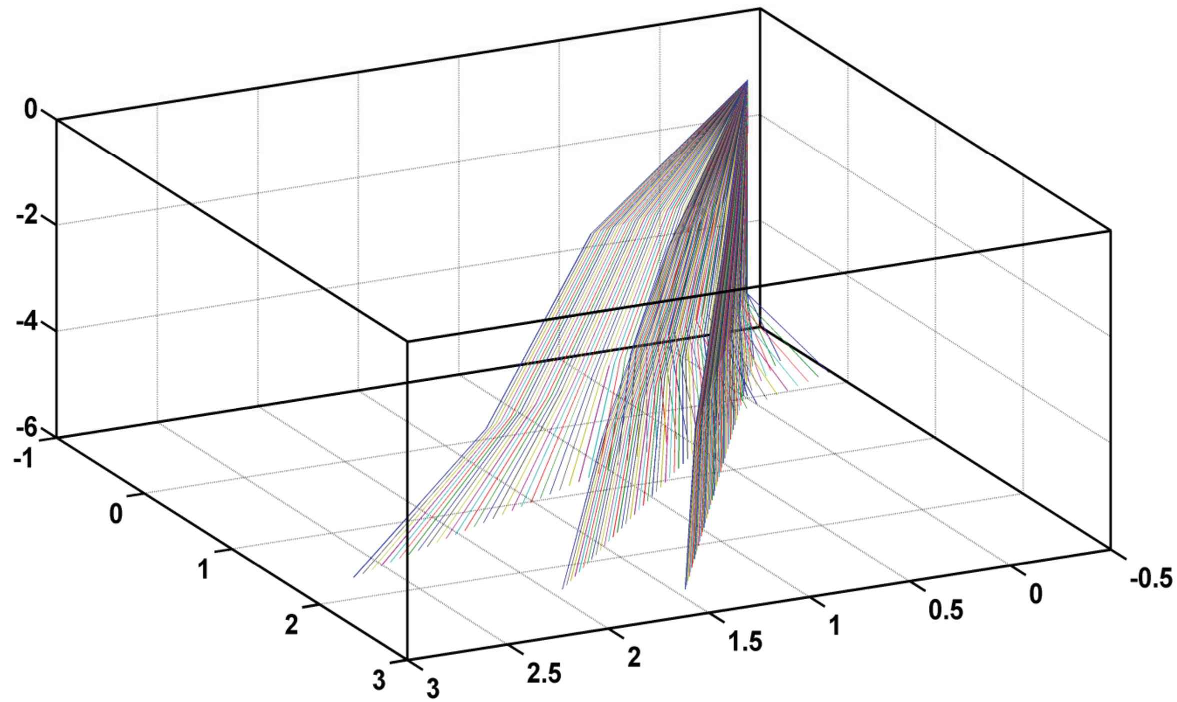


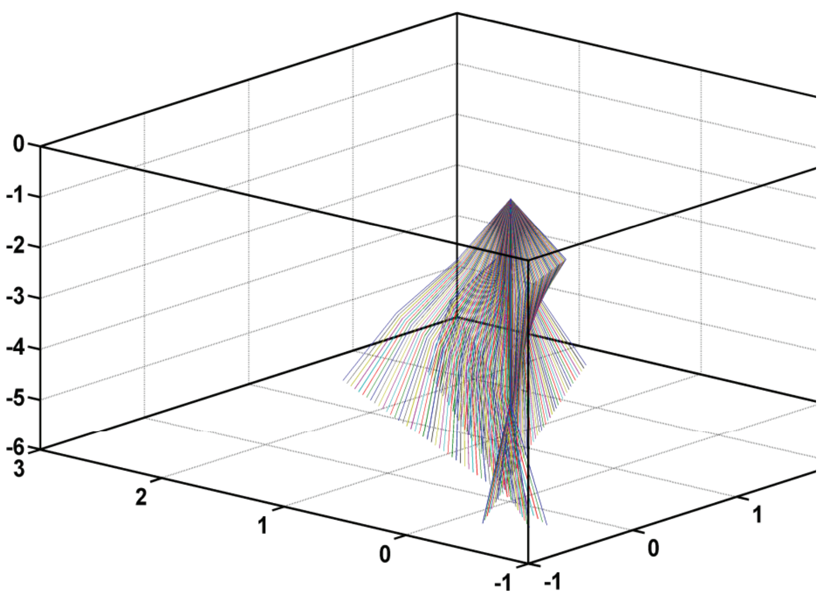
Spread of "Spaced" Azimuths: Surface Receivers

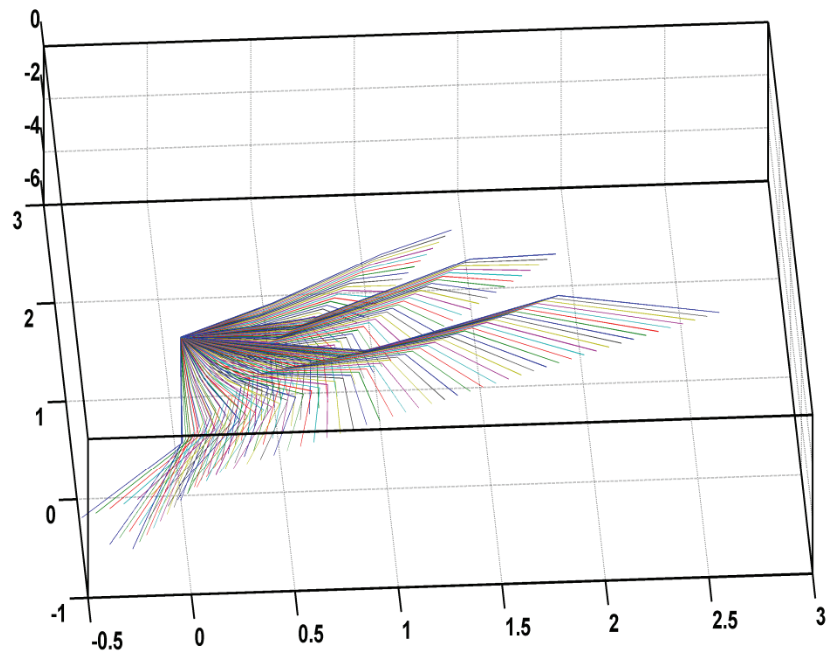
Dependent Variables Θ and Φ



3 Layered Orthorhombic Media: Linearized qP Rays To Reference Depth







More Conventional Ray Tracing Method:

(1). Eikonal Equation → Hamilton-Jacobi Theory.

(2). 6 ODEs in the Three Spatial Dimensions and Three Slowness Dimensions.

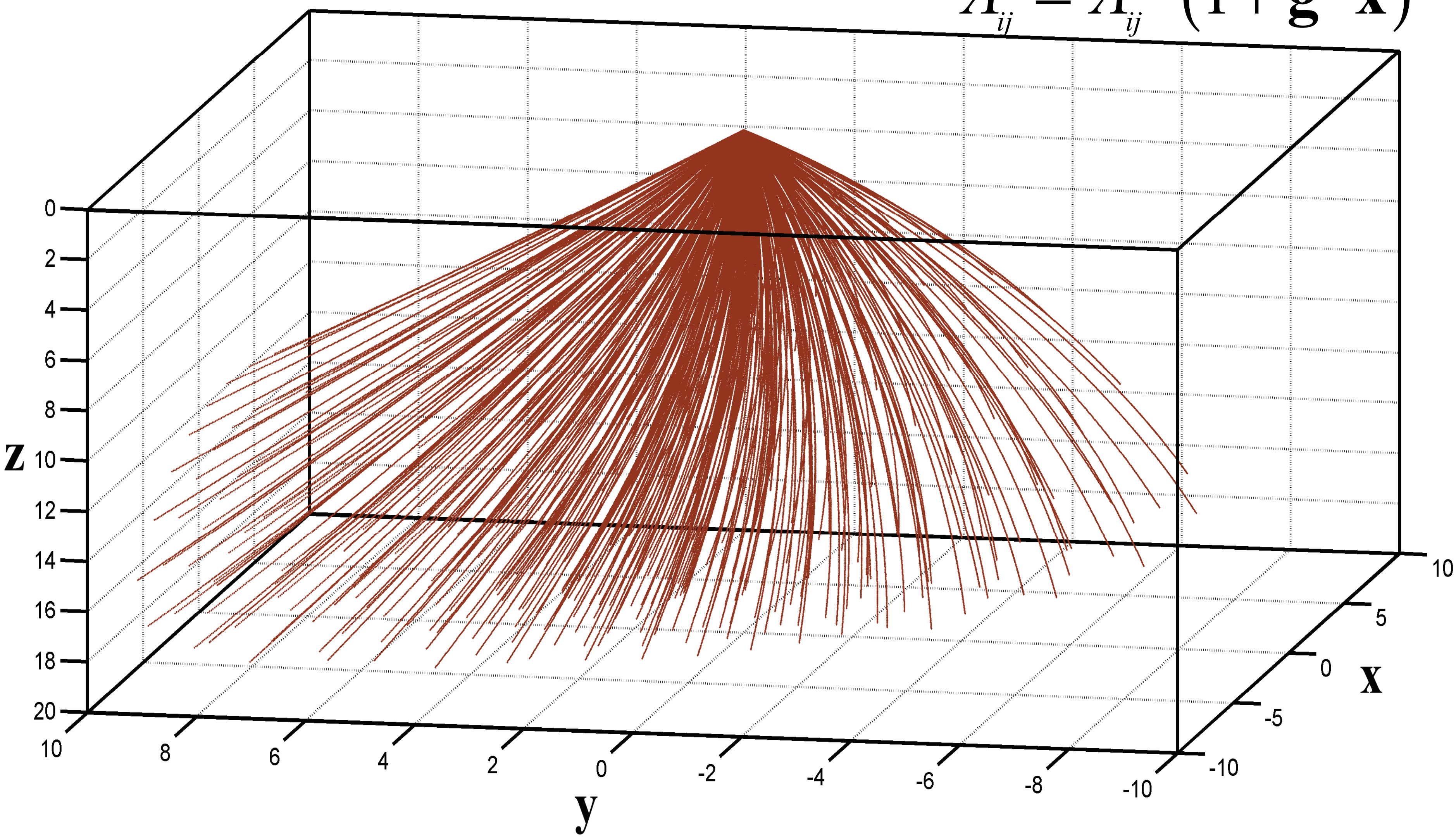
(3). Ray Tracing Process → Step by Time Increments.

(4). Initial Values of x_0 [$x(t_0)$] and p_0 [$p(t_0)$] Known.

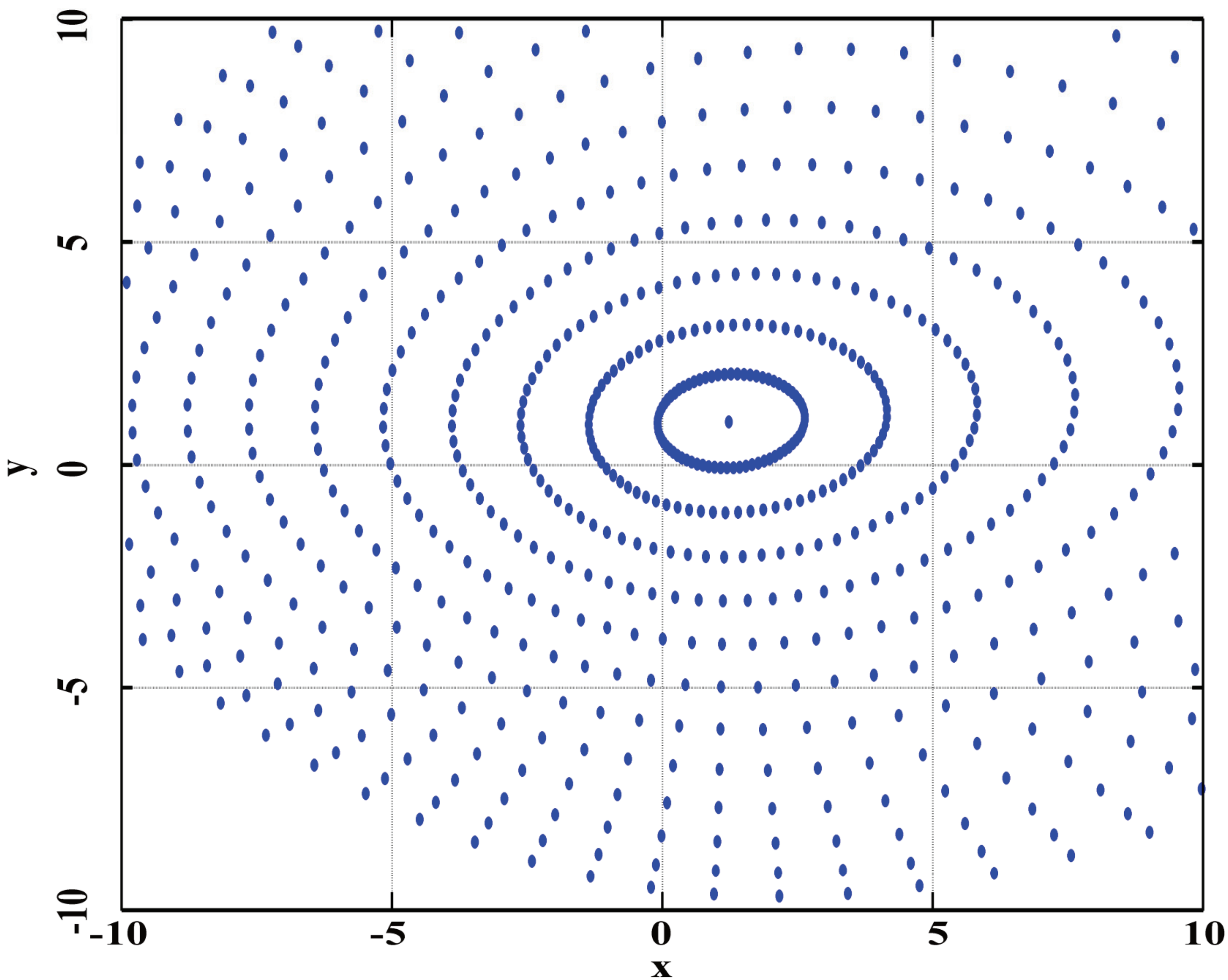
(5). Ray Tracing is Highly Dependent on Model Building Program.

Ray Cone Shot to Reference Depth:

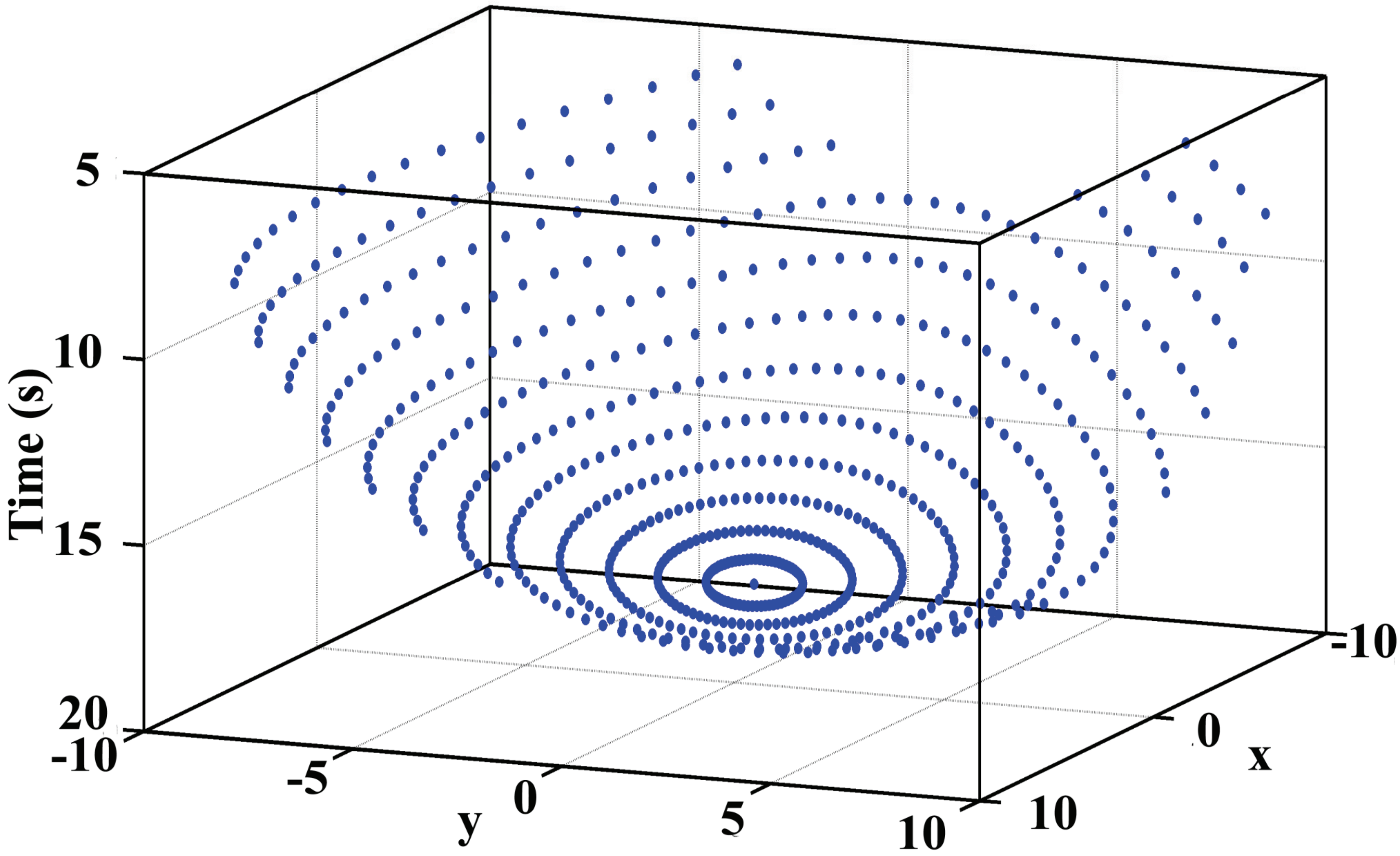
$$A_{ij} = A_{ij}^{(0)} \left(1 + \mathbf{g} \cdot \mathbf{x}\right)^2$$



Scatter Plot: At Common Depth 20km



Time to Reference Depth



Summary

• 2D Anisotropic Ray Tracing Revisited.
• Basics of 3D Anisotropic Ray Tracing Introduced.

From Here, Where?

(1). Write a 3D Model Building Program.
(2). Modify Existing 3D Program -
Proprietary.
(3). *Monastery (Cloistered). Make*
Chokecherry Wine

Acknowledgements

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