Full Waveform Inversion (FWI) with wave-equation migration (WEM) and well control

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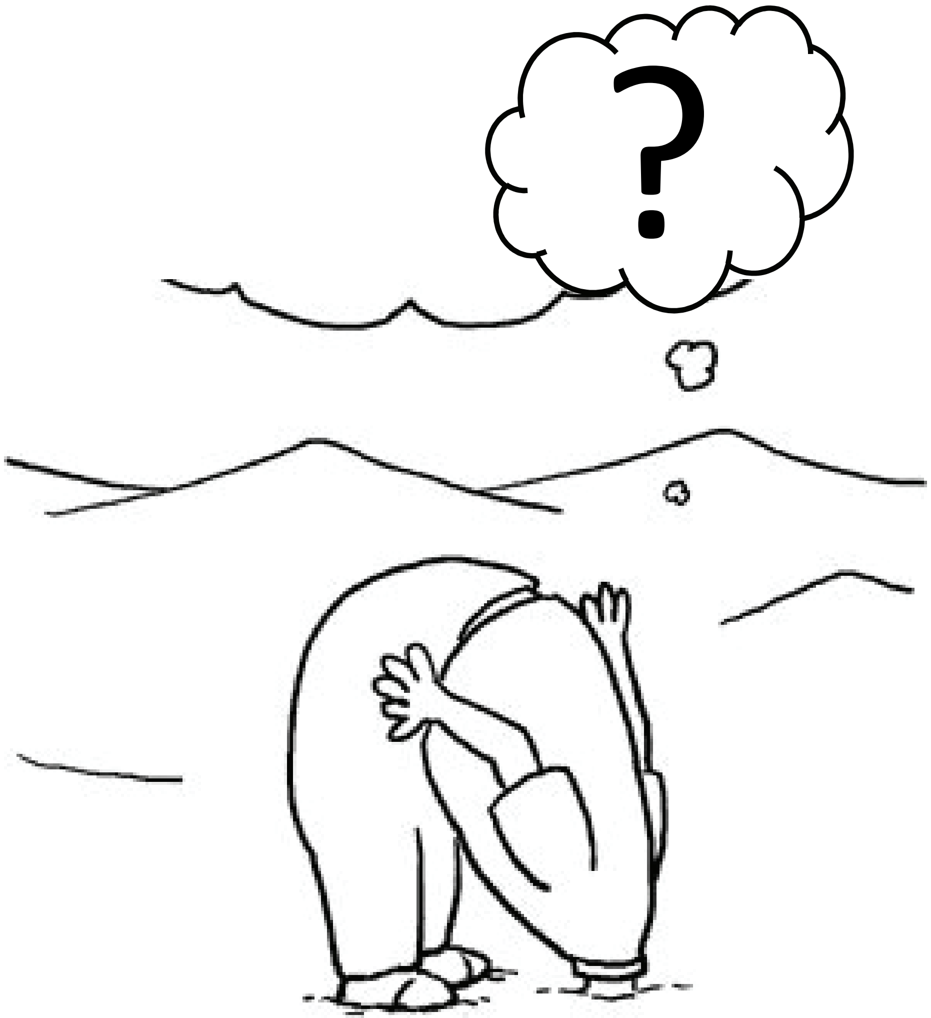
Outline

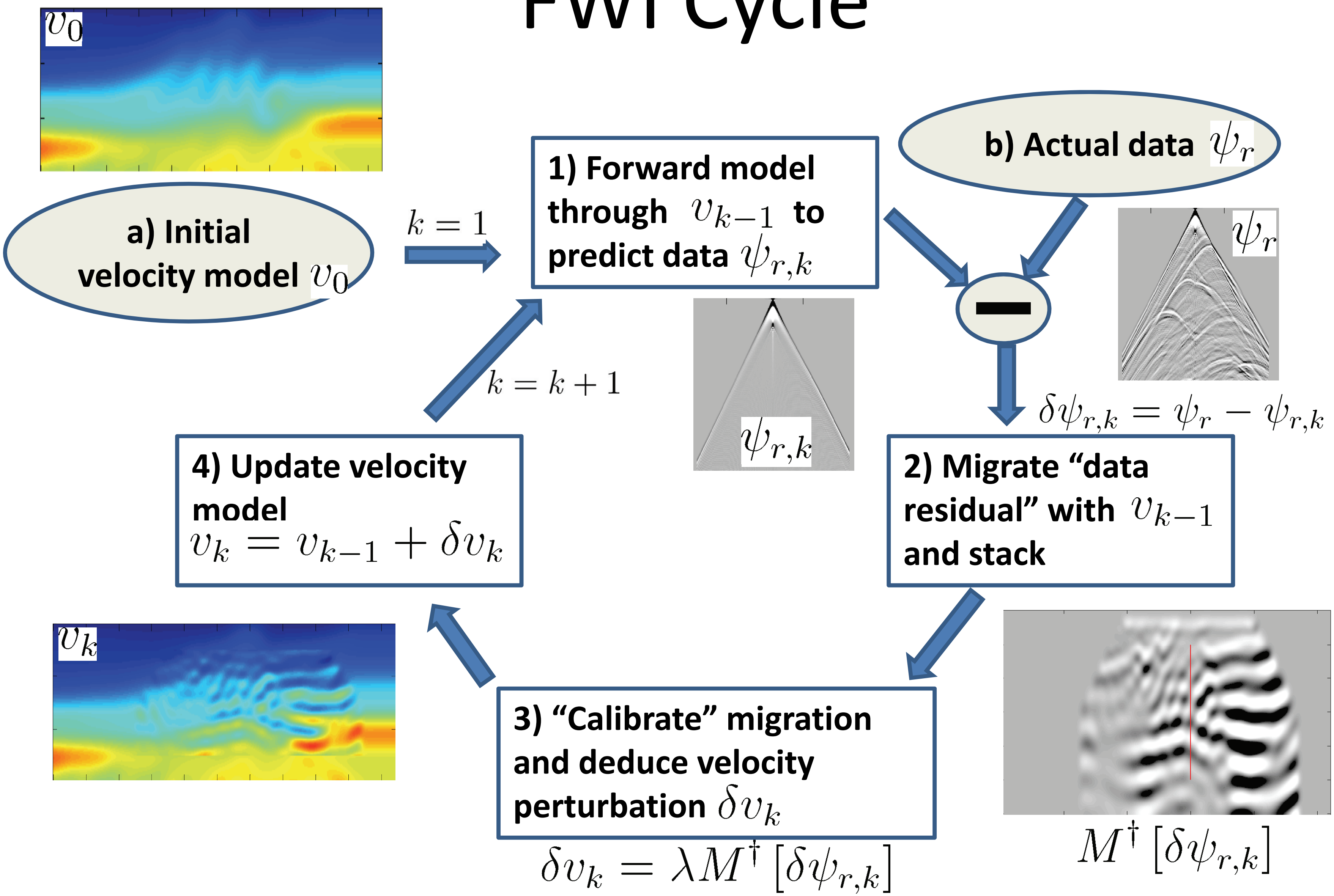
The FWI cycle
The fundamental theorem of FWI
Understanding the theorem
What kind of migration?
Calibrating the migration
A numerical experiment with Marmousi
Conclusions

Initial impressions of FWI



Second impression of FWI

FWI Cycle



v_0
a) Initial velocity model v_0
$k = 1$
1) Forward model through v_{k-1} to predict data $\psi_{r,k}$
b) Actual data ψ_r
$\psi_{r,k}$
ψ_r
$k = k + 1$
$\delta\psi_{r,k} = \psi_r - \psi_{r,k}$
4) Update velocity model $v_k = v_{k-1} + \delta v_k$
2) Migrate "data residual" with v_{k-1} and stack
v_k
3) "Calibrate" migration and deduce velocity perturbation δv_k
$\delta v_k = \lambda M^\dagger [\delta\psi_{r,k}]$
$M^\dagger [\delta\psi_{r,k}]$

Fundamental Theorem of FWI

Theorem (Tarantola, Lailly): Given real acoustic data and an approximate velocity model, then a linearized velocity model update, $dvel$, is given by

$$dvel(x,z) = \lambda \int \underbrace{\sum_s \sum_r \omega^2 \psi_s(x,z,\omega) \delta\psi^*_{r(s)}(x,z,\omega) d\omega}_{\text{A prestack migration}}$$

λ a scalar to be determined ("step length")

$\psi_s(x,z,\omega)$ monochromatic, forward propagated (downward continued), shot model

$\delta\psi_{r(s)}(x,z,\omega)$ monochromatic, reverse propagated (downward continued), data residual

Understanding the FTFWI

Interpretation: A prestack migration of the *data residual* is proportional to the desired update to the velocity model.

This is because the gradient of the data misfit function can be shown to be a type of prestack migration.

A cross correlation imaging condition arises naturally and there is a frequency squared factor. In the time domain this is a type of reverse –time migration (RTM).

$$\underbrace{\int \sum_{s,r} \omega^2 \psi_s(x,z,\omega) \delta\psi_r^*(x,z,\omega) d\omega}_{\text{Frequency domain}} \Rightarrow \underbrace{- \int \sum_{s,r} \partial_t \Psi_s(x,z,t) \partial_t \Psi_r(x,z,T-t) dt}_{\text{TIme domain}}$$

Understanding the FTFWI

Frequency dependence of λ :

- FTFWI assumes that the source wavelet is known.
- An unknown wavelet is equivalent to a complex-valued (i.e. amplitude and phase), frequency-dependent scalar.

So we write

$$dvel(x,z) = \int \lambda(\omega) \sum_s \sum_r \psi_s(x,z,\omega)\delta\psi_r^*(x,z,\omega)d\omega$$

where the factor ω^2 has been absorbed into λ

What kind of migration?

• The direct interpretation of the FTFWI requires a prestack reverse-time migration (RTM) using time-differentiated wavefields.

• However, experience suggests that all depth migrations can produce comparable results.

• With λ allowed to be complex and frequency-dependent, there seems no reason that a depth-stepping wave-equation migration (WEM) should not be used.

$$dvel(x, z) = \int \lambda(\omega) M^{\dagger}[\delta\psi_r(x, z, \omega)] d\omega$$

where $M^{\dagger}$ is a generalized migration operator and we expect that λ depends on both the source wavelet and the type of migration.

Calibrating the migration

FTFWI: Find the scalar λ such that

$$v_k(x,z) = v_{k-1} + \lambda M^\dagger[\delta\psi_k](x,z)$$

produces the best forward modelled data.

Standard practice finds λ in a 1D search called a "line search".

Calibrating the migration

This paper: Find the scalar λ such that

$$\int \left(v_{true}(x_w, z) - v_k(x_w, z)\right)^2 dz = \min$$

where x_w is the well location at which v_{true} is known.

Defining the velocity residual $\delta v_k(x_w, z) = v_{true}(x_w, z) - v_{k-1}(x_w, z)$

$$\int \left(\delta v_k(x_w, z) - \lambda(\omega) M^\dagger[\delta\psi_k](x, z, \omega)\right)^2 dz = \min$$

So we will match the migrated data residual to the velocity residual at the well. This is a process very similar to standard impedance inversion.

Calibrating the migration

Our explicit calibration procedure is:

1. Convolve the migrated stack with a Gaussian smoother
2. Determine the best (least squares) scalar to match the smoothed migrated trace at the well to the residual velocity at the well.
3. Determine the best (least squares) constant-phase rotation to match the scaled, smoothed, migrated trace to the residual velocity at the well.
4. Apply the amplitude scalar and phase rotation to the entire smoothed stack to estimate $dvel$.

Many other, more sophisticated calibration methods are possible.

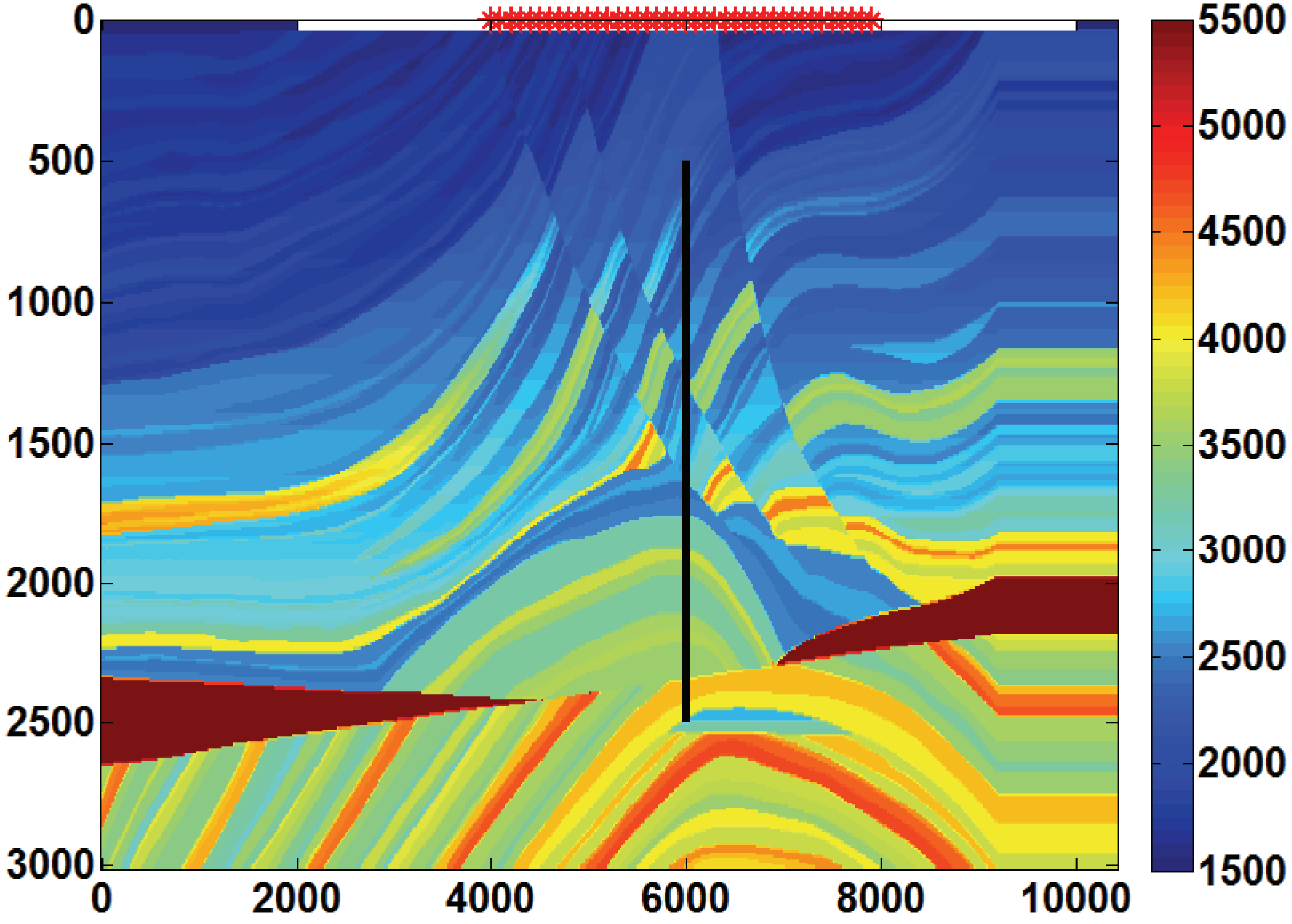
Numerical Experiment

Using data created from the Marmousi model, we implemented the four steps of FWI as:

1. Modelling using acoustic finite difference tools in Matlab.
2. Migration of data residual using PSPI in Matlab.
3. Calibration by matching to data residual to velocity residual at well using least-squares amplitude and constant-phase rotations.
4. Update using addition.

Marmousi Model

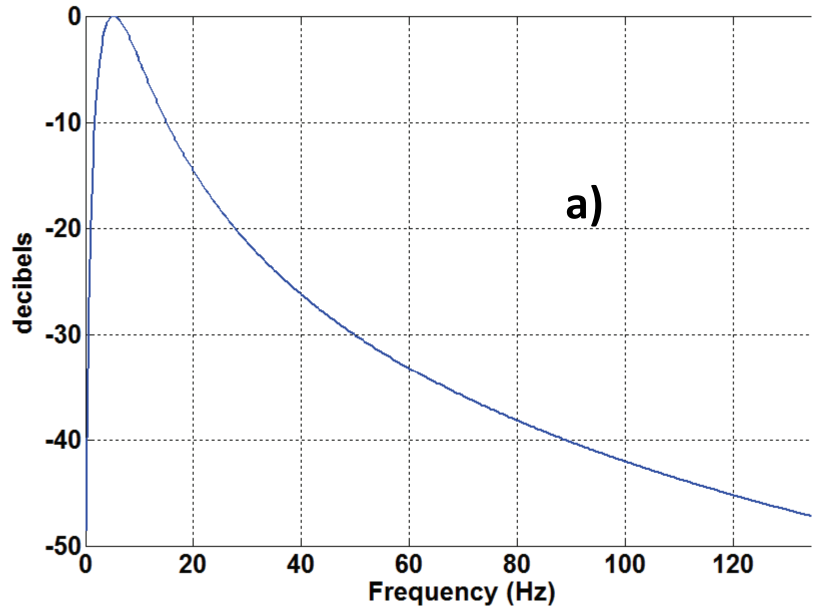
with shots (red), receivers (white), and well (black)



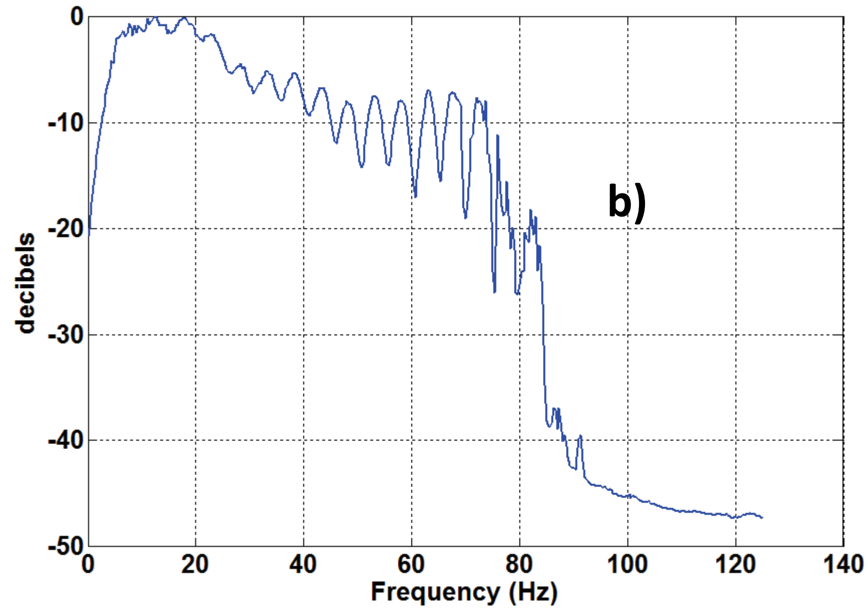
40 shots, split spread, far offset 2000 m.

Modelling

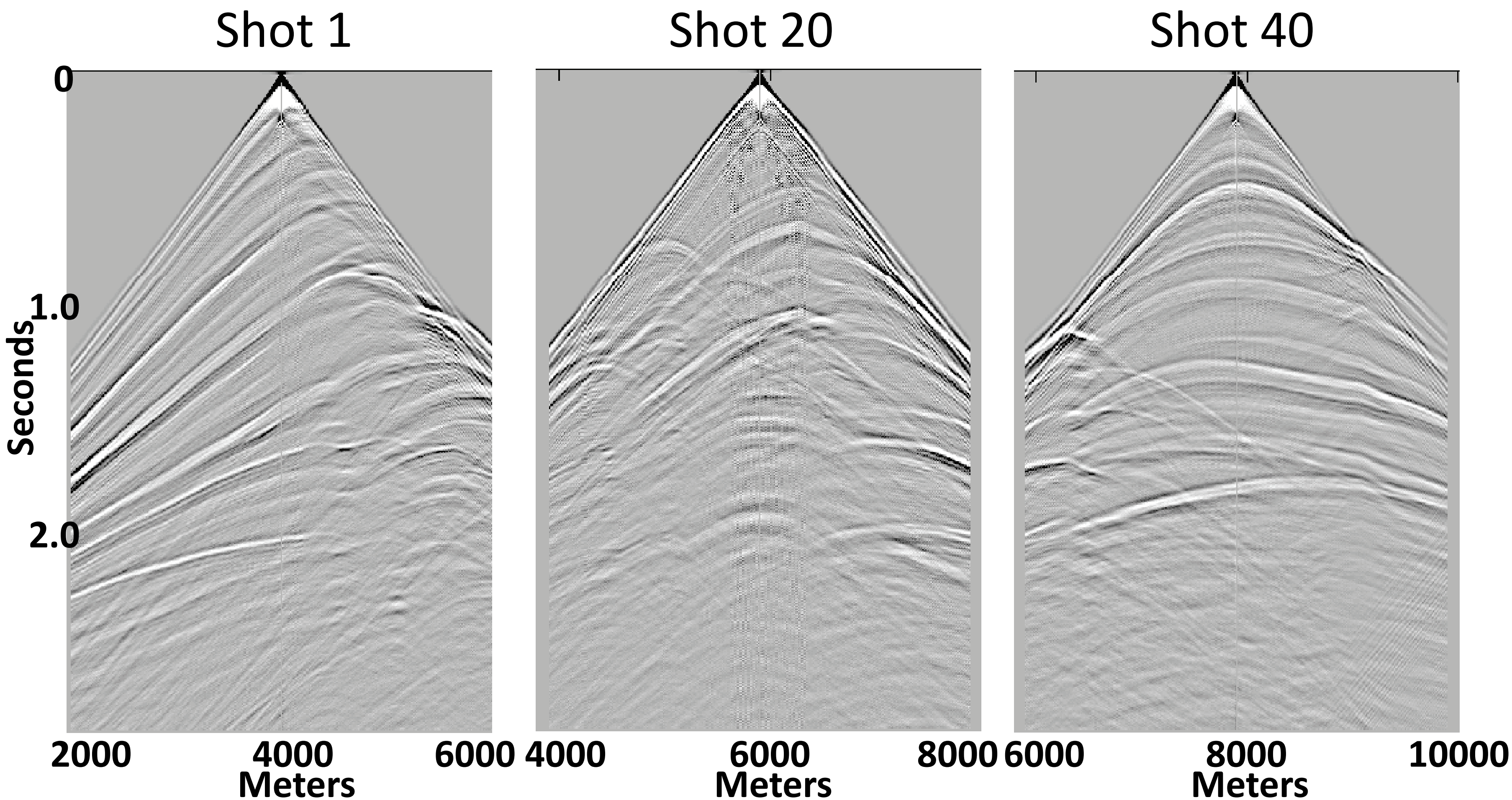
Spectrum of source wavelet (5 Hz dominant)



Spectrum of modelled data

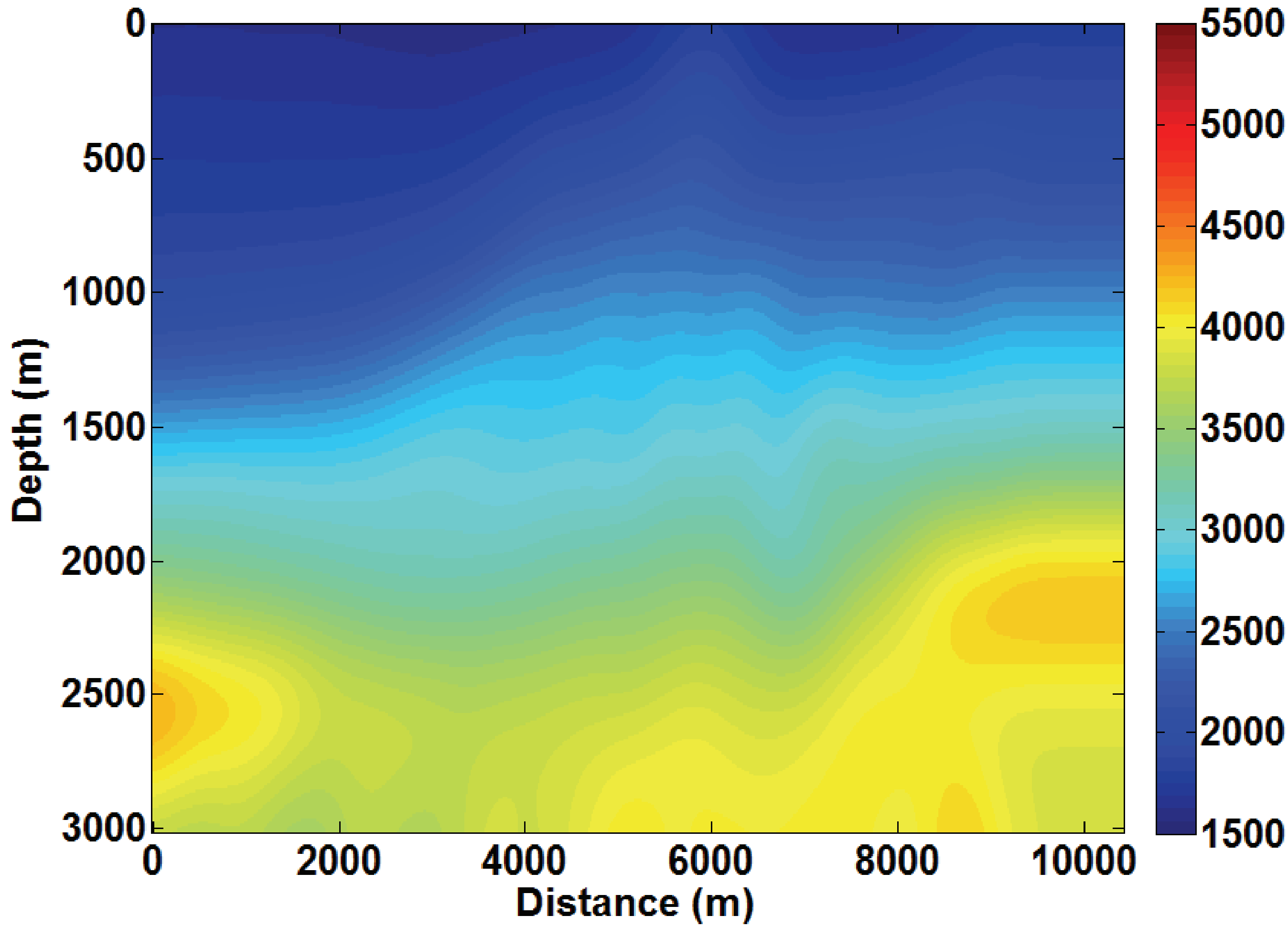


Sample Shots

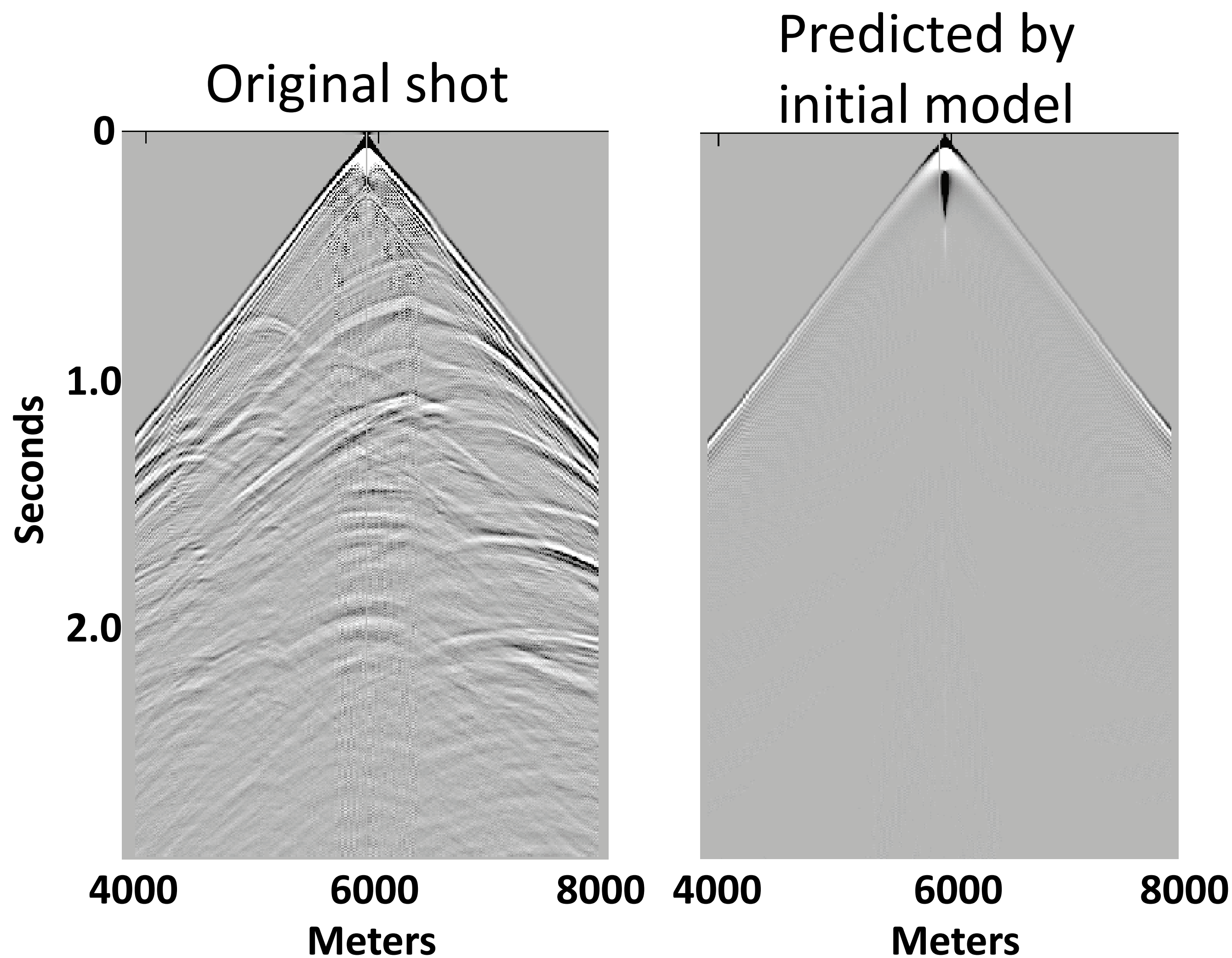


Initial Velocity Model

Gaussian smoother 580m width

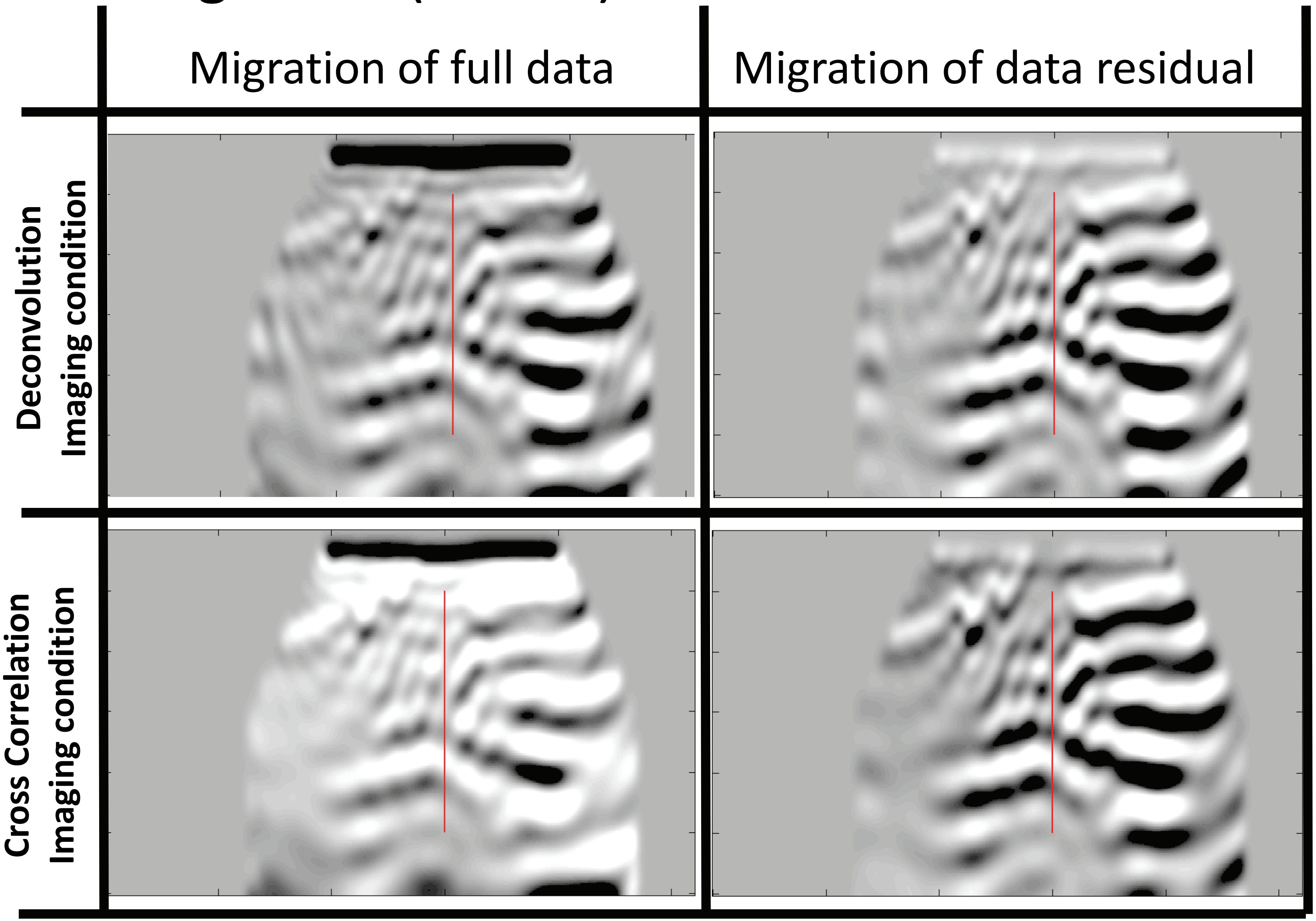


Shot 20



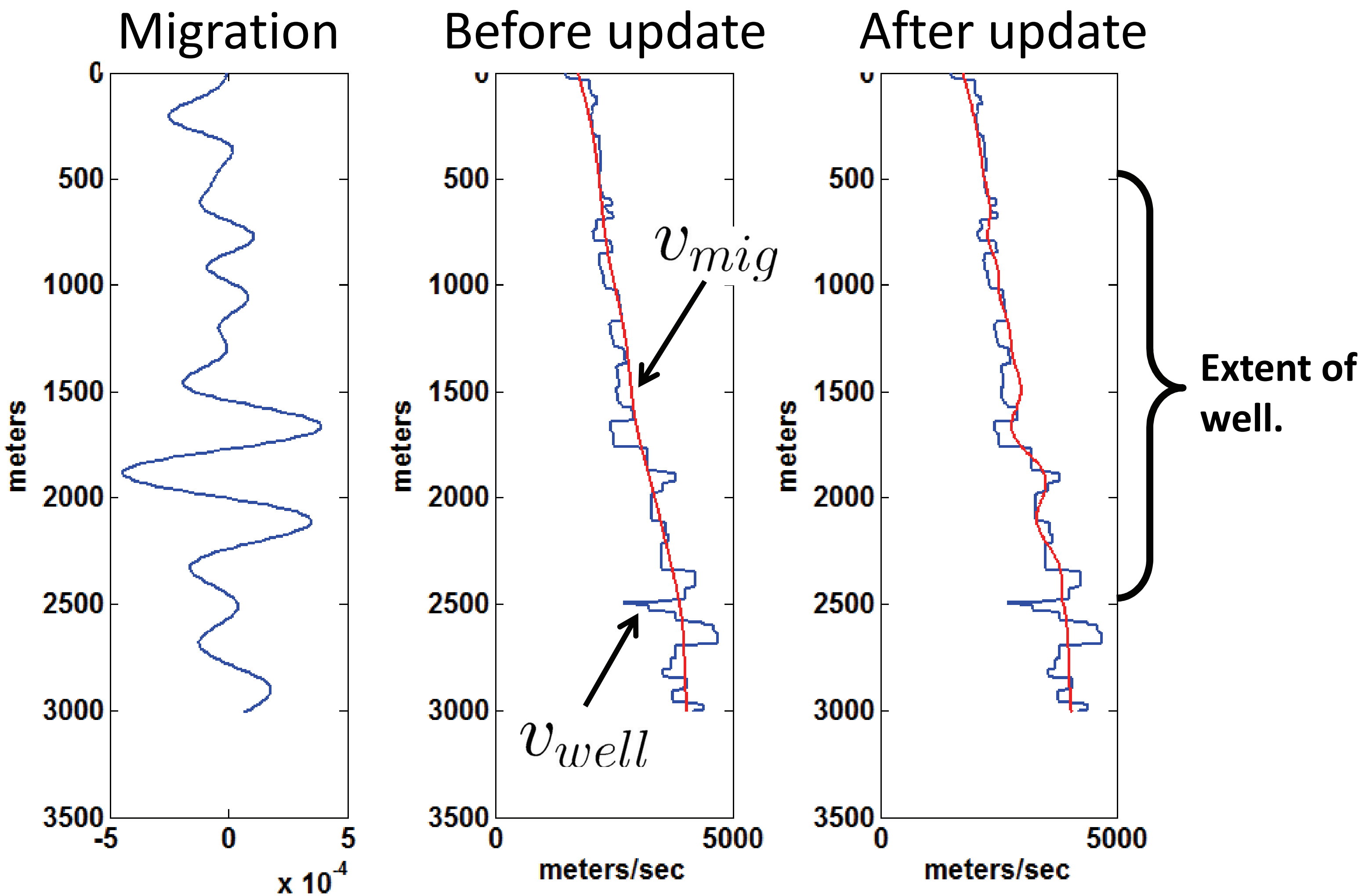
Original shot
Predicted by
initial model
0
1.0
2.0
Seconds
4000
6000
8000
Meters
4000
6000
8000
Meters

First Migration (0-5 Hz) with and without modelling



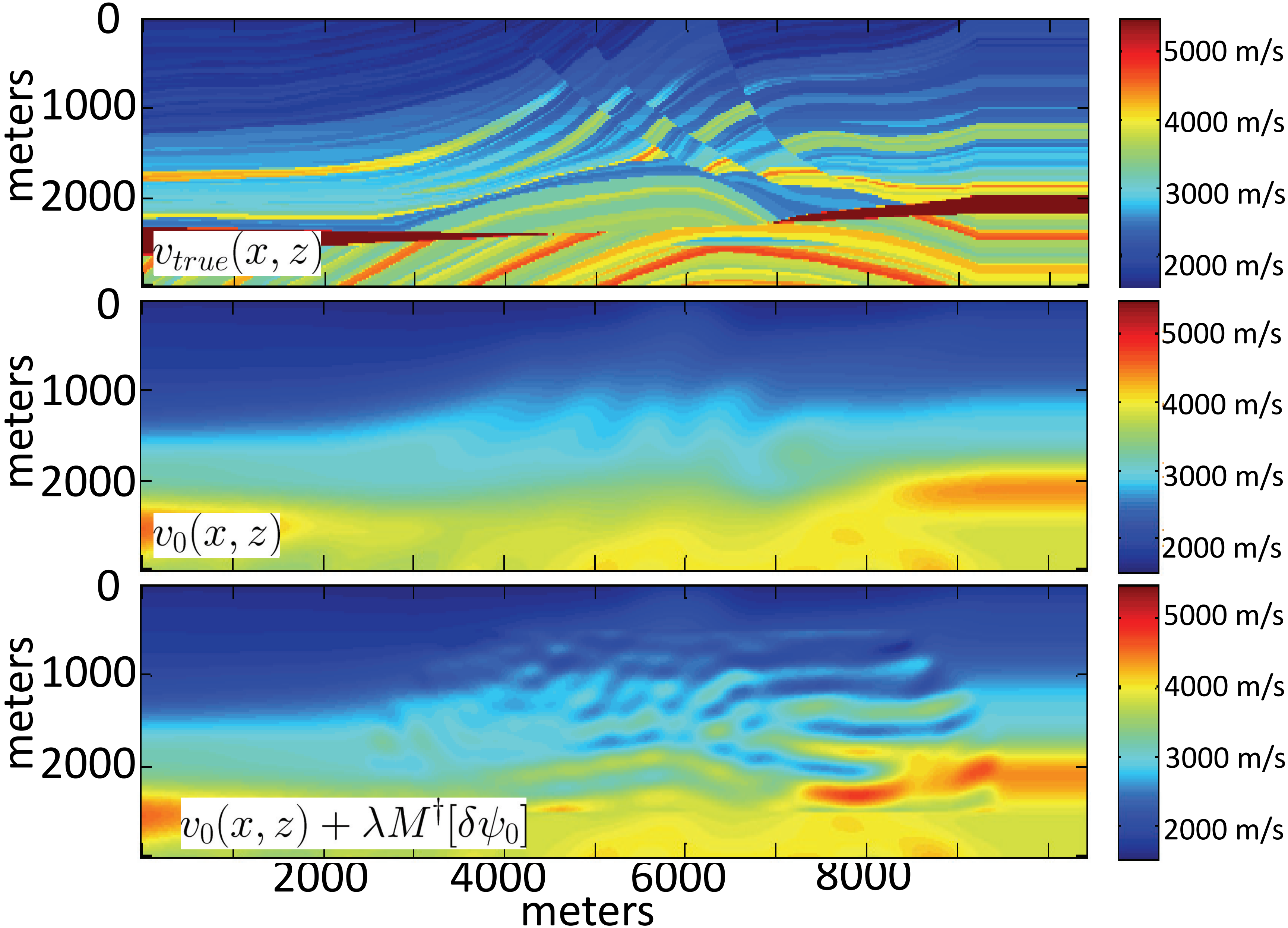
Calibration at the well

First iteration 0-5 Hz.

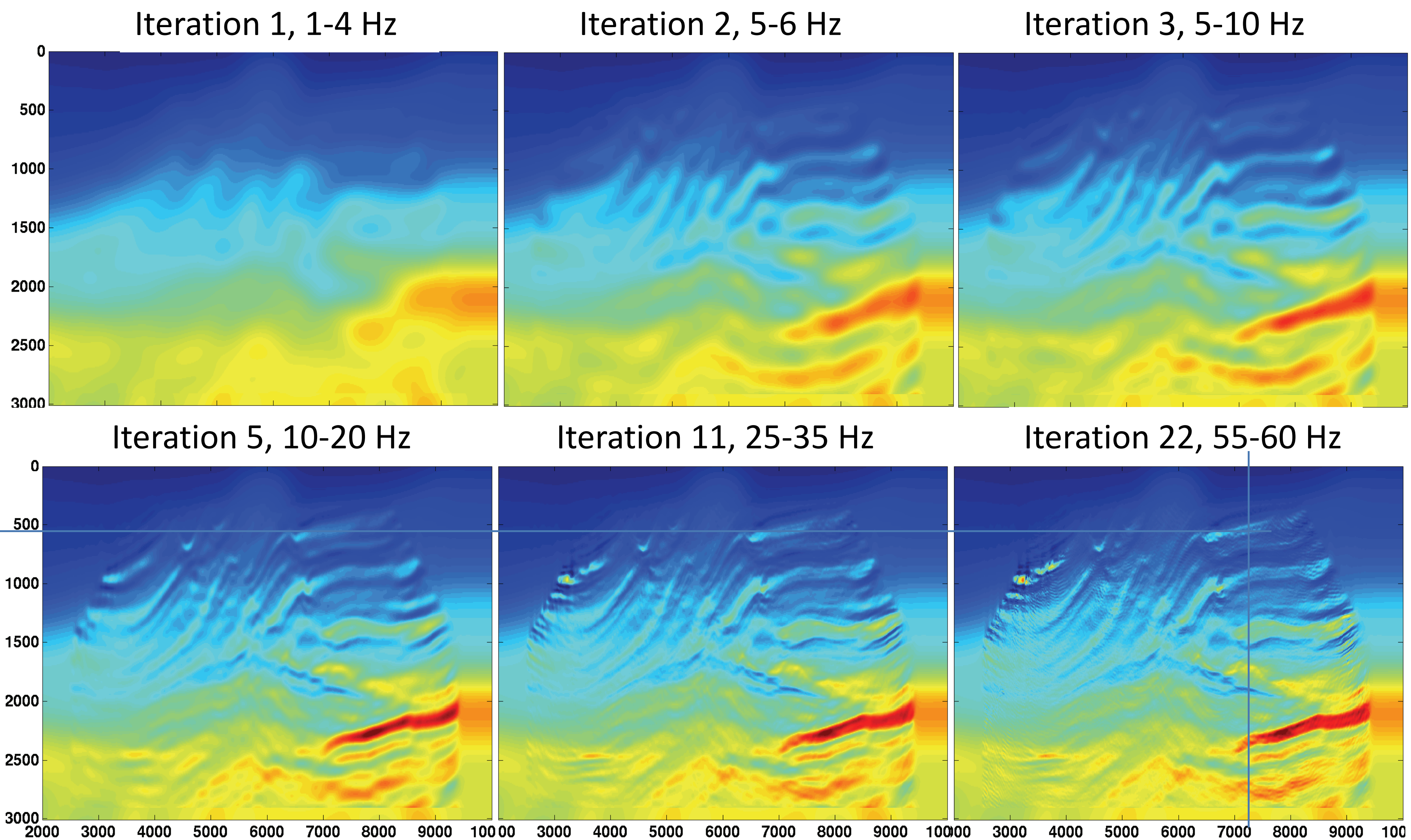


Blue is the exact velocity at the well. Red is the migration velocity.

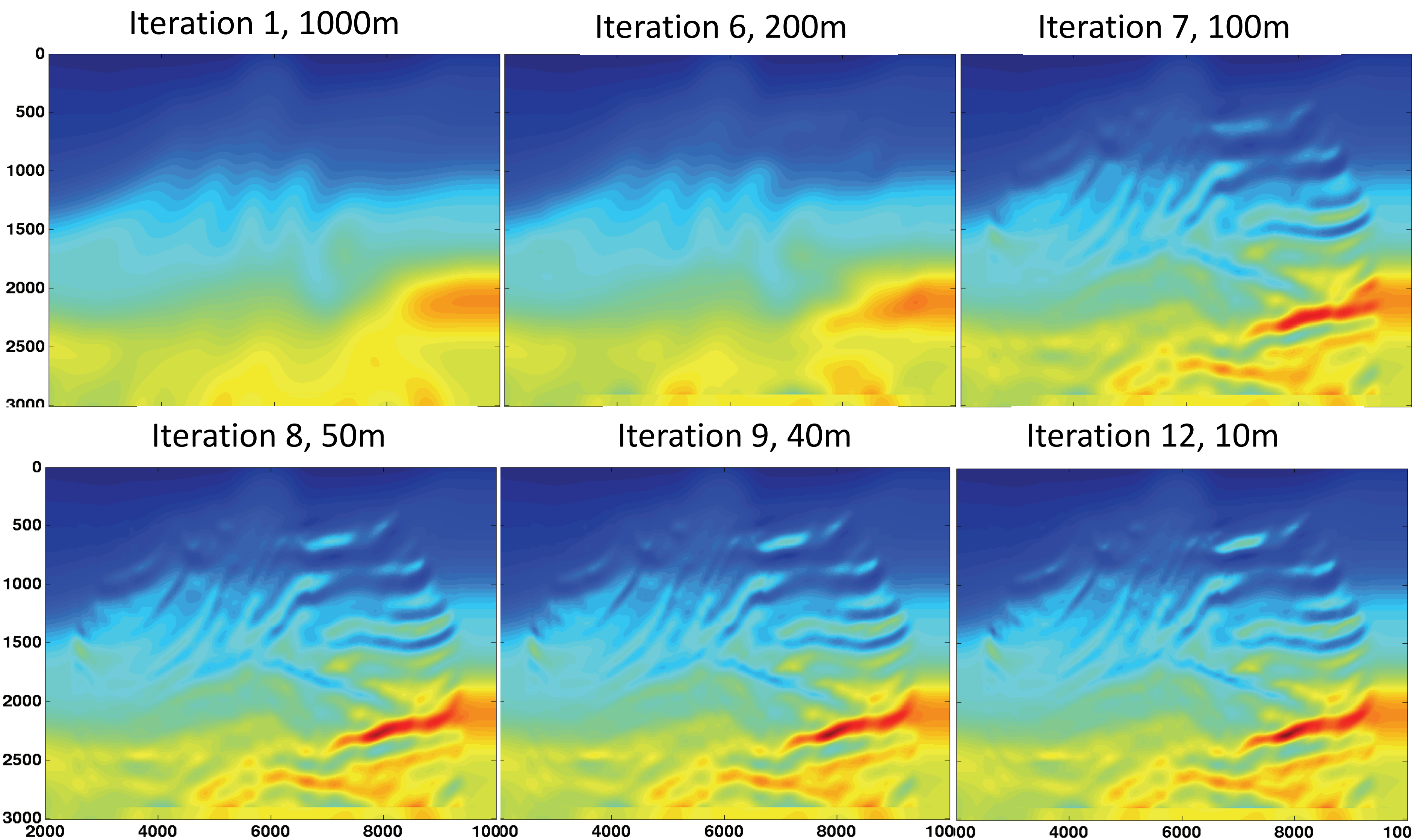
After 1 Iteration 0-5 Hz



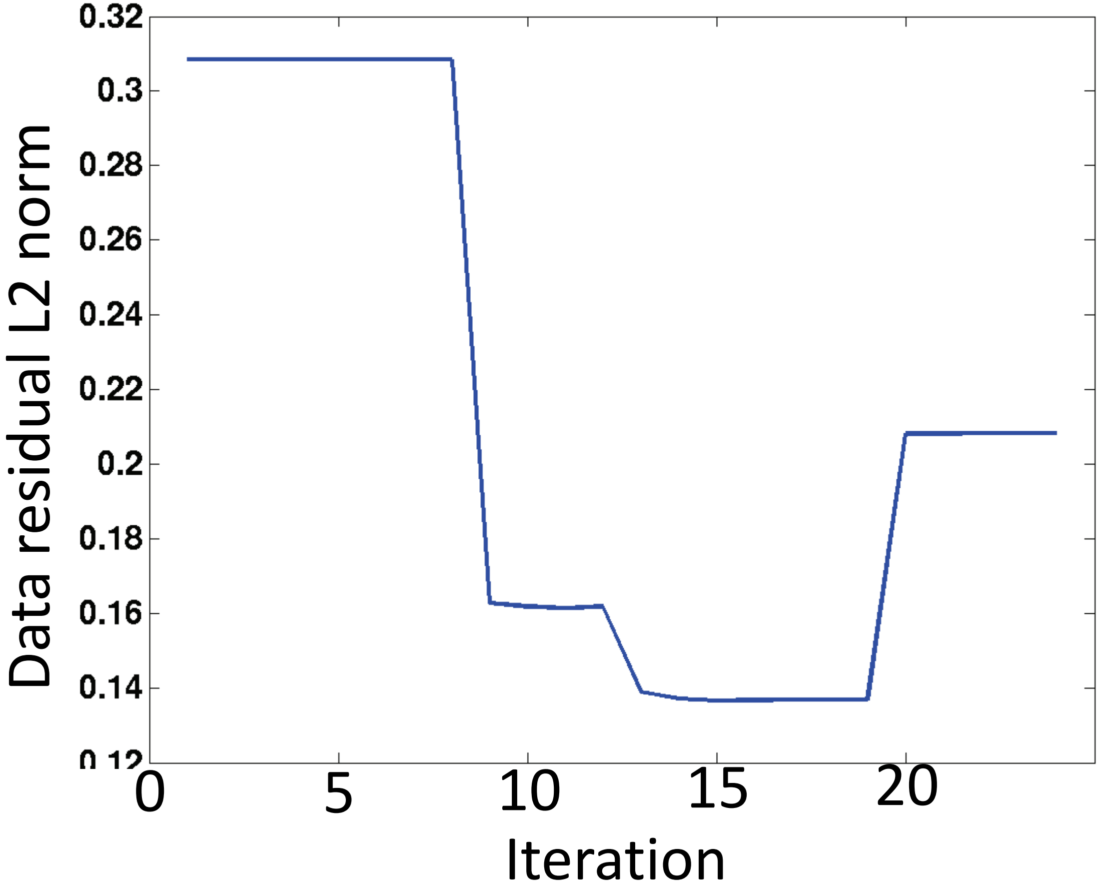
Frequency Iteration



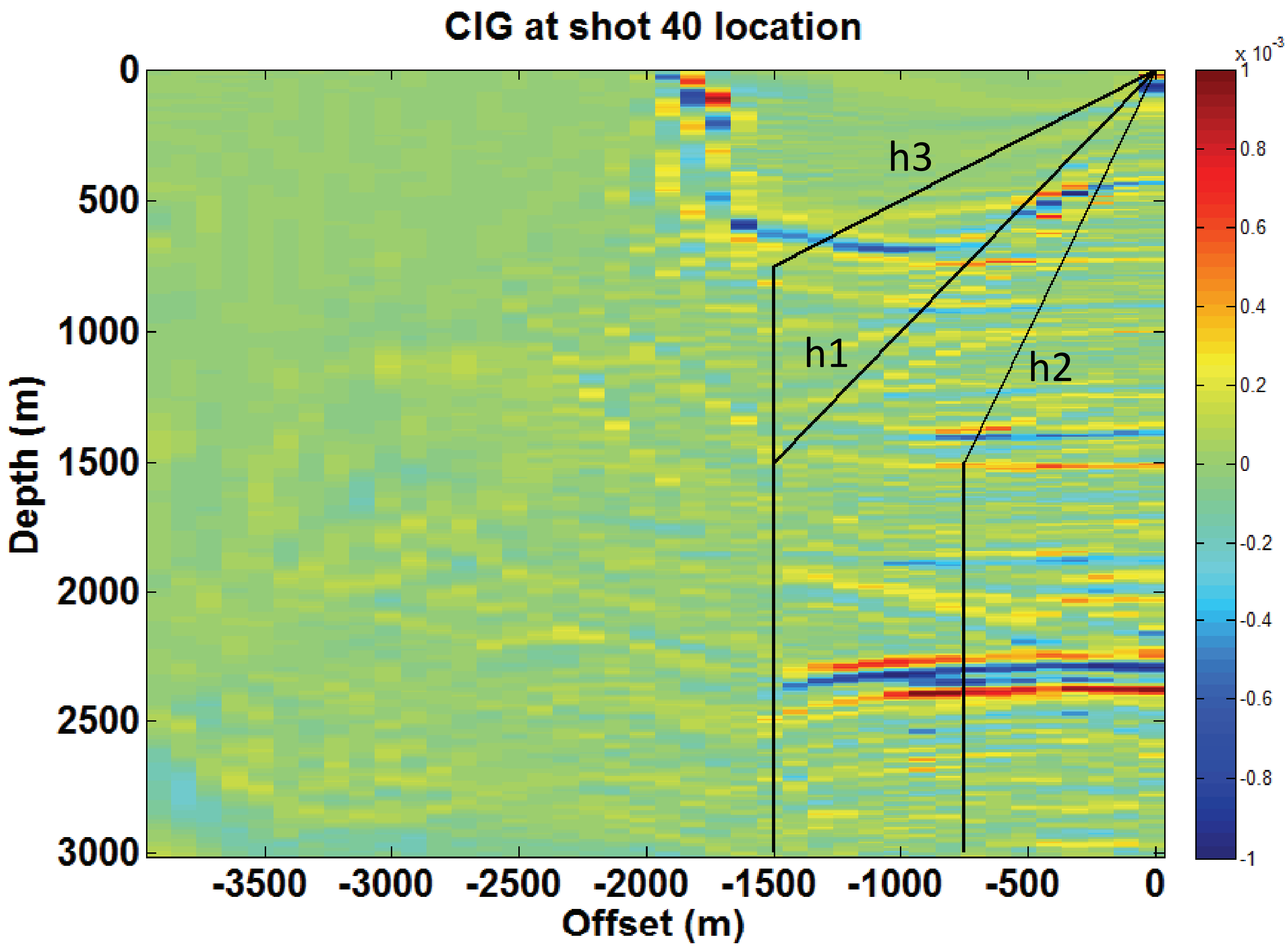
Smoother Iteration 1-40 Hz normal mute



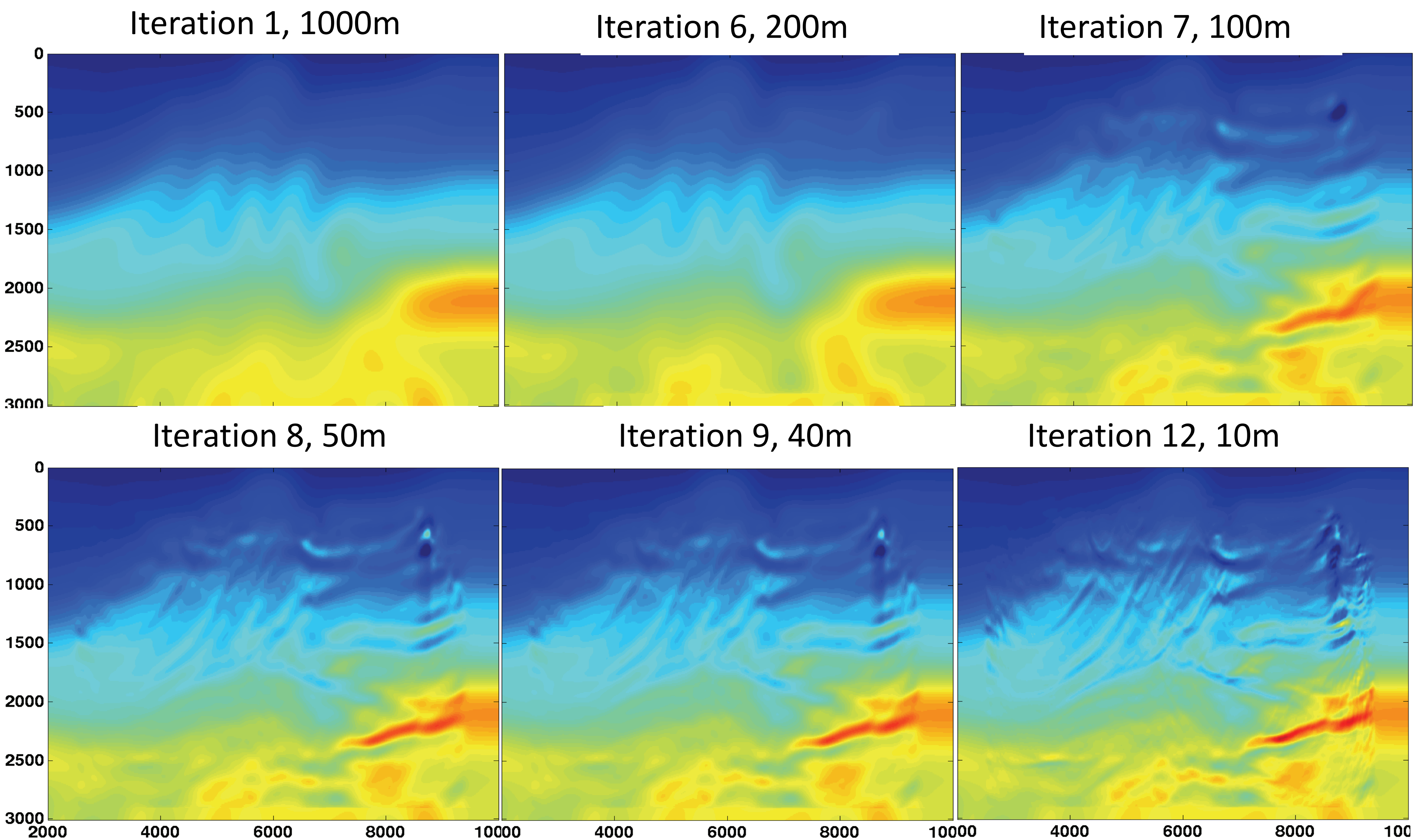
Data Residual L2 Norms



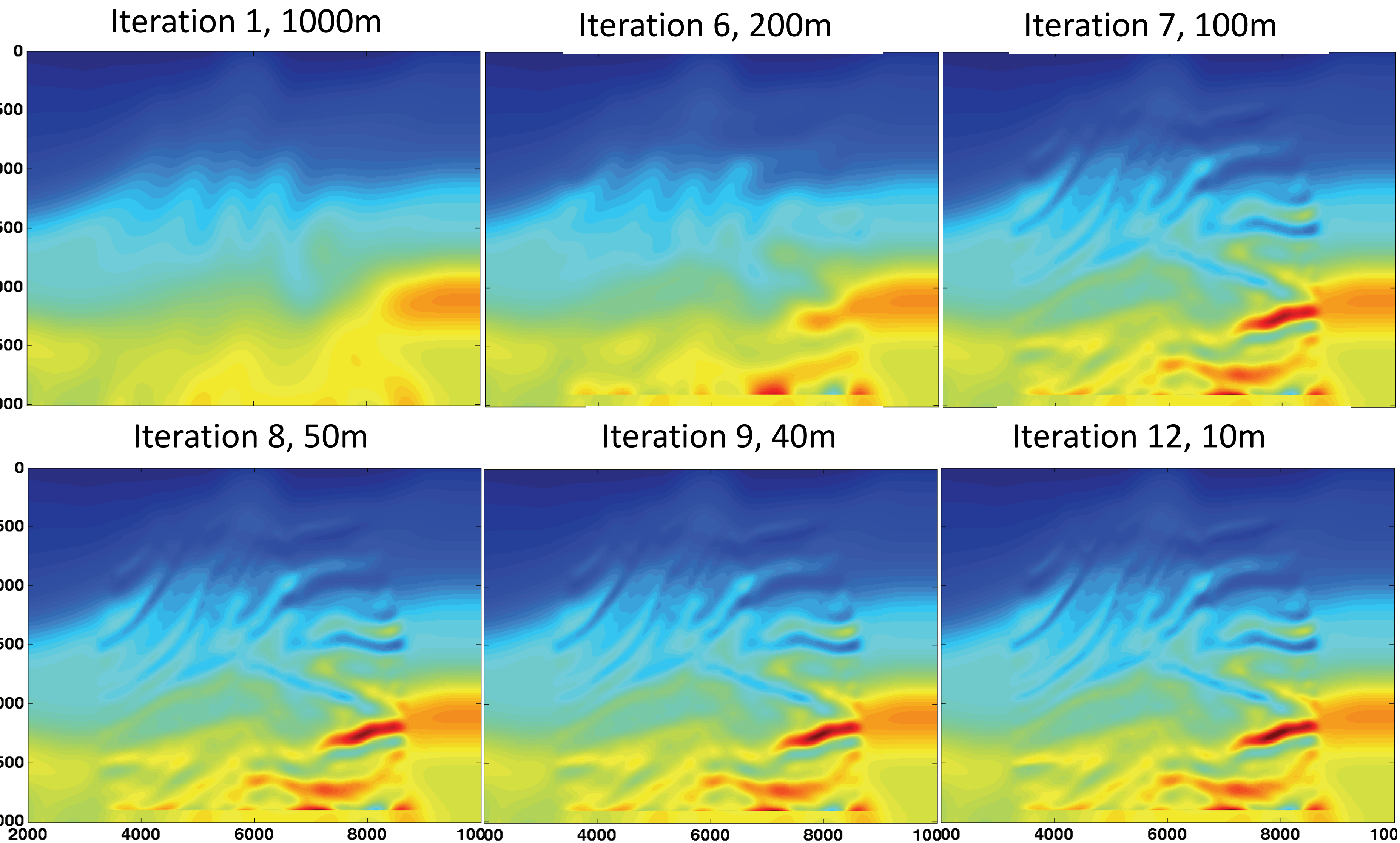
Common Image Gather



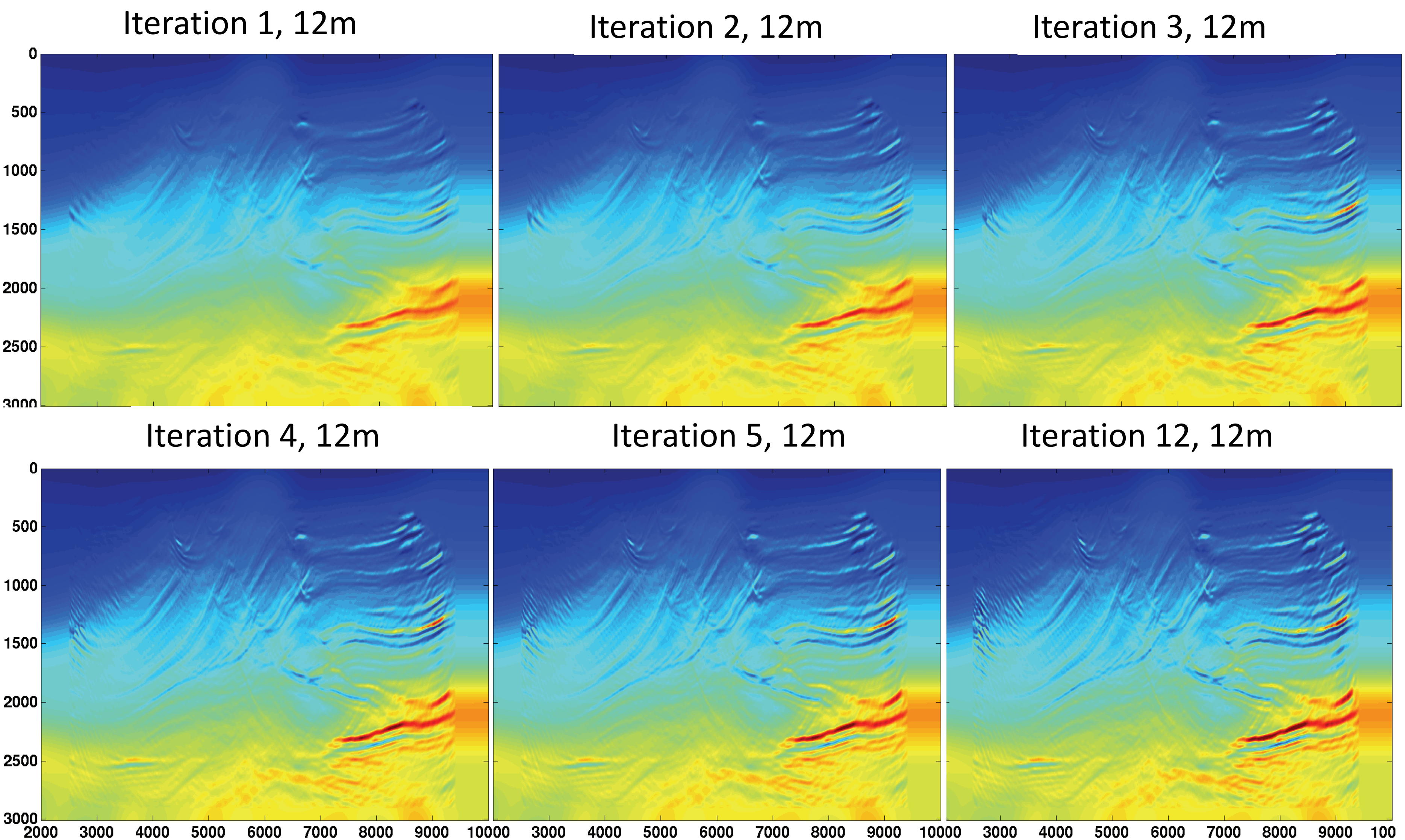
Smoother Iteration 1-40 Hz wide mute



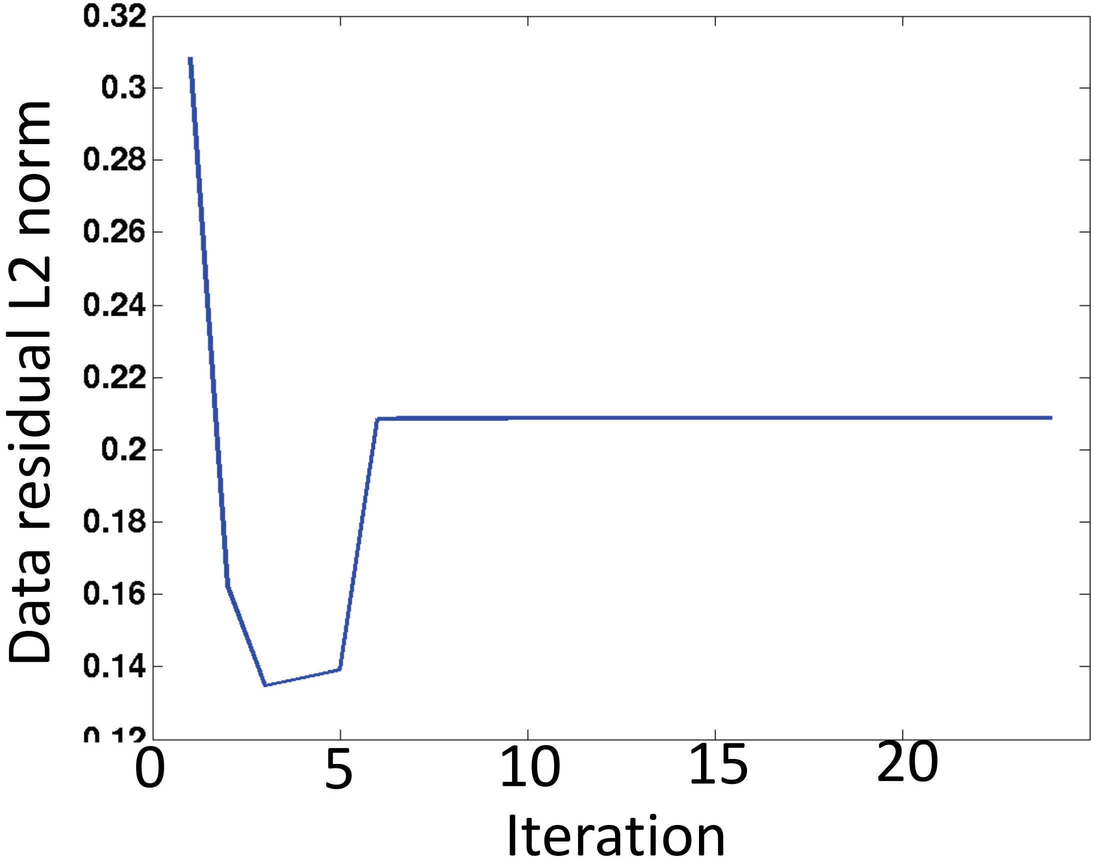
Smoother Iteration 1-40 Hz harsh mute



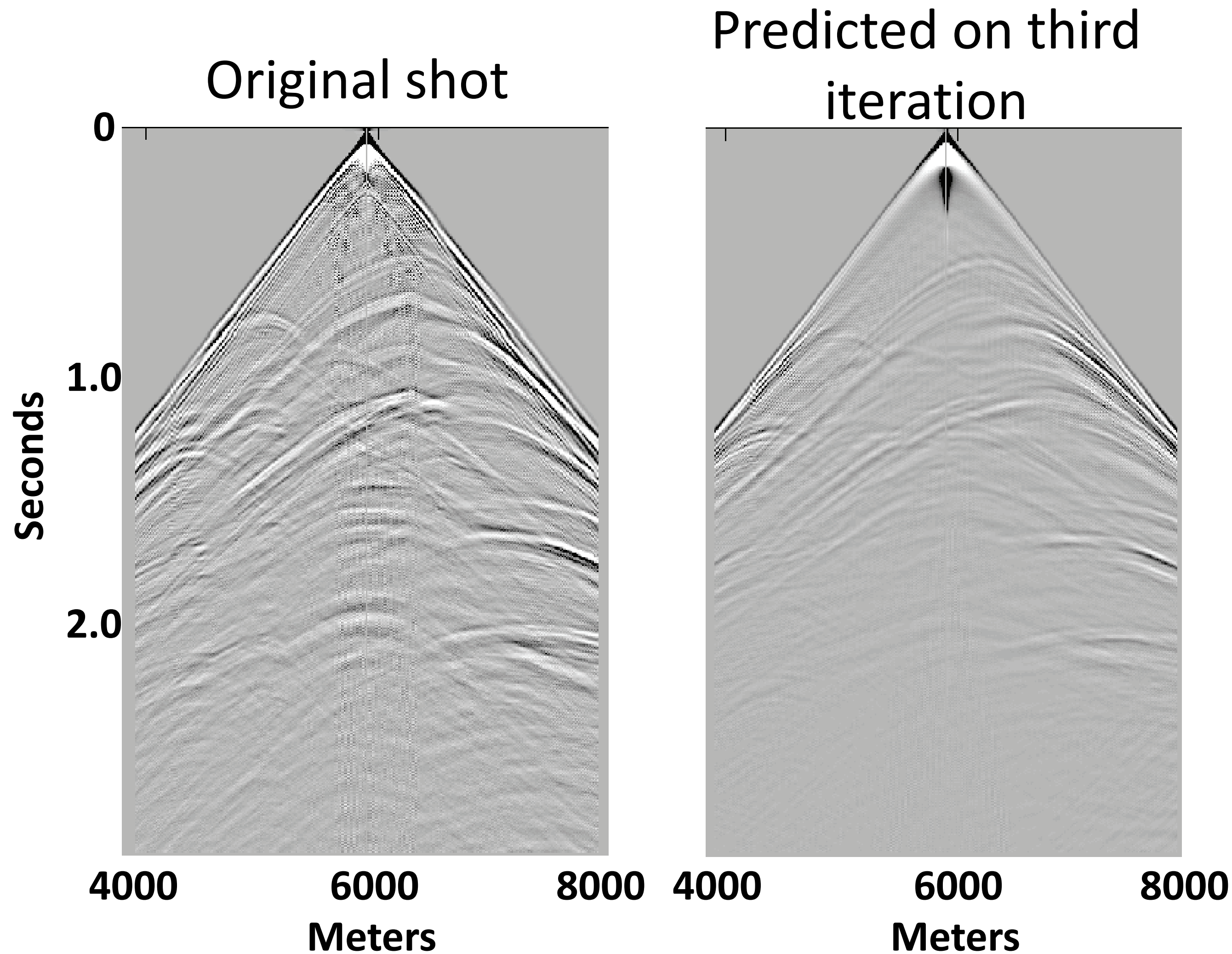
1-40 Hz Wide Mute, smoother fixed at 12 m



Data Residual L2 Norms

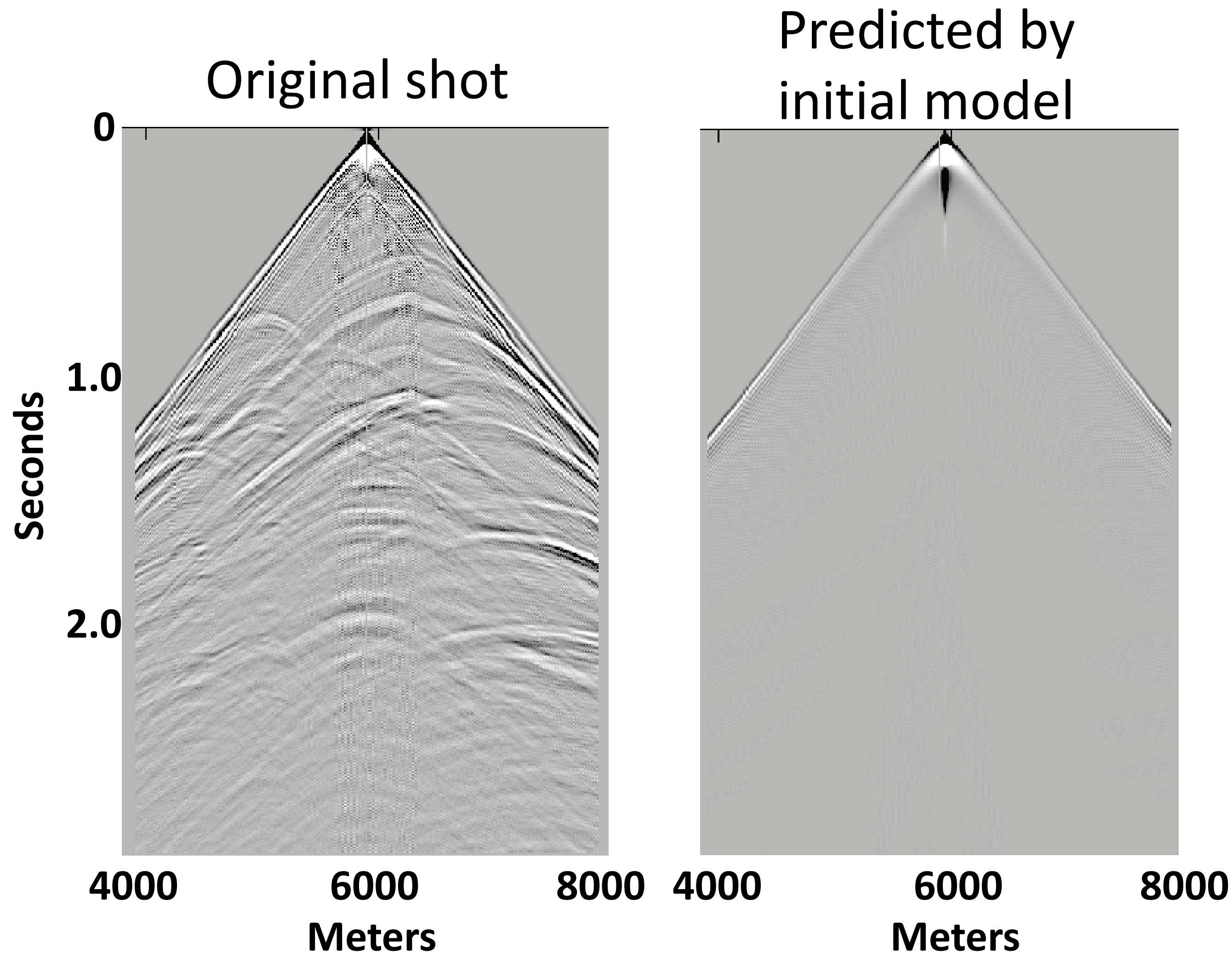


Shot 20



Original shot
Predicted on third iteration
0
1.0
2.0
Seconds
4000
6000
8000
Meters
4000
6000
8000
Meters

Shot 20


Original shot
Predicted by initial model
0
1.0
2.0
Seconds
4000
6000
8000
Meters
4000
6000
8000
Meters

Conclusions

The FTFWI suggests a generalized inversion scheme with many possible variations.

FWI is an iterative modelling, migration, and calibration process.

FWI can be done with migration algorithms other than RTM.

Calibration generally involves a wavelet estimation and scaling, and is reminiscent of impedance inversion.

The process demonstrated here is not claimed to be optimal, but is suggestive of many other variants.

Acknowledgements

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mitacs

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Calibrating the migration

FTFWI: Find the scalar λ such that

$$v_k(x,z) = v_{k-1} + \lambda M^\dagger[\delta\psi_k](x,z)$$

produces the best forward modelled data.

This paper: Find the scalar λ such that

$$\int \left(v_{true}(x_w,z) - v_k(x_w,z)\right)^2 dz = \min$$

where x_w is the well location at which v_{true} is known.

Calibrating the migration

Matching condition at the well

$$\int \left(v_{true}(x_w, z) - v_k(x_w, z)\right)^2 dz = \min$$

Substituting for v_k

$$\int \Big(v_{true}(x_w, z) - v_{k-1} - \lambda M^{\dagger}[\delta\psi_k](x, z)\Big)^2 dz = \min$$

Defining the velocity residual

$$\int \Big(\delta v_k(x_w, z) - \lambda M^{\dagger}[\delta\psi_k](x, z)\Big)^2 dz = \min$$

So we will match the migrated data residual to the velocity residual at the well. This is a process very similar to standard impedance inversion.

Weird