A brief comparison of the frequency spectra from the Hussar 2011 and Priddis 2012 shoots and the theoretical predictions of the Sharpe Hollow Cavity Model (SHCM)

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Outline

I A brief introduction and derivation of the Sharpe Hollow Cavity Model

II Exploring the effects of certain variables in the SHCM which include:
- The choice of pressure pulse
- Depth
- Offset
- Cavity radius
- Rigidity

III Investigating the frequency content of the data obtained in the Hussar 2011 and Priddis 2012 experiments

IV Comparison of the frequency spectra from both experiments with the predictions of the SHCM

V Attempt to draw some conclusions regarding the validity of the SHCM

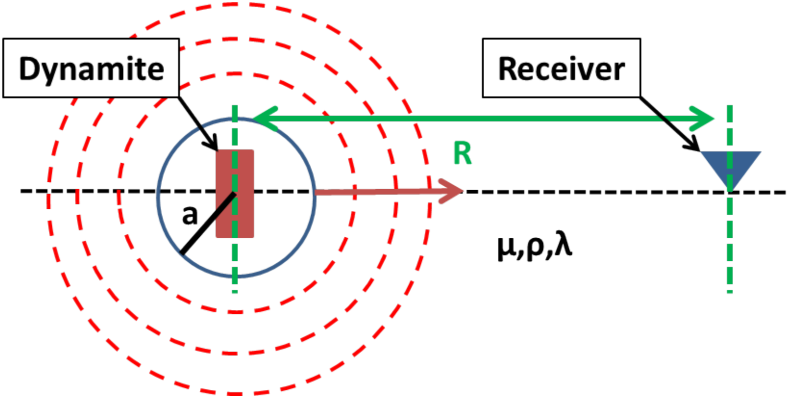
Introduction

- Dynamite is a cheap, commonly used means of elastic wave production in exploration seismology
- Understanding the nature of dynamite explosions and the resulting waves can result in vast improvements in surveys that utilize dynamite
- The nature of propagating waves near the source of a dynamite explosion is a poorly understood phenomenon in Physics due to the nonlinear nature of the subsurface in close proximity to the explosion
- There are numerous mathematical models that have been developed which attempt to account for this nonlinear behavior and predict wave behavior in the far-field
- In this study, we investigate the viability of of the SHCM in predicting the behavior of compressional waves emitted from dynamite

The Sharpe Hollow Cavity Model

Theorem

An explosive pressure source can be modeled by a hollow cavity that is being acted on from the inside by a uniformly distributed pressure pulse.



The Sharpe Hollow Cavity Model

- According to Sharpe, the area inside the cavity represents the region in which emitted waves do not behave linearly
- Compressional waves are assumed to emanate directly from the outside wall of the cavity in a spherical form
- The size of the cavity is directly proportional to the charge size

Derivation of the Sharpe Hollow Cavity Model

By exploiting the spherical symmetry of the proposed problem, the wave equation can be written as:

$$\frac{\partial^2 (r\Phi)}{\partial t^2} = v^2 \frac{\partial^2 (r\Phi)}{\partial r^2}. \tag{1}$$

The solution, Φ, in this case must be both divergent and decreasing with time in order to represent a spherical waveform. Sharpe proposed the following:

$$\Phi = \frac{1}{r} e^{-int} \tag{2}$$

where t is time, and r is the distance from the source.

Using this solution for Φ and solving for the displacement, ie, $u = \partial\Phi/\partial r$, results in the displacement equation:

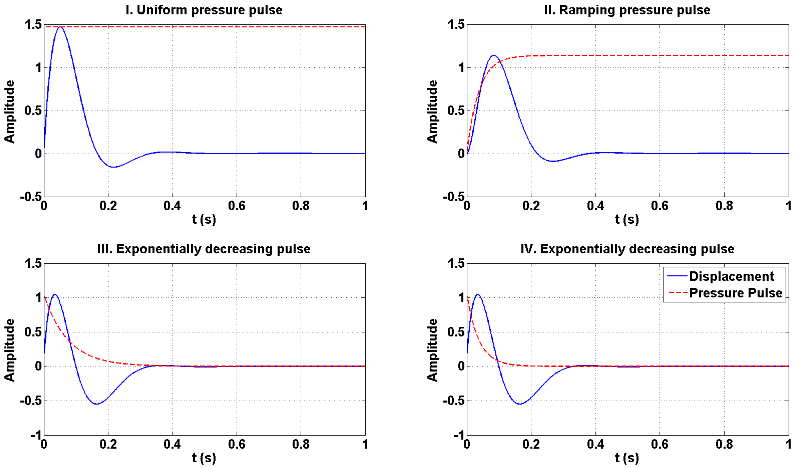
$$u = \frac{a^2 p_o}{2\sqrt{2}\mu r} e^{-\omega t/\sqrt{2}} \sin \omega t, \tag{3}$$

where r is the distance from the center of the source, a is the cavity radius, μ is the medium rigidity, p_o is a uniform pressure pulse, and ω is the angular frequency of the oscillating solution represented by

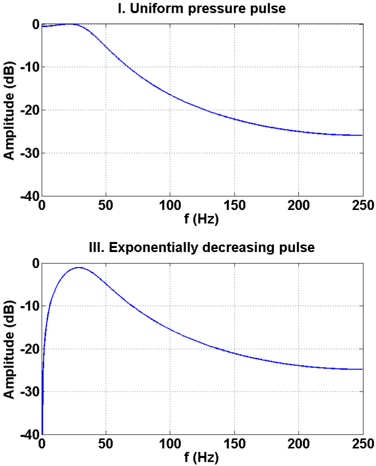
$$\omega = \frac{2\sqrt{2}v}{3a}, \tag{4}$$

where v is the velocity of the p-waves emitted by the source. Assigning different pressure pulses to the cavity can be accomplished via convolution of the desired pressure pulse with that of the displacement shown above.

Pressure pulse forms



Pressure pulse forms



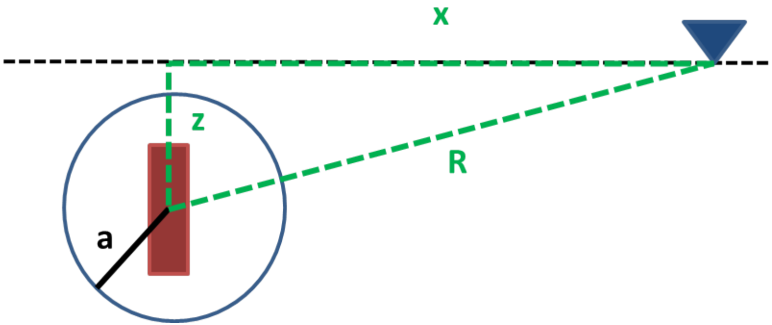
Predictions of the SHCM

$$u = \frac{a^2 p_o}{2\sqrt{2}\mu r} e^{-\omega t/\sqrt{2}} \sin \omega t, \tag{5}$$

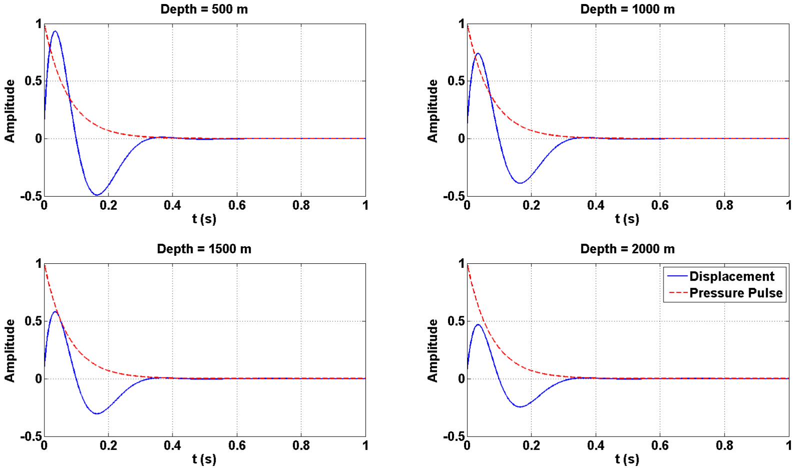
$$\omega = \frac{2\sqrt{2}v}{3a}, \tag{6}$$

- Amplitude response should INCREASE with larger charge sizes
- Dominant frequency should DECREASE with increased charge size
- Amplitude should DECREASE with increased rigidity
- A low frequency roll off should be present if in fact a decreasing exponential best represents a dynamite explosion

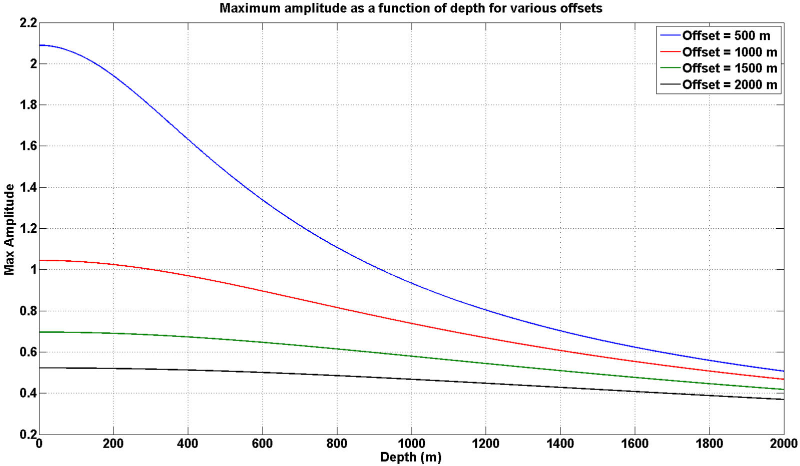
The role of charge depth



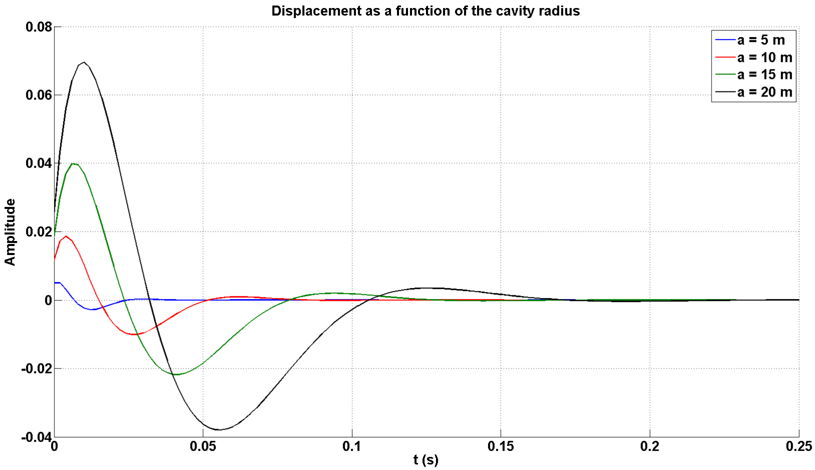
The effect of charge depth on displacement



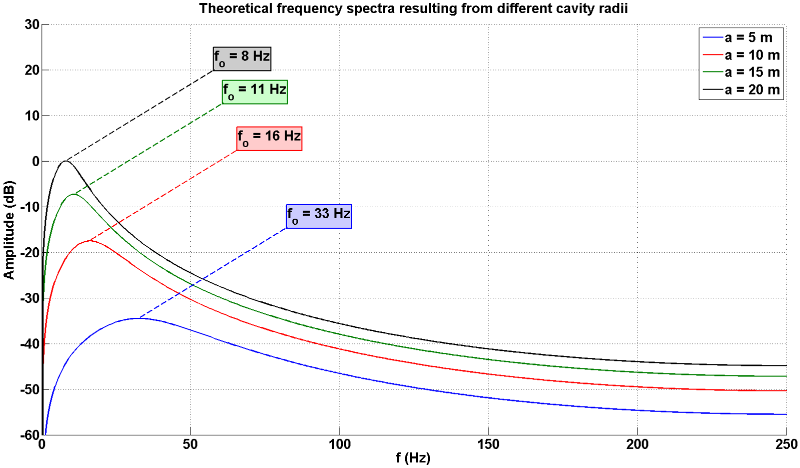
The effect of charge depth and offset on displacement



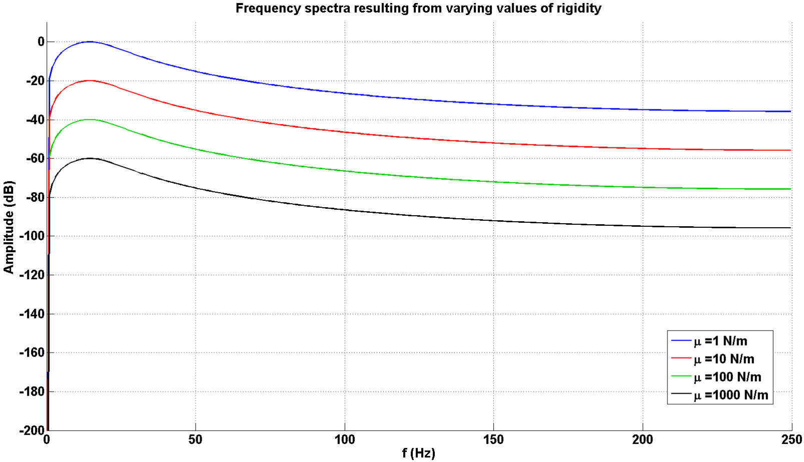
The effect of cavity radius on displacement



The effect of cavity radius on the frequency spectra



The effect of medium rigidity on the frequency spectra



Charge size and cavity radius

In previous studies, we found that charge size is related to cavity radius via a cubic relationship such that

$$m = ca^3 \tag{7}$$

where m is the charge size in kilograms, a is the cavity radius in meters, and c is the constant in kilograms per meter cubed that links them. If we assume the dominant frequency is given by

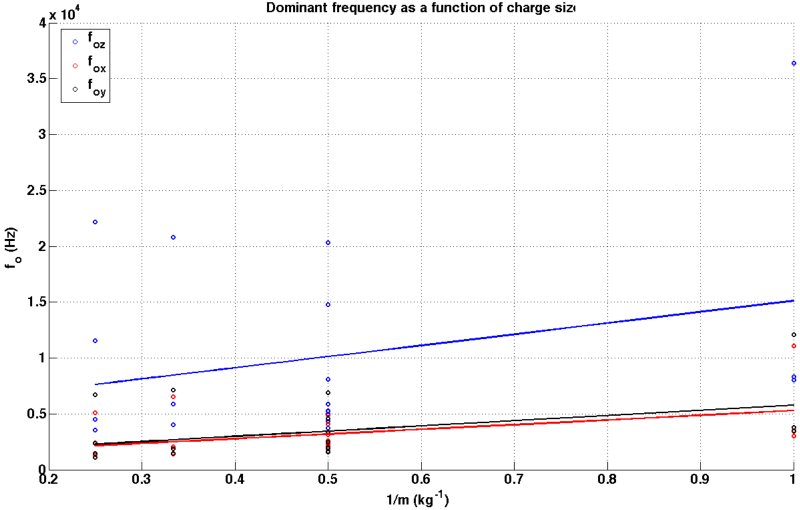
$$f_o = \frac{\omega}{2\pi} = \frac{\sqrt{2}v}{3a} \tag{8}$$

Then we can use Equation 7 with Equation 8 to obtain the following:

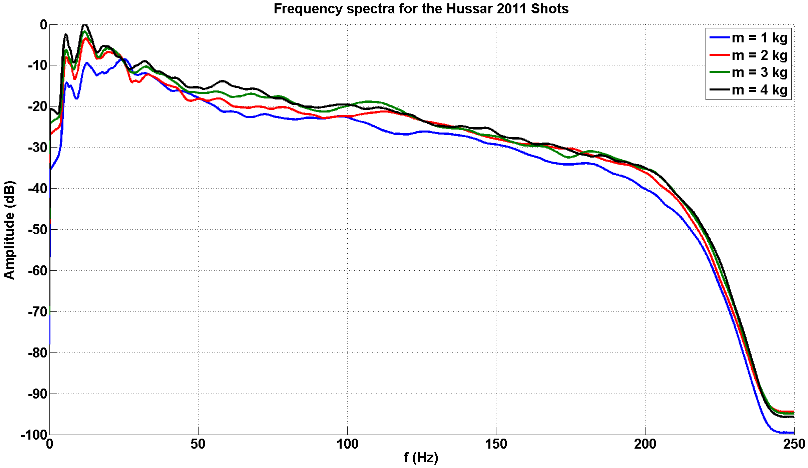
$${f_o}^3 = c\left(\frac{\sqrt{2}v}{3}\right)^3 \frac{1}{m} \tag{9}$$

where v is the wave velocity in meters per second. From this, we should be able to estimate c from our data.

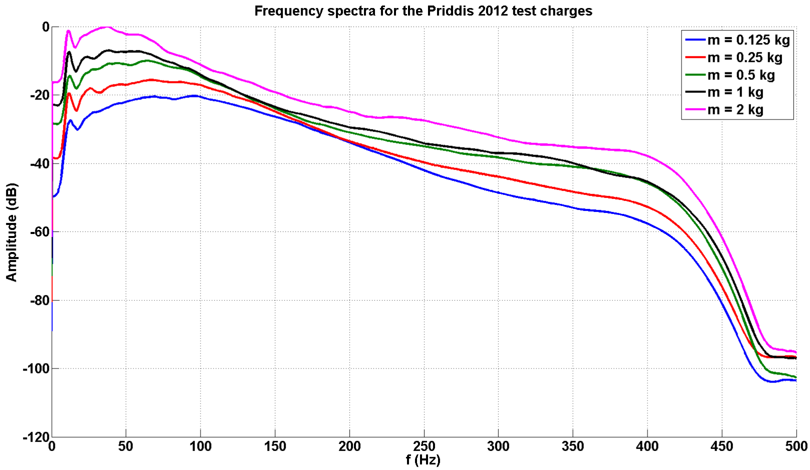
Estimating the value of c in Hussar



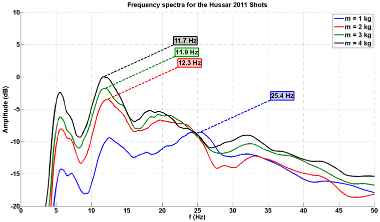
Data from the Hussar 2011 experiment

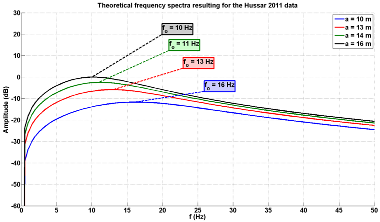


Data from the Priddis 2012 experiment



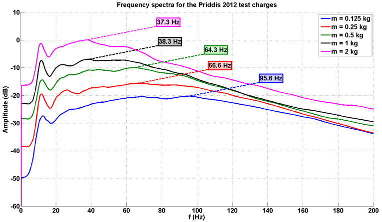
Hussar comparison with the SHCM

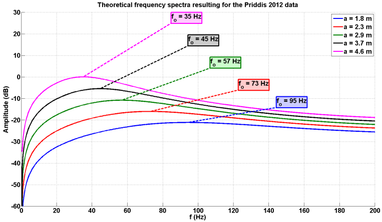




$c = 0.001\ kg/m^3$				
	Measured Data		*Theoretical*	
m (kg)	**fo (Hz)**	**A (dB)**	**fo (Hz)**	**A (dB)**
1	25.4	-8	16	-11
2	12.3	-4	13	-7
3	11.9	-3	11	-3
4	11.7	0	10	0

Hussar comparison with the SHCM





$c = 0.015\ kg/m^3$				
	Measured Data		*Theoretical*	
m (kg)	**fo (Hz)**	**A (dB)**	**fo (Hz)**	**A (dB)**
0.125	95.6	-20	95	-20
0.25	66.6	-17	73	-17
0.5	64.3	-10	57	-10
1	38.3	-8	43	-7
2	37.3	0	35	0

Conclusions

- The SHCM is a viable model for predicting that nature of waves emitted from a dynamite explosion
- A decreasing exponential represents a reasonable physical model for the pressure pulse of a dynamite explosion due to the observation of the low-frequency roll-off in both the data and the theory
- Dominant frequency decreases with increased charge size
- Amplitude response is larger for bigger charge sizes
- Using smaller charge sizes results in a loss in high frequency content
- We need to determine a more accurate link between charge size and cavity radius to move forward with this model
- We may be able to improve on this model using numerical rather than analytical techniques since we're not constrained by certain assumptions solving this problem numerically

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