Viscoelastic scattering potentials and full waveform inversion sensitivities

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Outline

- Motivations: Scattering and exploration seismology
- Scattering theory
- Elastic scattering potential
- Viscoelastic scattering potential
- Viscoelastic sensitivities
- Conclusion and future direction

Motivations

- Quantification of the wave propagation in complex realistic media.

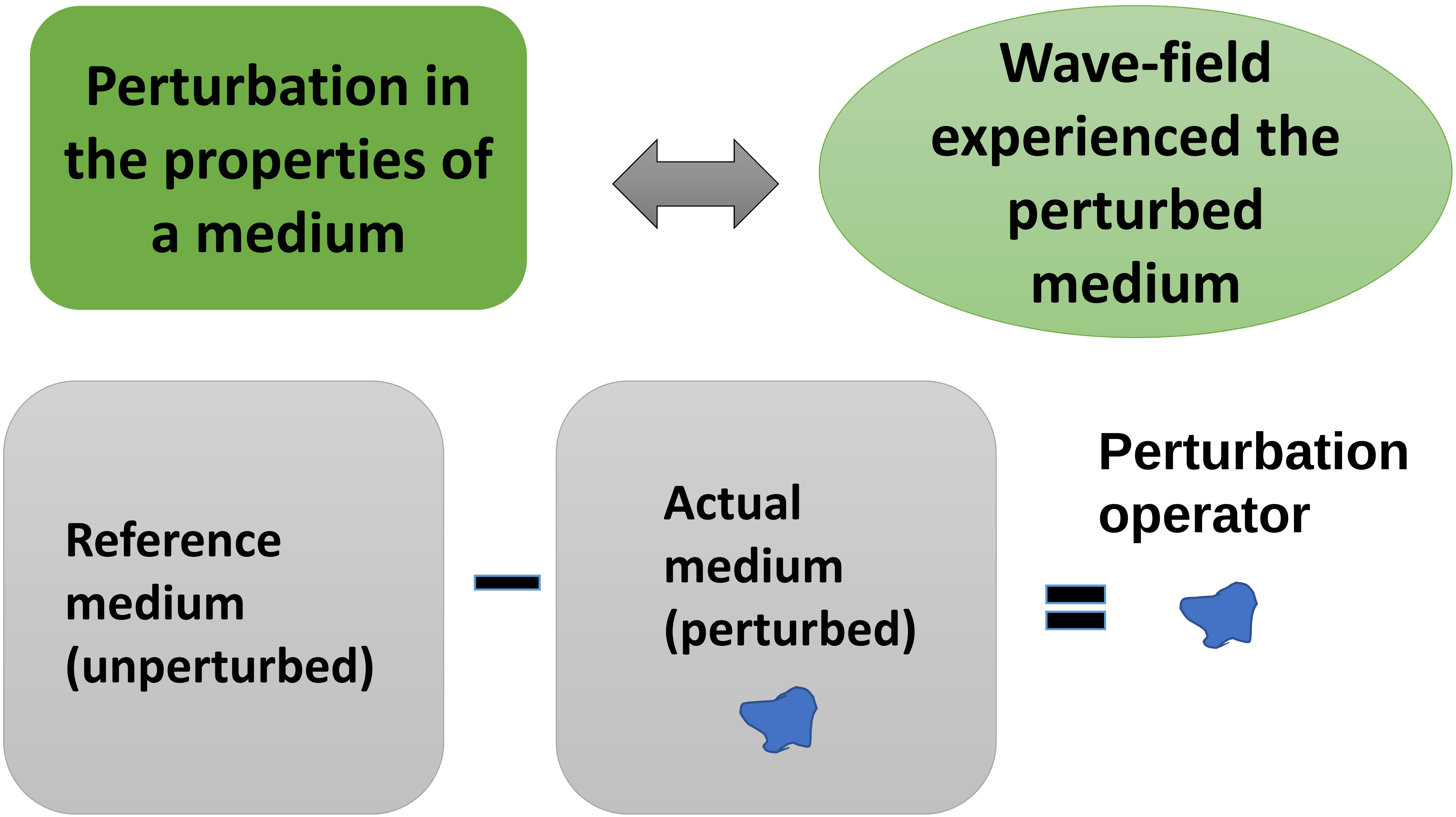
- Modeling of inhomogeneous wave propagation.

- Direct inverse scattering for seismic data processing.

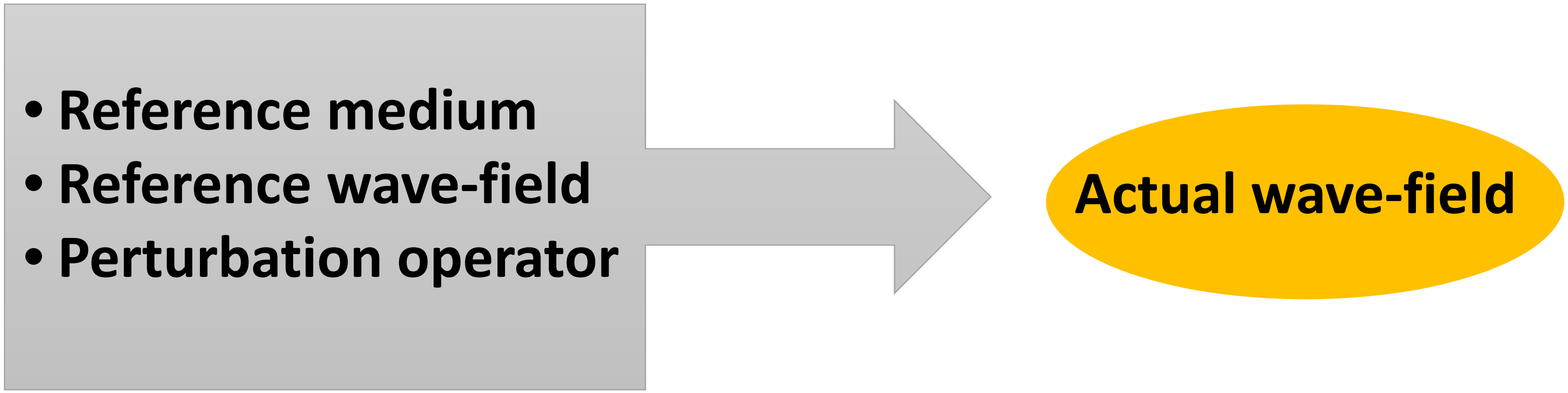
- Inversion in attenuating media

- Mathematical form of full waveform inversion updates and sensitivities for determining Q_P and Q_S.

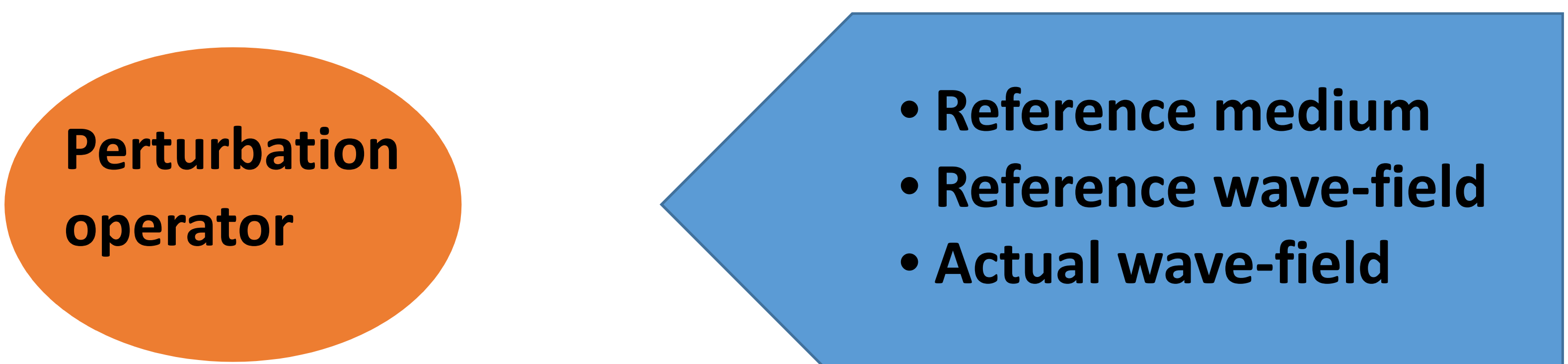
Scattering theory



Forward Scattering



Backward Scattering



Lippmann–Schwinger equation

$$\mathbf{L}G = -\delta(\mathbf{r} - \mathbf{r}_s)$$ Actual medium

$$\mathbf{L}_0 G_0 = -\delta(\mathbf{r} - \mathbf{r}_s)$$ Reference medium

$$\mathbf{V} \equiv \mathbf{L} - \mathbf{L}_0$$ Scattering operator

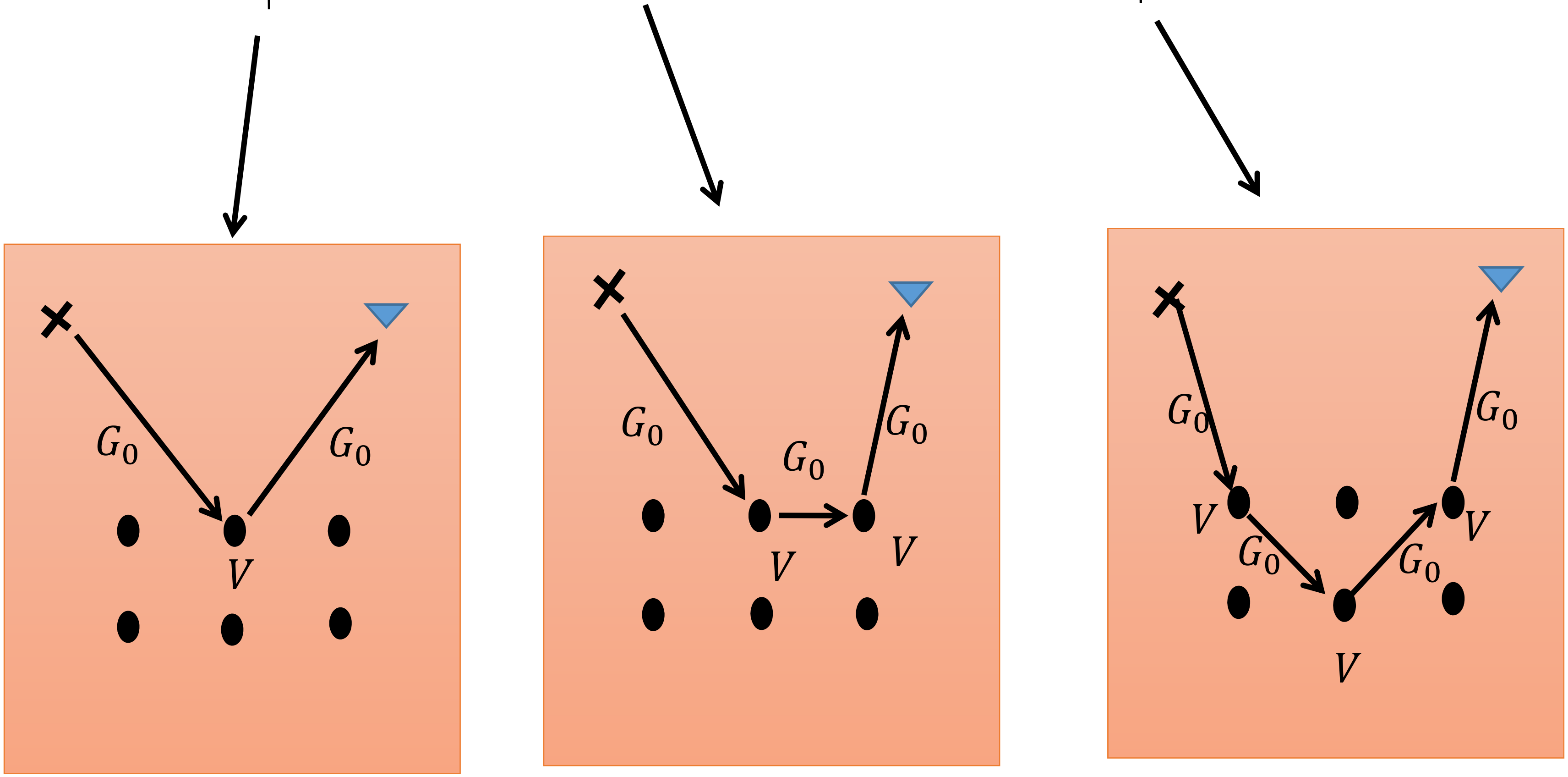
$$\mathbf{\Psi}_s \equiv \mathbf{G} - \mathbf{G}_0$$ Scattered field operator

Lippmann–Schwinger equation

$$\mathbf{\Psi}_s = \mathbf{G} - \mathbf{G}_0 = \mathbf{G}_0\mathbf{V}\mathbf{G}$$

Born Series

$$D = \underbrace{G_0 V G_0} + \underbrace{G_0 V G_0 V G_0} + \underbrace{G_0 V G_0 V G_0 V G_0} + \ldots$$



Elastic scattering potential

For isotropic, elastic earth the wave equation is given by

$$\mathcal{L}_E(\mathbf{x},\omega)\mathbf{u}(\mathbf{x},\omega) = 0$$

$$(\mathcal{L}_E)_{ij} = \delta_{ij}(\rho\omega^2 + \partial_k \mu \partial_k) + \partial_i \lambda \partial_j + \partial_j \mu \partial_i$$

Scattering potential

$$\mathcal{V} = \mathcal{L} - \mathcal{L}_0$$

Elastic scattering potential

$$(\mathcal{V}_E)_{ij} = \rho_0 \left\{ \omega^2 a_\rho \delta_{ij} + V_{P_0}^2 \partial_i a_\gamma \partial_j + V_{S_0}^2 \left(\delta_{ij} \partial_k a_\mu \partial_k - 2\partial_i a_\mu \partial_j + \partial_j a_\mu \partial_i \right) \right\}$$

Perturbation parameters

$$a_\rho = \frac{\rho - \rho_0}{\rho} = \frac{\Delta\rho}{\rho}, \quad a_\gamma = \frac{\gamma - \gamma_0}{\gamma} = \frac{\Delta\gamma}{\gamma}, \quad a_\mu = \frac{\mu - \mu_0}{\mu} = \frac{\Delta\mu}{\mu}$$

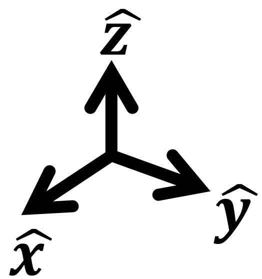
Bulk modulus $\gamma = \lambda + 2\mu$

P- and S-velocity

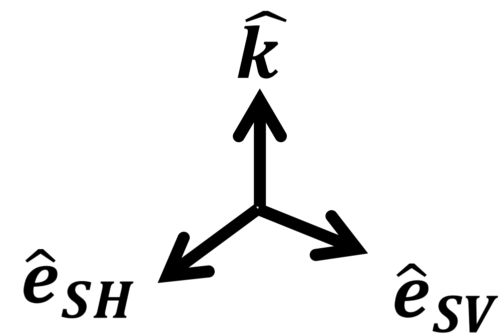
$$V_P = \sqrt{\rho^{-1}(\lambda + 2\mu)} \quad V_S = \sqrt{\rho^{-1}\mu}$$

Scattering Potential transformation

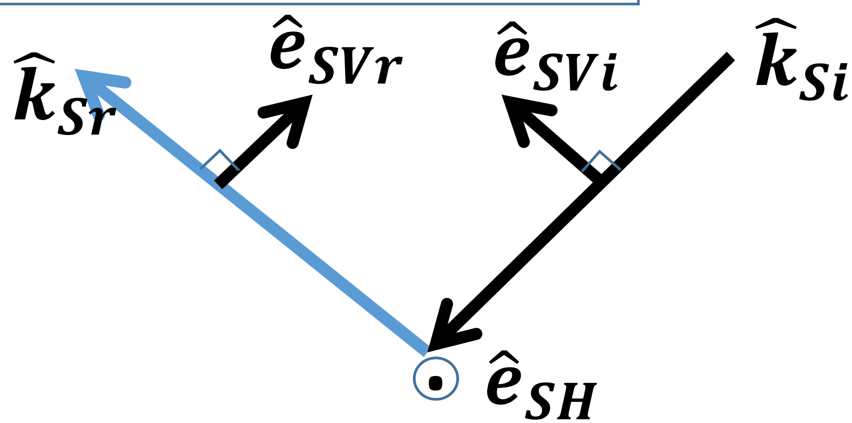
(Robert H. Stolt, Professor Arthur B. Weglein **Seismic Imaging and Inversion, 2012**)



$$\begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{pmatrix}$$



$$\begin{pmatrix} \mathcal{V}_{PP} & \mathcal{V}_{PSH} & \mathcal{V}_{PSV} \\ \mathcal{V}_{SHP} & \mathcal{V}_{SHSH} & \mathcal{V}_{SHSV} \\ \mathcal{V}_{SVP} & \mathcal{V}_{SVSH} & \mathcal{V}_{SVSV} \end{pmatrix}$$

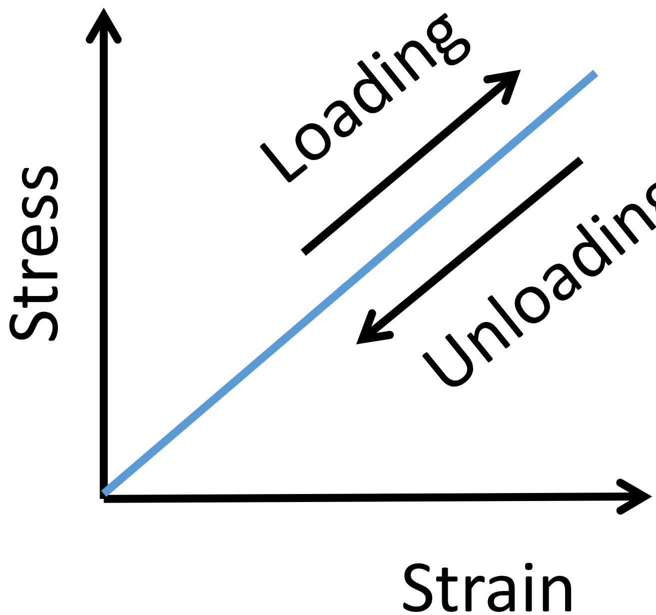


$$\hat{\mathbf{e}}_{SH} = \frac{\mathbf{k}_{Sr} \times \mathbf{k}_{Si}}{|\mathbf{k}_{Sr} \times \mathbf{k}_{Si}|}$$

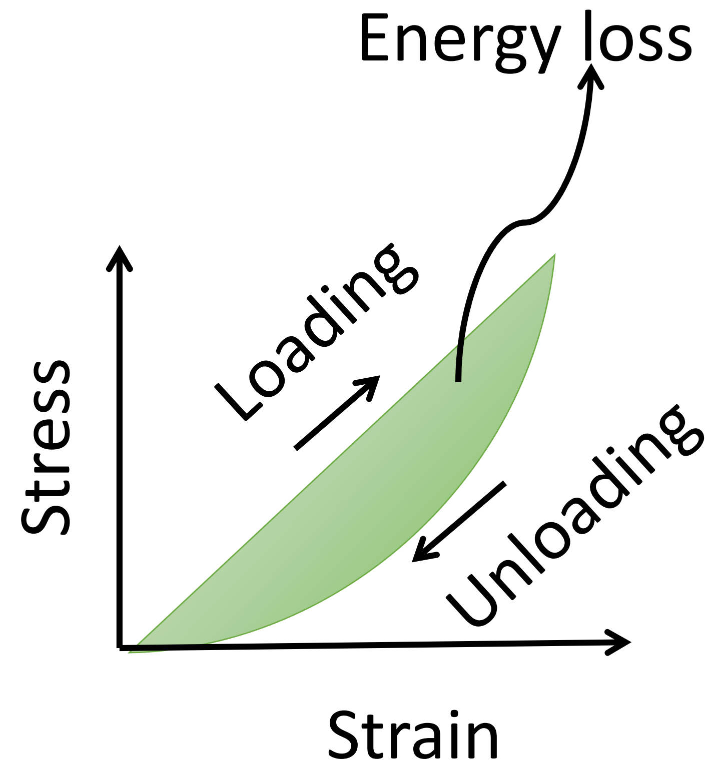
$$\hat{\mathbf{e}}_{SV} = \hat{\mathbf{k}}_S \times \hat{\mathbf{e}}_{SH}$$

Viscoelastic Scattering Potential

Elastic



Viscoelastic



1) Viscoelastic Scattering Operator

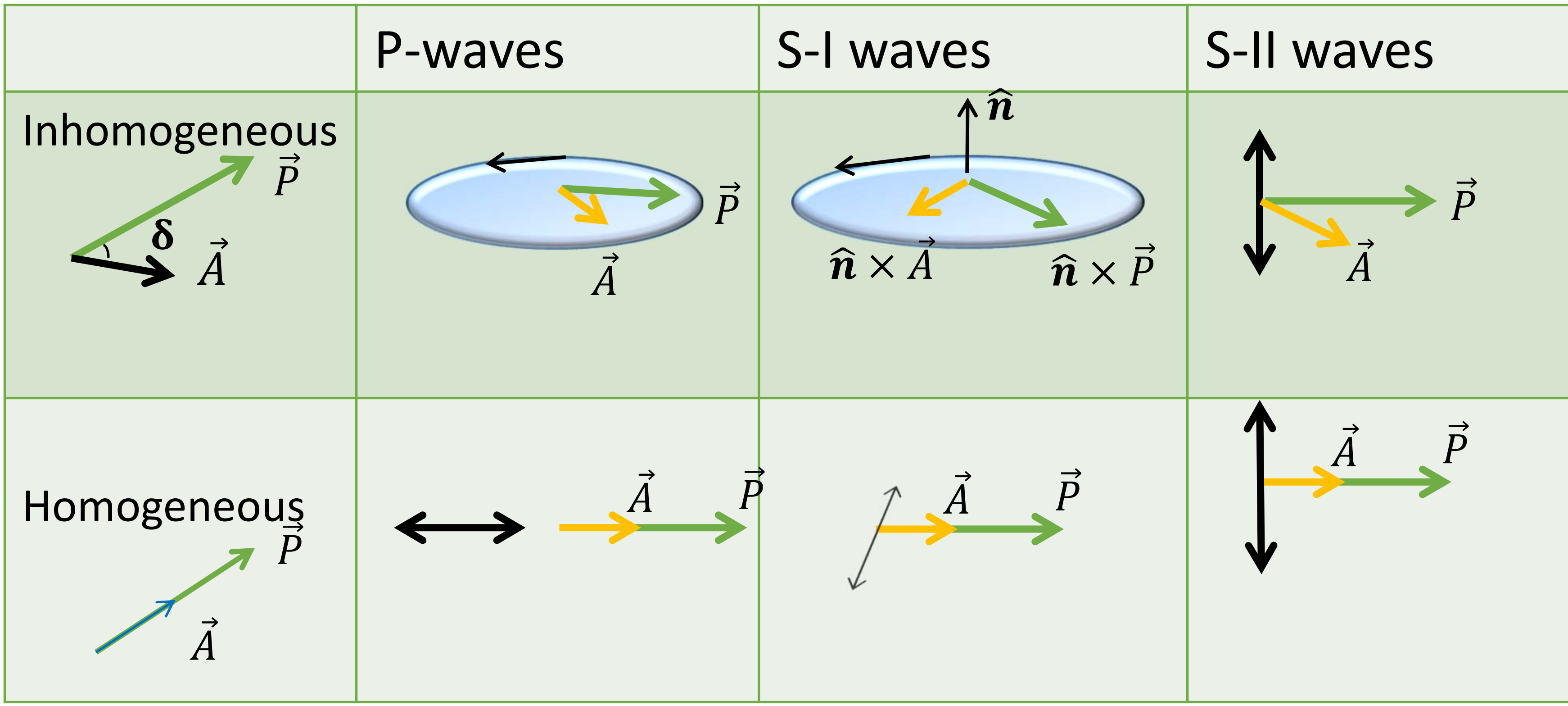
$$V_{ve}(\mathbf{r},\omega) = L_{ve}(\mathbf{r},\omega) - L_{ve_0}(\mathbf{r},\omega)$$

$$\begin{aligned}\rho_0^{-1}(V_{ve})_{ij} = A_\rho \omega^2 \delta_{ij} + \alpha_{H_0}^2 \partial_i \left\{A_\rho + 2A_{\alpha_H} + iQ_{HP_0}^{-1} A_{Q_{HP}}\right\} \partial_j + \\ \delta_{ij}\beta_{H_0}^2 \partial_k \left\{A_\rho + 2A_{\beta_H} + iQ_{HS_0}^{-1} A_{Q_{HS}}\right\} \partial_k - \\ 2\beta_{H_0}^2 \partial_i \left\{A_\rho + 2A_{\beta_H} + iQ_{HS_0}^{-1} A_{Q_{HS}}\right\} \partial_j + \\ \beta_{H_0}^2 \partial_j \left\{A_\rho + 2A_{\beta_H} + iQ_{HS_0}^{-1} A_{Q_{HS}}\right\} \partial_i,\end{aligned}$$

2) Definition of orthogonal frame constructed by displacement vectors

Inhomogeneous waves

$$\vec{K} = \vec{P} - i\vec{A}$$

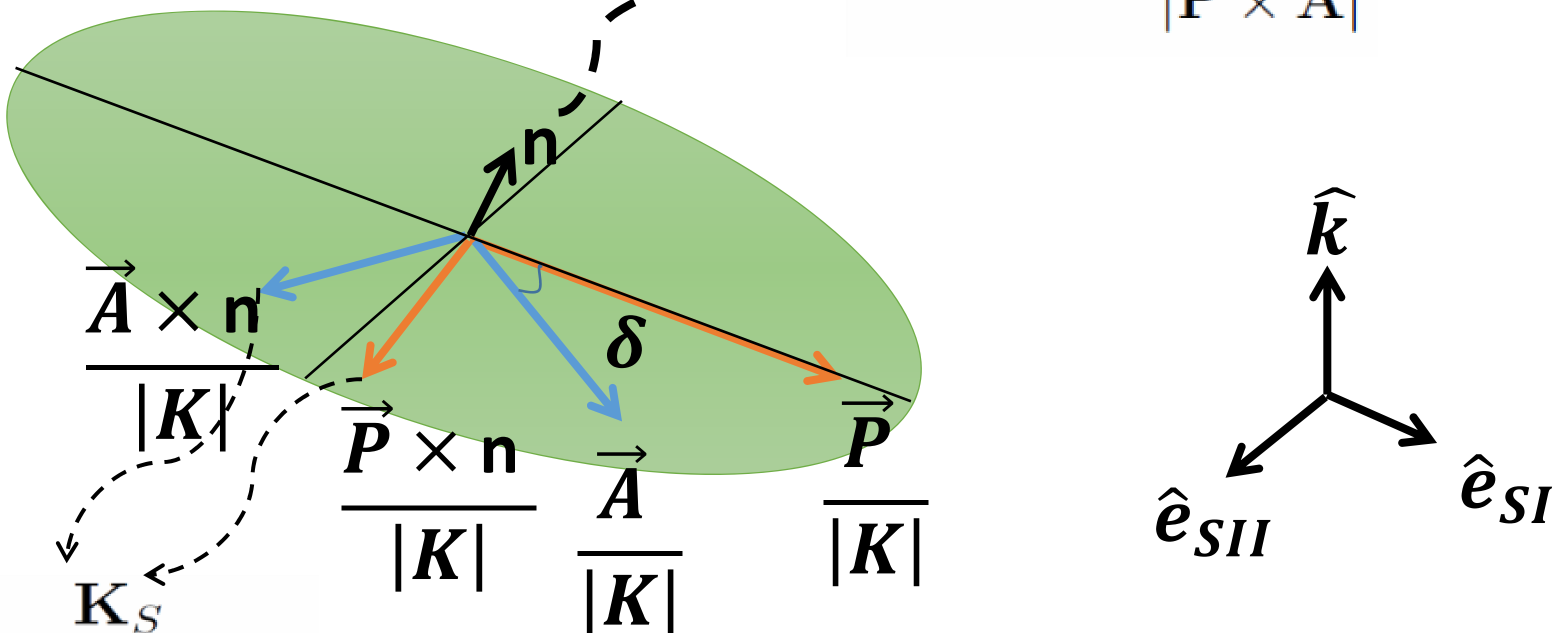


Framework for inhomogeneous waves

Roger D. Borcherdt, **Viscoelastic Waves in Layered Media, (2009)**

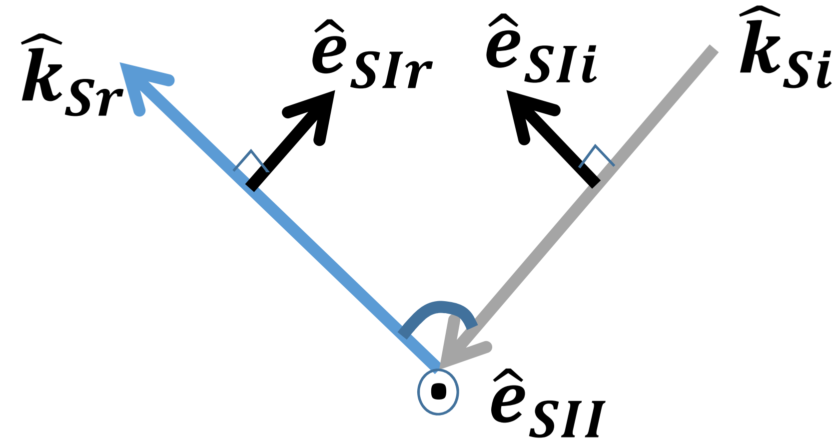
$$\vec{u}_R = \left|\vec{G}_0 k_S\right| \exp\left[-\vec{A}_{SI} \bullet \vec{r}\right] \left(\frac{\vec{P}_{SI} \times \hat{n}}{|k_S|} \cos[\zeta_{SI}(t) + \psi_S] + \frac{\vec{A}_{SI} \times \hat{n}}{|k_S|} \sin[\zeta_{SI}(t) + \psi_S] \right)$$

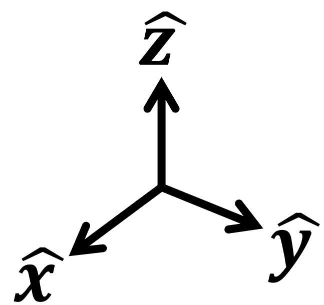
$$\mathbf{e}_{S_{II}} = \mathbf{n} = \frac{\mathbf{P} \times \mathbf{A}}{|\mathbf{P} \times \mathbf{A}|}$$



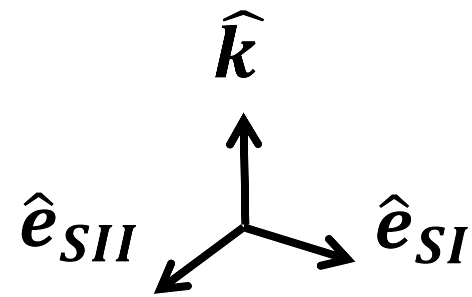
$$\mathbf{e}_{S_I} = \frac{\mathbf{K}_S}{|\mathbf{K}_S|} \times \mathbf{n}$$

Transformation of Scattering Potential





$$\begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{pmatrix}$$



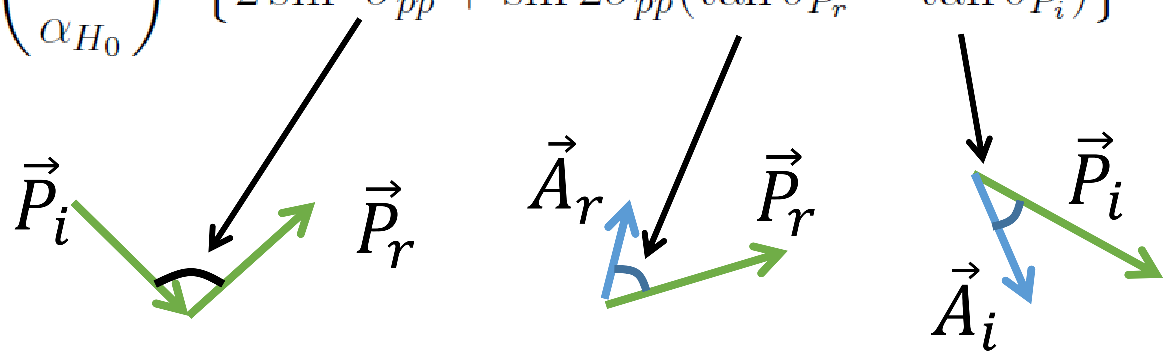
$$\begin{pmatrix} \mathcal{V}_{PP} & \mathcal{V}_{PS_{II}} & \mathcal{V}_{PS_{I}} \\ \mathcal{V}_{S_{II}P} & \mathcal{V}_{S_{II}S_{II}} & \mathcal{V}_{S_{II}S_{I}} \\ \mathcal{V}_{S_{I}P} & \mathcal{V}_{S_{I}S_{II}} & \mathcal{V}_{S_{I}S_{I}} \end{pmatrix}$$

Viscoelastic P-P scattering element

$$ {}^{P}_{P}\mathbb{V}_{ve} = {}^{P}_{P}\mathbb{V}_{e} + iQ^{-1}_{HP_0}\rho_0\left\{\mathbb{F}^{\rho}A_{\rho} - 2A_{\alpha_H} + A_{Q_{HP}} + \mathbb{F}^{\beta_H}A_{\beta_H}\right\} - iQ^{-1}_{HS_0}\rho_0\left(\frac{\beta_{H_0}}{\alpha_{H_0}}\right)^2 \sin^2\sigma A_{Q_{HS}} $$

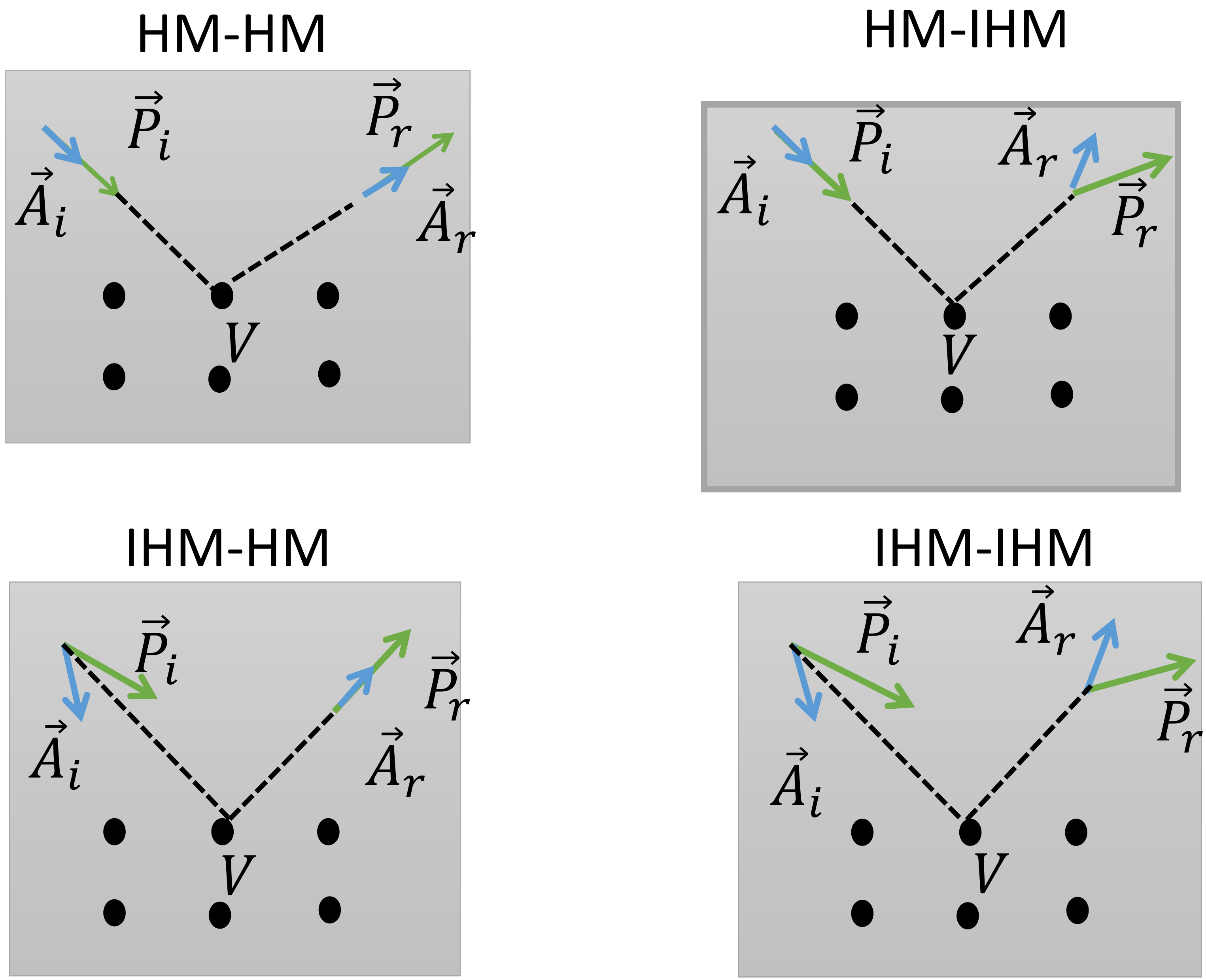
$$ \mathbb{F}^{\rho} = 1 + \frac{1}{2}\sin\sigma_{pp}(\tan\delta_{P_r} + \tan\delta_{P_i}) - \left(\frac{\beta_{H_0}}{\alpha_{H_0}}\right)^2\left\{2\sin^2\sigma_{pp} + \sin 2\sigma_{pp}(\tan\delta_{P_r} - \tan\delta_{P_i})\right\} $$

$$ \mathbb{F}^{\beta_H} = \left(\frac{\beta_{H_0}}{\alpha_{H_0}}\right)^2\left\{2\sin^2\sigma_{pp} + \sin 2\sigma_{pp}(\tan\delta_{P_r} - \tan\delta_{P_i})\right\} $$



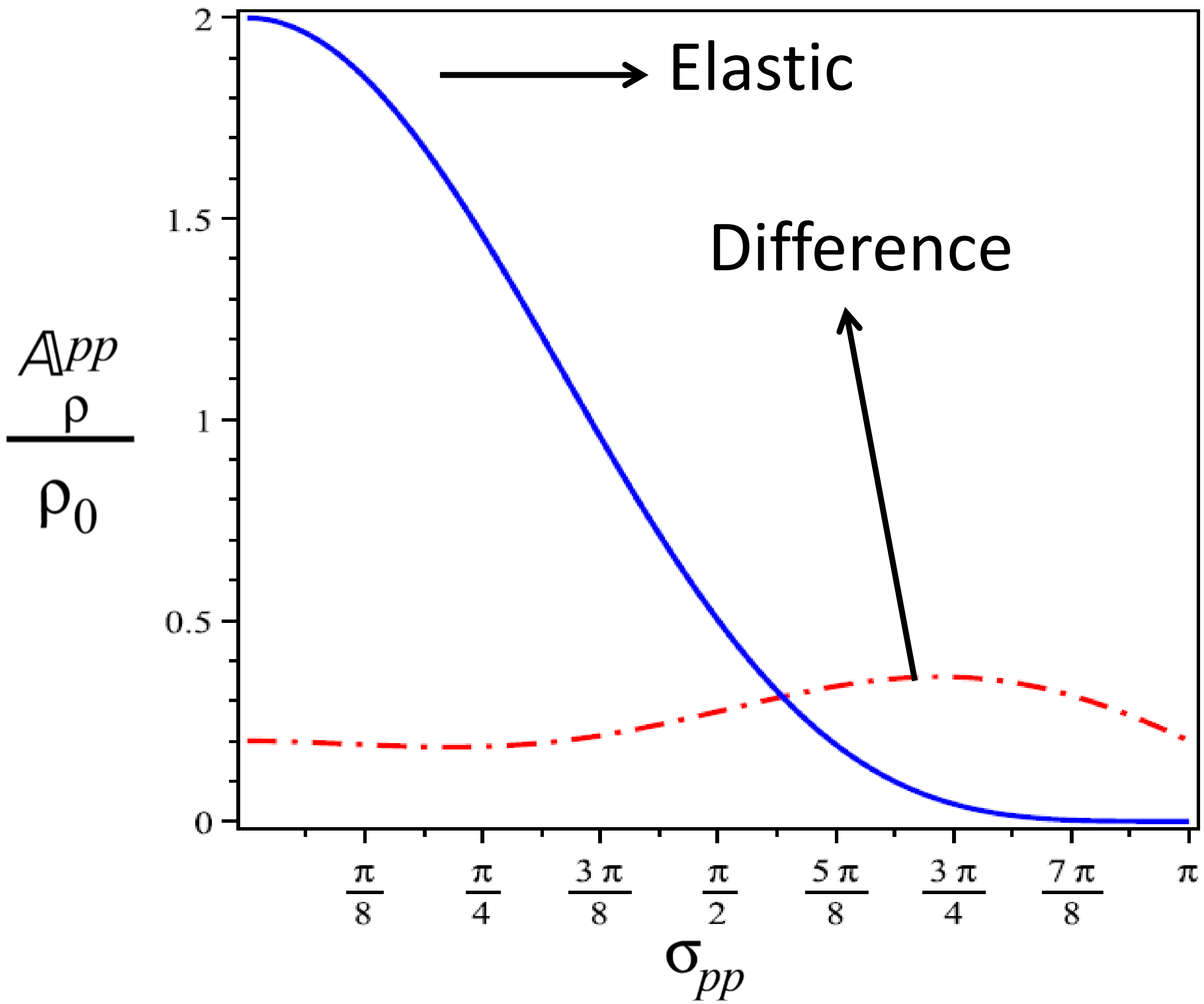
Elastic P-P scattering element

$$ {}^{P}_{P}\mathbb{V}_{e} = -\rho_0\left\{1 + \cos\sigma_{pp} - 2\left(\frac{\beta_{H_0}}{\alpha_{H_0}}\right)^2\sin^2\sigma_{pp}\right\}A_{\rho} + 2\rho_0 A_{\alpha_H} + 2\rho_0\left(\frac{\beta_{H_0}}{\alpha_{H_0}}\right)^2\sin^2\sigma_{pp}A_{\beta_H} $$


HM-HM
$\vec{P}_i$
$\vec{P}_r$
$\vec{A}_i$
$\vec{A}_r$
V
HM-IHM
$\vec{P}_i$
$\vec{A}_r$
$\vec{A}_i$
$\vec{P}_r$
V
IHM-HM
$\vec{P}_i$
$\vec{P}_r$
$\vec{A}_i$
$\vec{A}_r$
V
IHM-IHM
$\vec{P}_i$
$\vec{A}_r$
$\vec{P}_r$
$\vec{A}_i$
V

Scattering of homogeneous P-wave to inhomogeneous P-wave

$$\mathbb{V}_{ve} = (\mathbb{A}_{ve}^{\rho})A_{\rho} + (\mathbb{A}_{ve}^{\alpha_H})A_{\alpha_H} + (\mathbb{A}_{ve}^{\beta_H})A_{\beta_H} + (\mathbb{A}_{ve}^{Q_{HP}})A_{Q_{HP}} + (\mathbb{A}_{ve}^{Q_{HS}})A_{Q_{HS}}$$

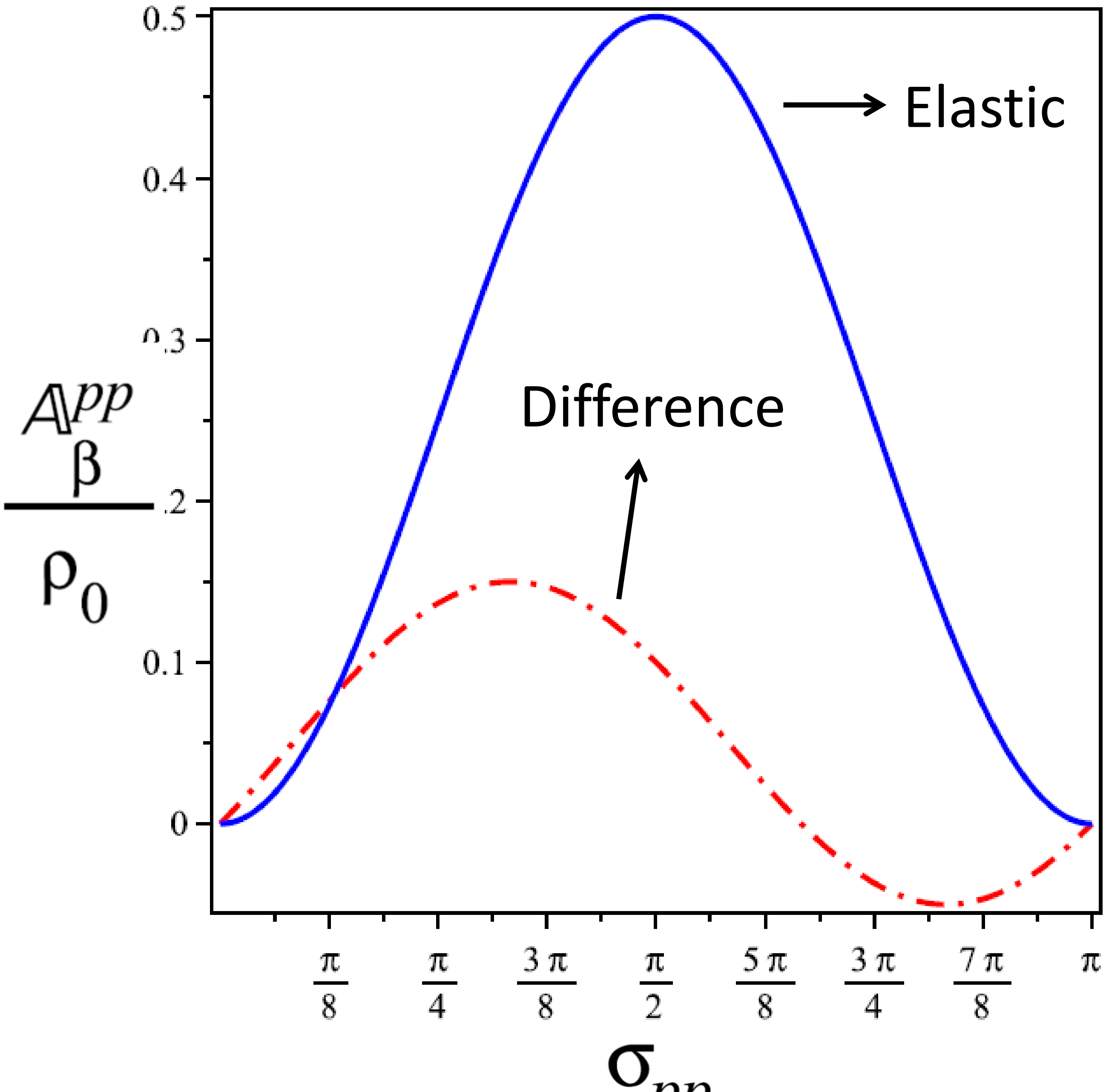


$$\delta_{P_r} = \frac{\pi}{3}$$

$$\delta_{P_i} = 0$$

$$Q_{HP_0}^{-1} = 0.2$$

$$\frac{\beta_{H_0}}{\alpha_{H_0}} = \frac{1}{2}$$



Elastic
Difference
0.5
0.4
0.3
0.2
0.1
0
π/8
π/4
3π/8
π/2
5π/8
3π/4
7π/8
π
σ_{pp}
$\frac{\mathbb{A}^{pp}_{\beta}}{\rho_0}$

VISCOELASTIC SENSITIVITIES

Sensitivity of the P-P field to Q_S

$$\frac{\partial \mathrm{PP}_{ve}^{Q_S}(\mathbf{x}_g, \mathbf{x}_s, \omega)}{\partial A_{Q_{HS}}(\mathbf{x})} =$$

$$-iQ_{HS_0}^{-1}\rho_0\left(\frac{\beta_{H_0}}{\alpha_{H_0}}\right)^2 \sin^2\sigma G_L^P(\mathbf{x}_g, \mathbf{x}, \omega) G_R^P(\mathbf{x}, \mathbf{x}_s, \omega)$$

Conclusions and future direction

1) Construction of the scattering potential for a viscoelastic medium based on Stolt and Weglein method.
2) We explicitly demonstrate that the components of the scattering potential related to the scattering of SII-waves to P- and SI waves are zero. In other words SII-waves can only scattered to SII-waves.
3) We have shown that viscoelastic scattering potential can be expressed as elastic scattering potential plus an additional perturbing term related to the inhomogeneity of waves and quality factors of P- and S-waves.

$$V_{ve} = V_e + iQ_{HP}^{-1}\mathbb{G}(\sigma, \delta_r, \delta_i, \Delta A) + iQ_{HS}^{-1}\mathbb{F}(\sigma, \delta_r, \delta_i, \Delta A)$$

4) Investigation of sensitivities to make up the gradient for FWI.
5) Q estimation based on amplitude attenuation for inverse Q filtering.
6) Viscoelastic AVO approximations.

Acknowledgements

- CREWES sponsors.
- Kris Innanen.
- Shahin Moradi.
- Hassan Khaniani.

Thank you