Numerical analysis of scattering in a viscoelastic medium

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and Hassan Khaniani

Outline

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  - **Elastic versus Viscoelastic**
  - **Viscoelastic models**
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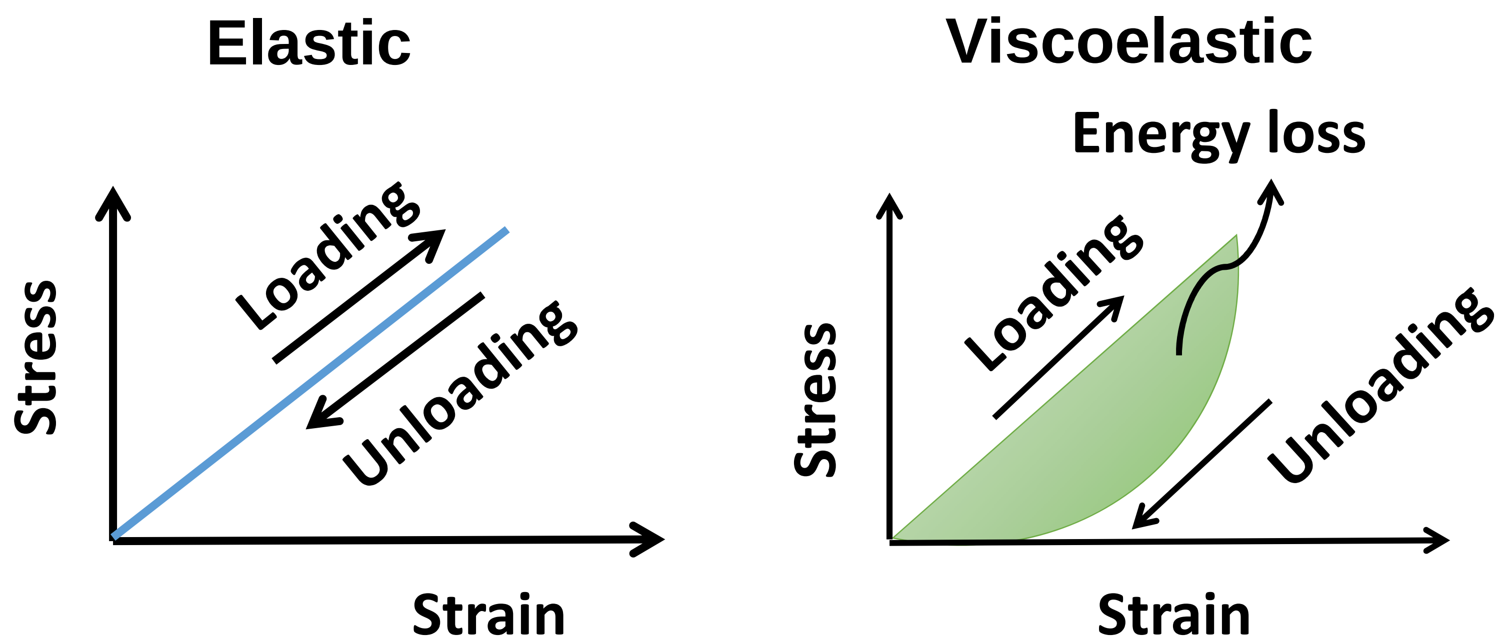
Motivation

- Characterize seismic wave propagation and scattering in viscoelastic media in the context of multicomponent survey data.

- Direct inverse scattering, Q estimation (AVF/AVA analysis), Q compensation.

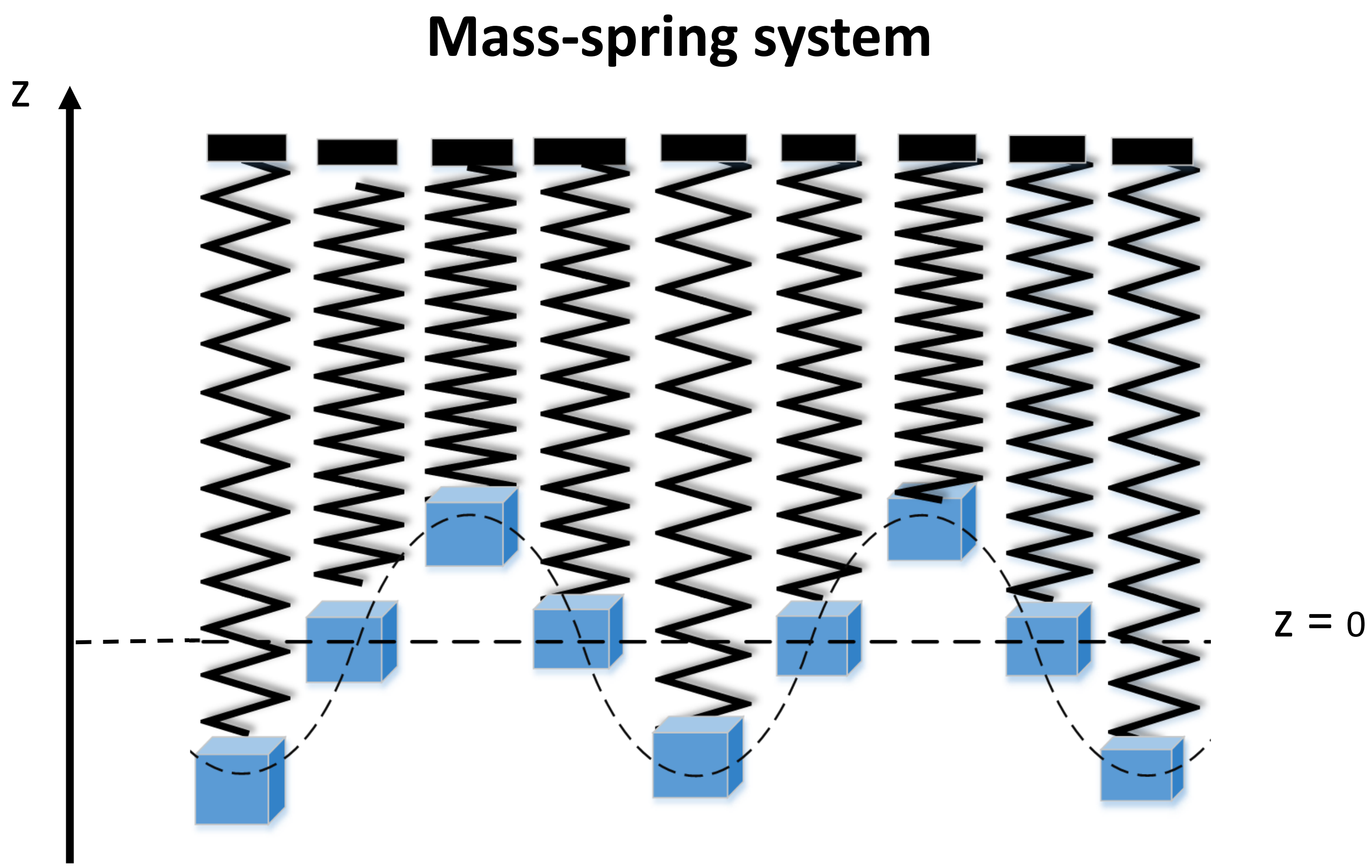
- Inverse scattering and full waveform inversion in attenuating media.

Elastic versus Viscoelastic



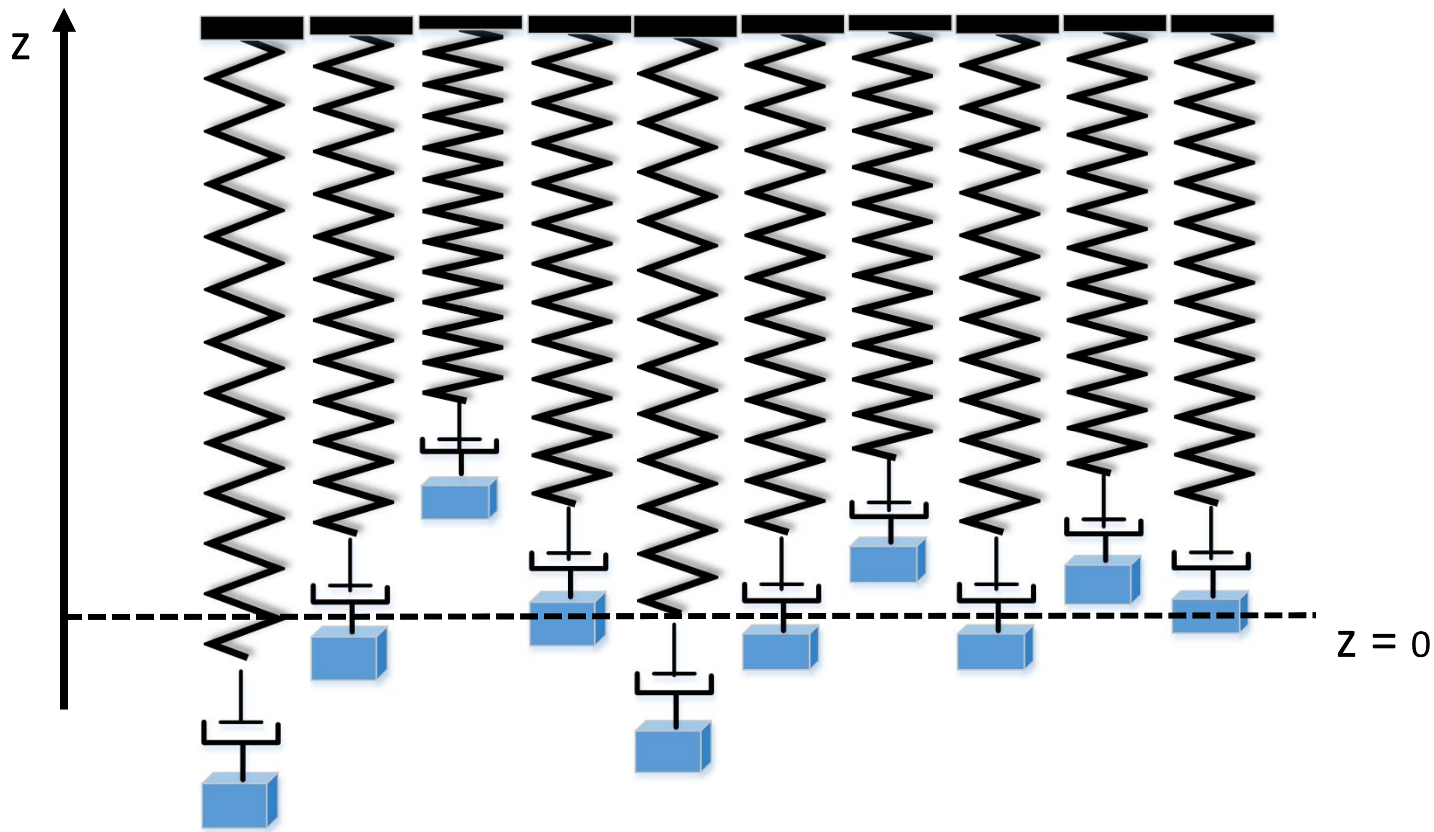
- Reciprocal of the quality factor (fractional energy loss): $Q^{-1} = -\frac{\Delta E}{2\pi E}$

Elastic model: undamped motion

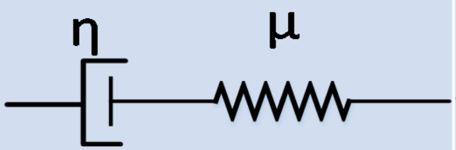
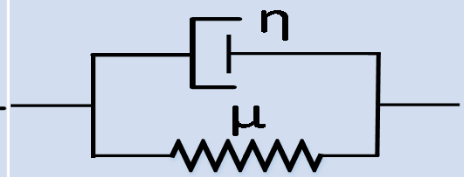
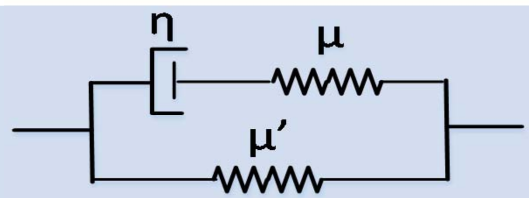


Viscoelastic model: damped motion

Spring dashpot system



Viscoelastic models

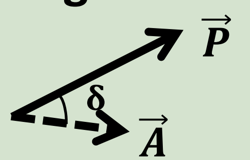
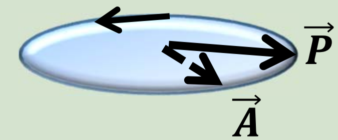
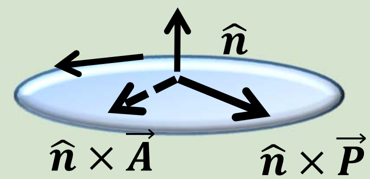
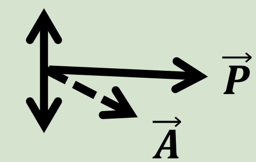
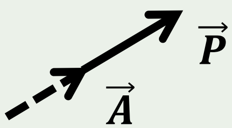
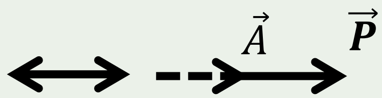
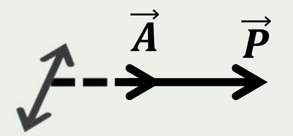
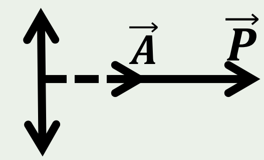
Maxwell Model	Kelvin Model	Standard linear Model
		
Viscoelastic Fluid	Viscoelastic Solid	Viscoelastic Medium
Same stresses Different strains	Same strains Different stresses	$\tau_\sigma = \frac{\eta}{\mu + \mu'}, \qquad \tau_\varepsilon = \frac{\eta}{\mu}$
$Q^{-1} = \frac{\mu}{\omega\eta}$	$Q^{-1} = \frac{\omega\eta}{\mu}$	$Q^{-1} = \frac{\omega(\tau_\sigma - \tau_\varepsilon)}{1 + \omega^2 \tau_\sigma \tau_\varepsilon}$

Viscoelastic waves*

Complex wave vector $\vec{\mathrm{K}} = \vec{\mathrm{P}} - \mathrm{i}\vec{\mathrm{A}}$

Attenuation vector

Propagation vector

	P-waves	S-I waves	S-II waves
Inhomogeneous 			
Homogeneous 			

* Borcherdt, R. D., 2009, Viscoelastic waves in layered media: Cambridge University Press.

Scattering theory

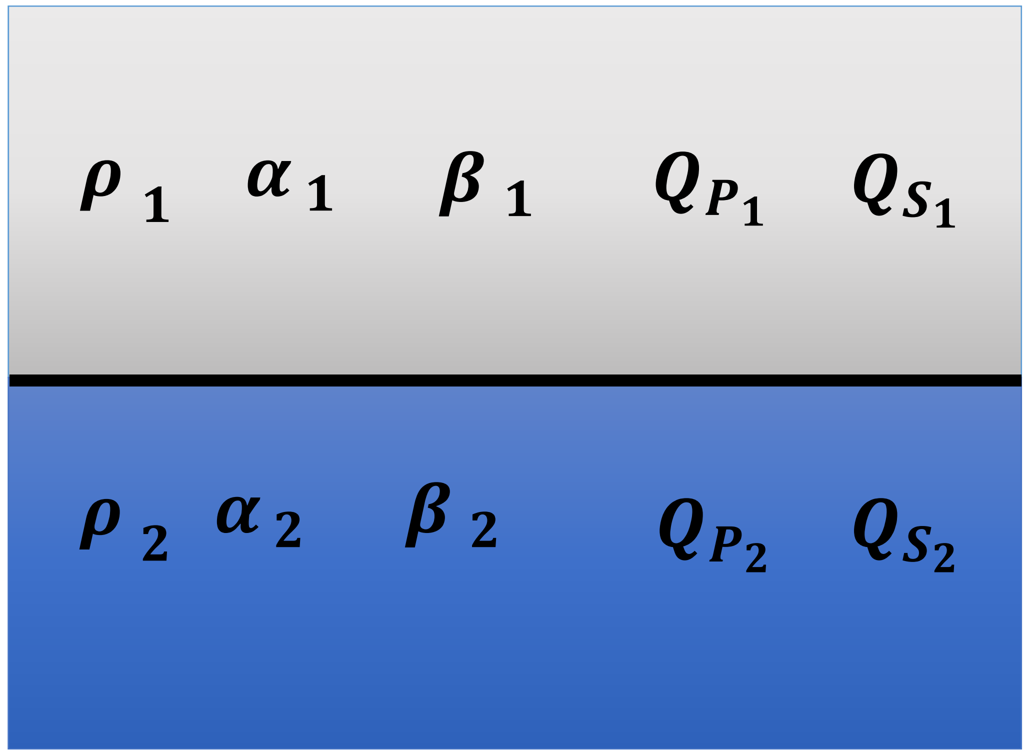
ρ Density

α P-wave velocity

β P-wave velocity

Q_P P-wave quality factor

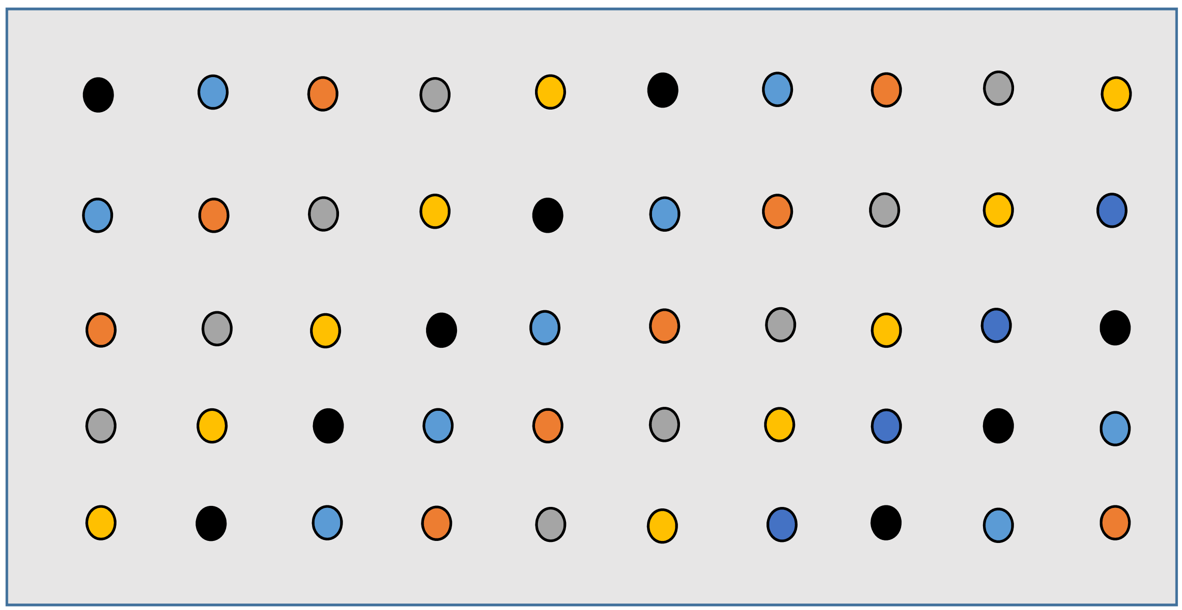
Q_S S-wave quality factor



Scattering theory

- Perturbation in P-wave velocity
- Perturbation in S-wave velocity
- Perturbation in density
- Perturbation in P-wave quality factor
- Perturbation in S-wave quality factor

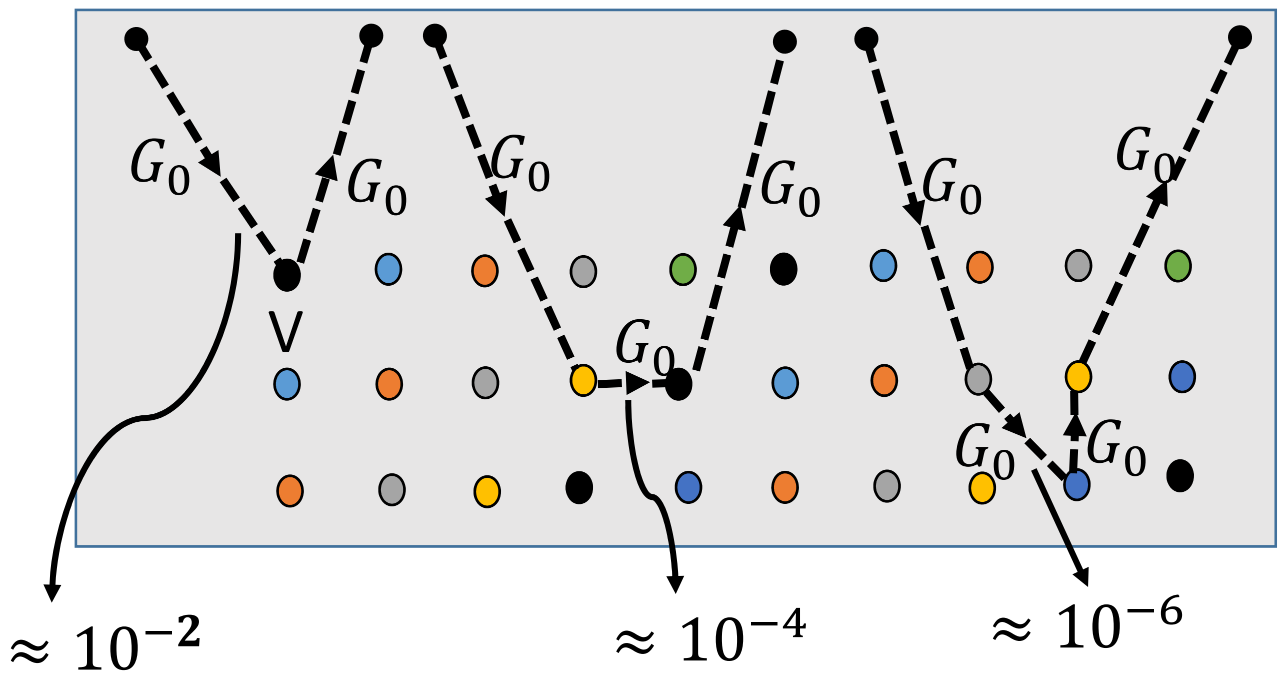
$$\frac{\delta\rho}{\bar{\rho}} = 2\left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right) \ll 1$$



Born series

Scattered wave =

$$G_0 V G_0 + G_0 V G_0 V G_0 + G_0 V G_0 V G_0 V G_0 + \cdots.$$



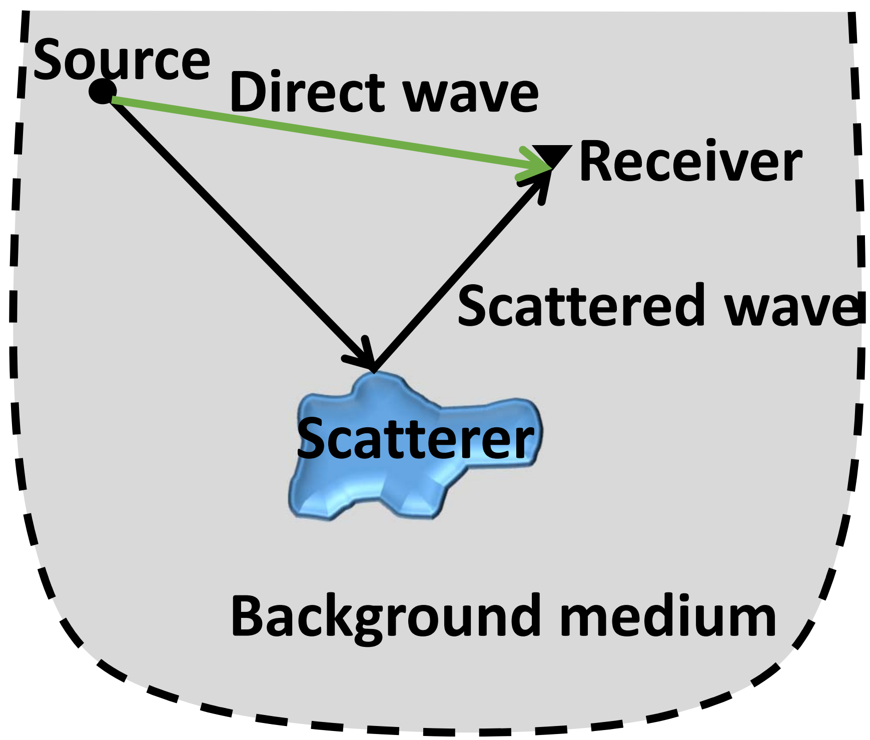
Max Born (1882-1970)
German physicist
Nobel prize in 1954

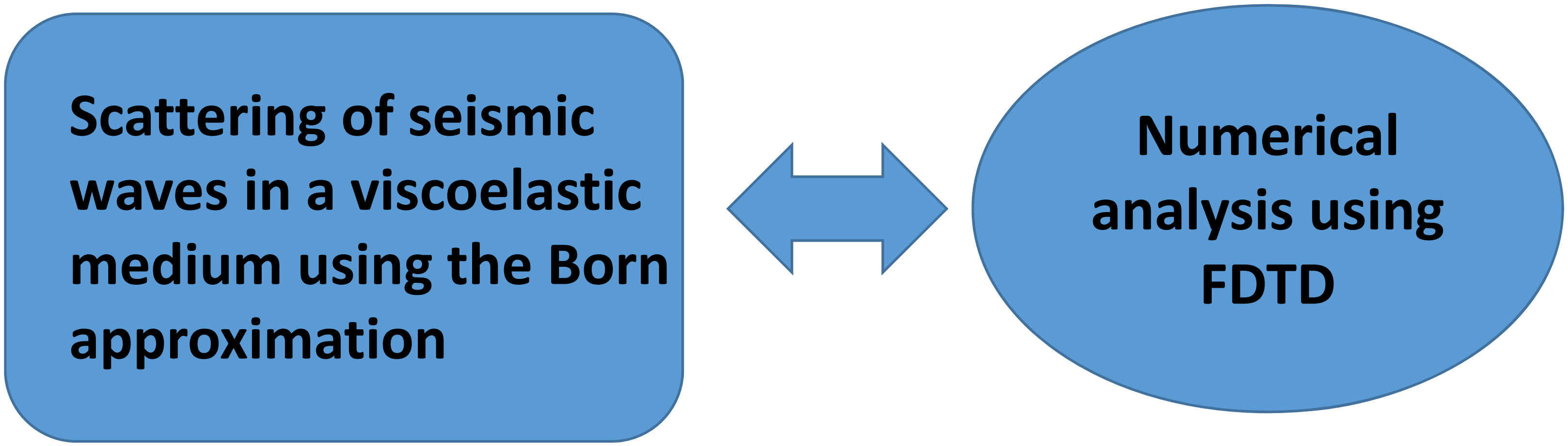
Born approximation: single scattering

- **Source and receiver located in background medium.**

- **Only primaries.**

Recorded wave =
Direct wave + Scattered wave





Shahpoor Moradi and Kris Innanen
Scattering of homogeneous and inhomogeneous viscoelastic waves from arbitrary heterogeneities
CREWES Research Report-Volume 26 (2014)

Shahpoor Moradi,Kris Innanen
Hassan Khaniani
Numerical analysis of scattering in a viscoelastic medium
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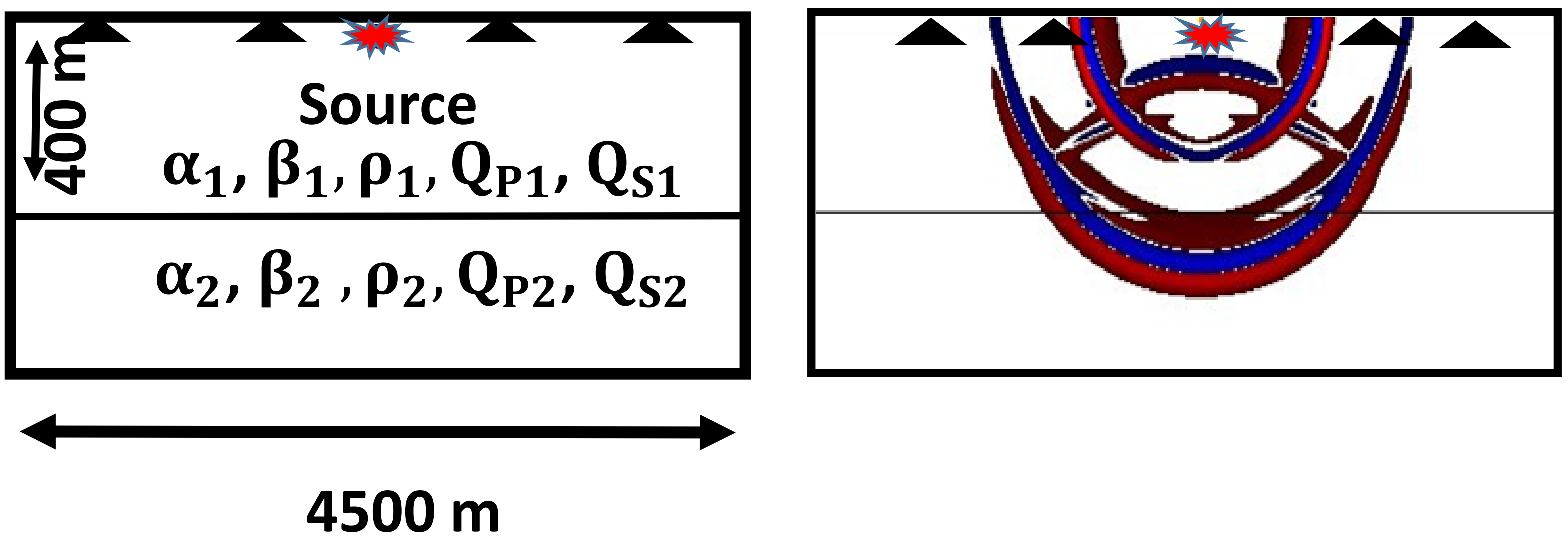
Numerical study of scattering using FDTD

Finite Difference Time Domain (FDTD), eighth-order accurate in space and second-order accurate in time, 45 Hz zero phase wavelet *



* Martin, R., and Komatitsch, D., 2009, Geophysical Journal International, 179, No. 1,333–344.
http://komatitsch.free.fr/README_seismic_cpml.html

Contrast in density (theory)

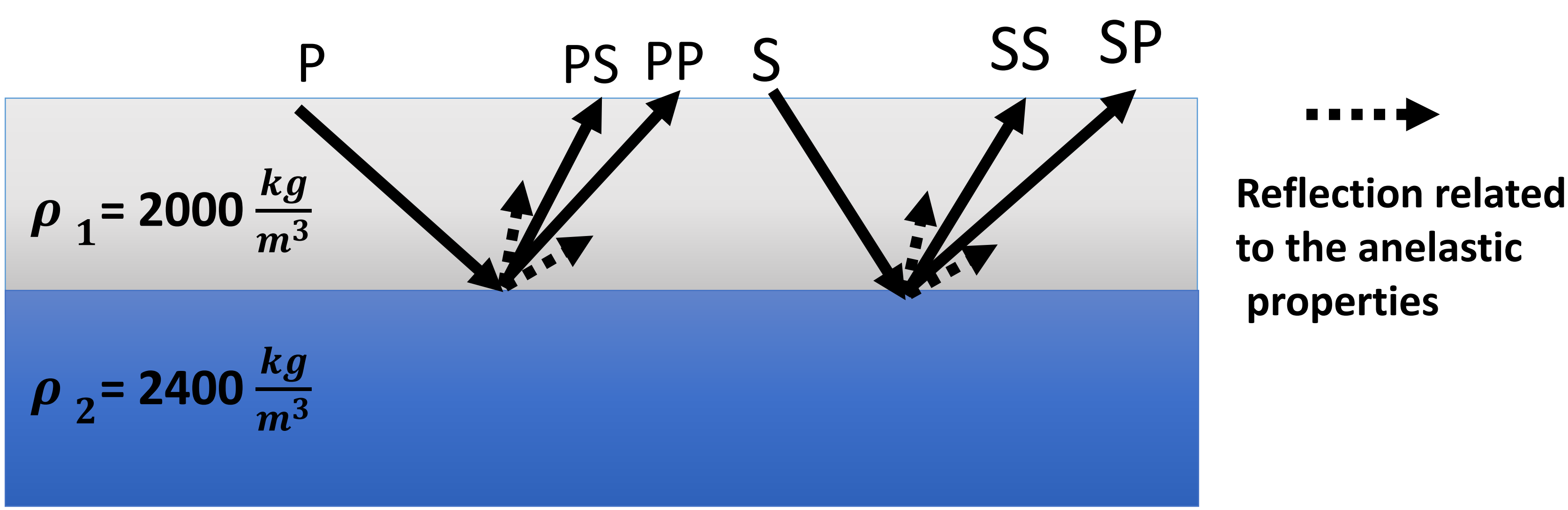
$${}^{P}_{P}\mathbb{V}_{visco} = \left({}^{P}_{P}\mathbb{V}^{\rho}_{e} + i\,{}^{P}_{P}\mathbb{V}^{\rho}_{ane}\right) A_{\rho}$$ P-to-P

$${}^{SI}_{P}\mathbb{V}_{visco} = \left({}^{SI}_{P}\mathbb{V}^{\rho}_{e} + i\,{}^{SI}_{P}\mathbb{V}^{\rho}_{ane}\right) A_{\rho}$$ S-to-P

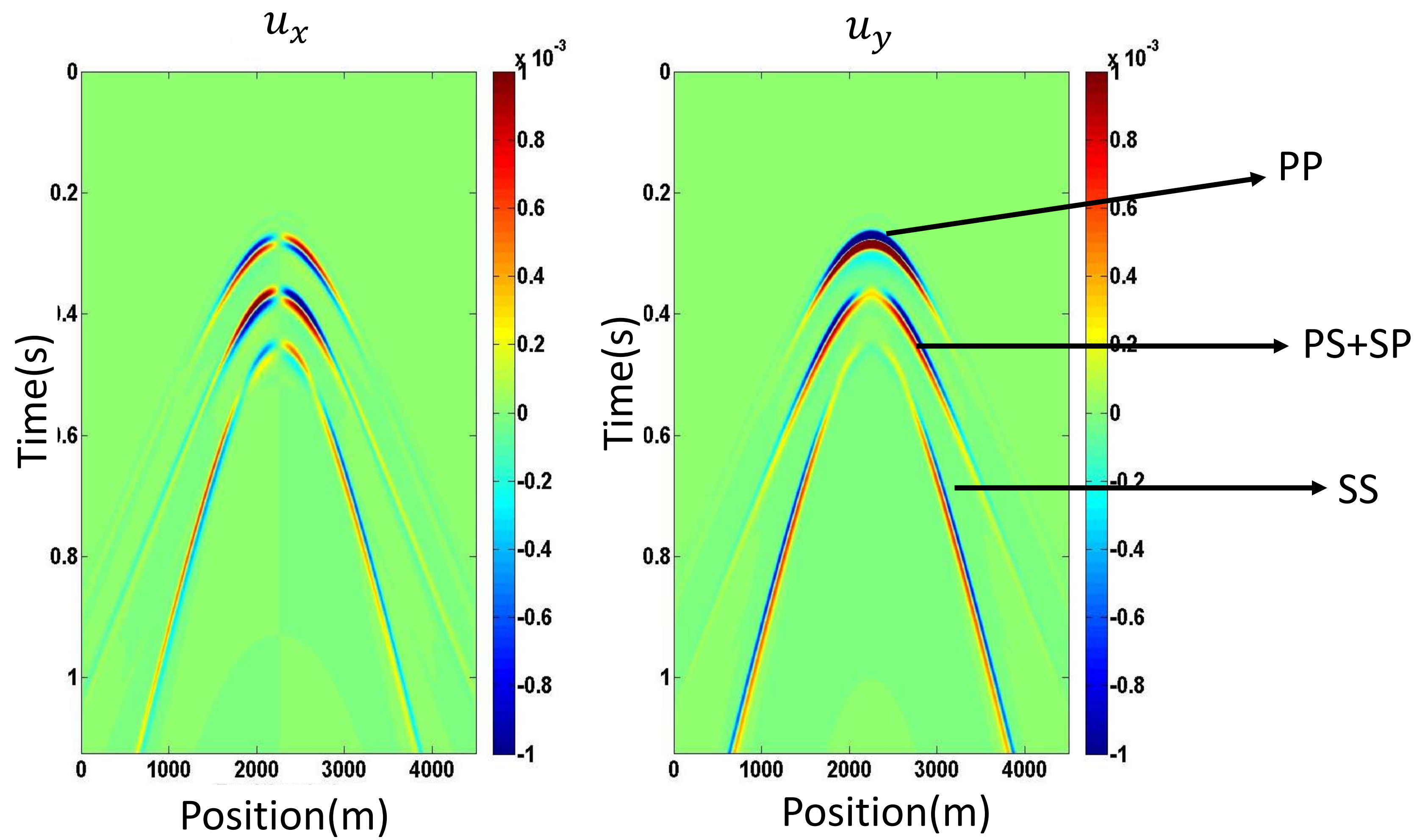
$${}^{P}_{SI}\mathbb{V}_{visco} = \left({}^{P}_{SI}\mathbb{V}^{\rho}_{e} + i\,{}^{P}_{SI}\mathbb{V}^{\rho}_{ane}\right) A_{\rho}$$ P-to-S

$${}^{SI}_{SI}\mathbb{V}_{visco} = \left({}^{SI}_{SI}\mathbb{V}^{\rho}_{e} + i\,{}^{SI}_{SI}\mathbb{V}^{\rho}_{ane}\right) A_{\rho}$$ S-to-S

Born approximation



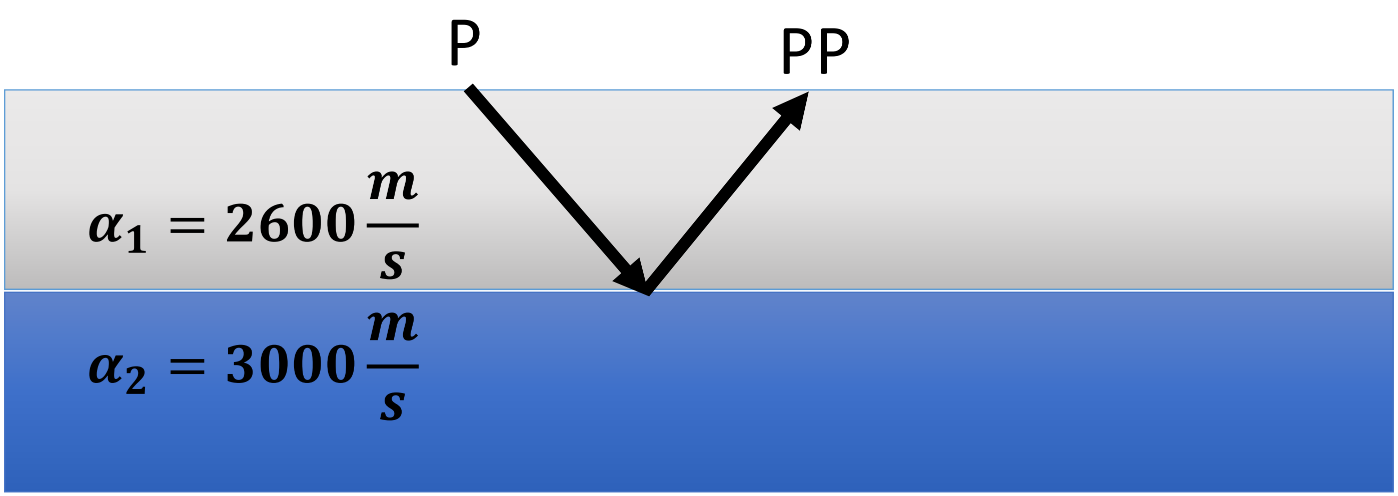
Contrast in density (numeric)



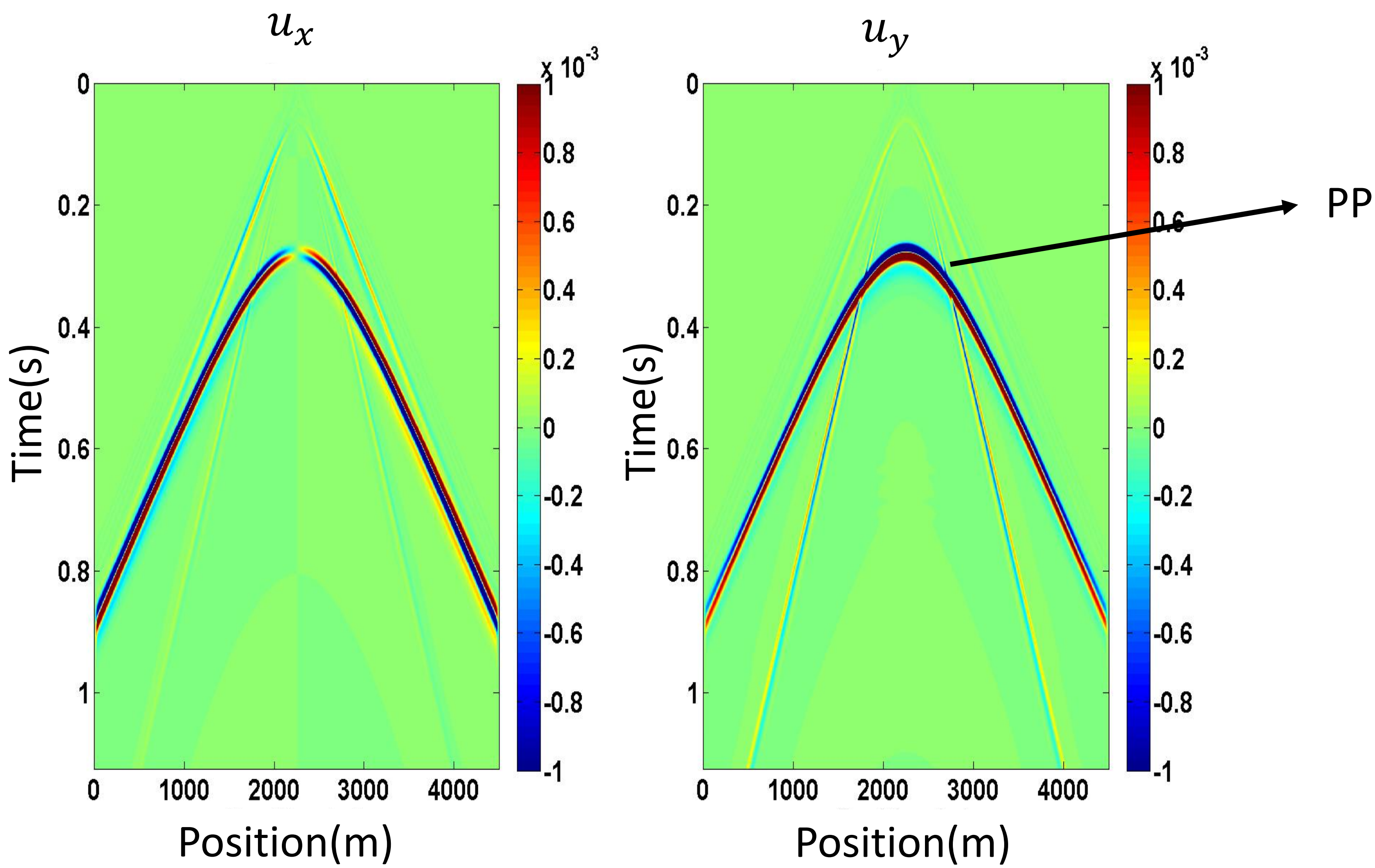
Contrast in P-wave velocity (theory)

$${}^{P}_{P}\mathbb{V}_{visco} = \left({}^{P}_{P}\mathbb{V}^{\alpha}_{e}\right) A_{\alpha}$$

Born approximation



Contrast in P-wave velocity (numeric)



Contrast in S-wave velocity (theory)

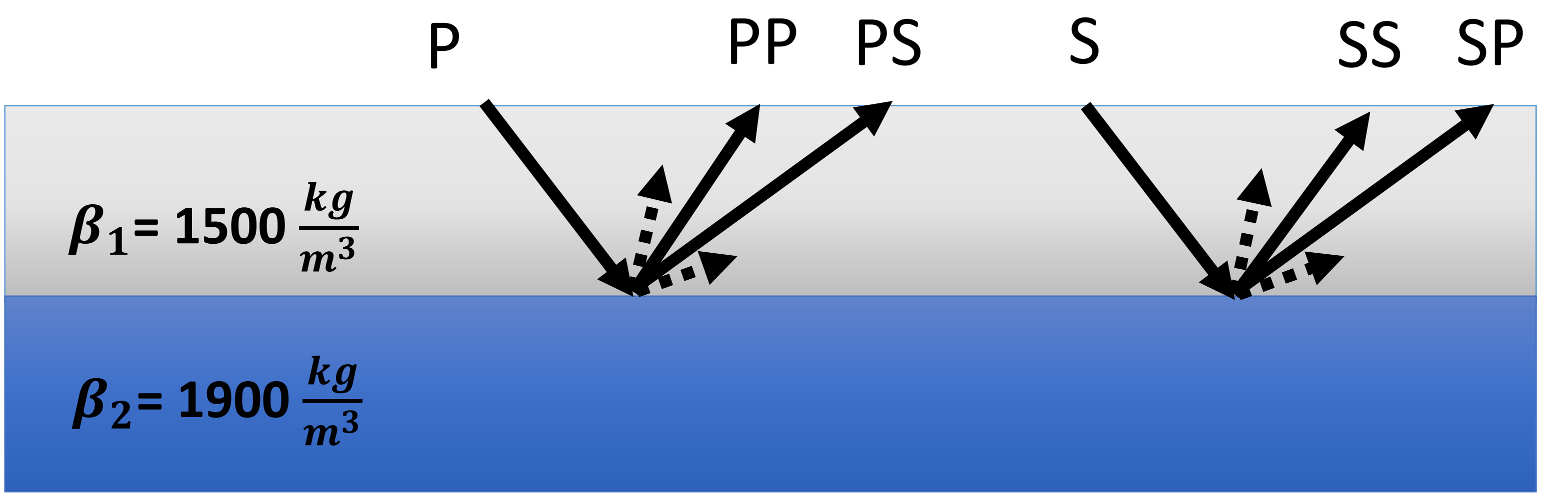
$$ {}^{P}_{P}\mathbb{V}_{visco} = \left({}^{P}_{P}\mathbb{V}^{\beta}_{e} + i\,{}^{P}_{P}\mathbb{V}^{\beta}_{ane}\right) A_{\beta} $$

$$ {}^{SI}_{P}\mathbb{V}_{visco} = \left({}^{SI}_{P}\mathbb{V}^{\beta}_{e} + i\,{}^{SI}_{P}\mathbb{V}^{\beta}_{ane}\right) A_{\beta} $$

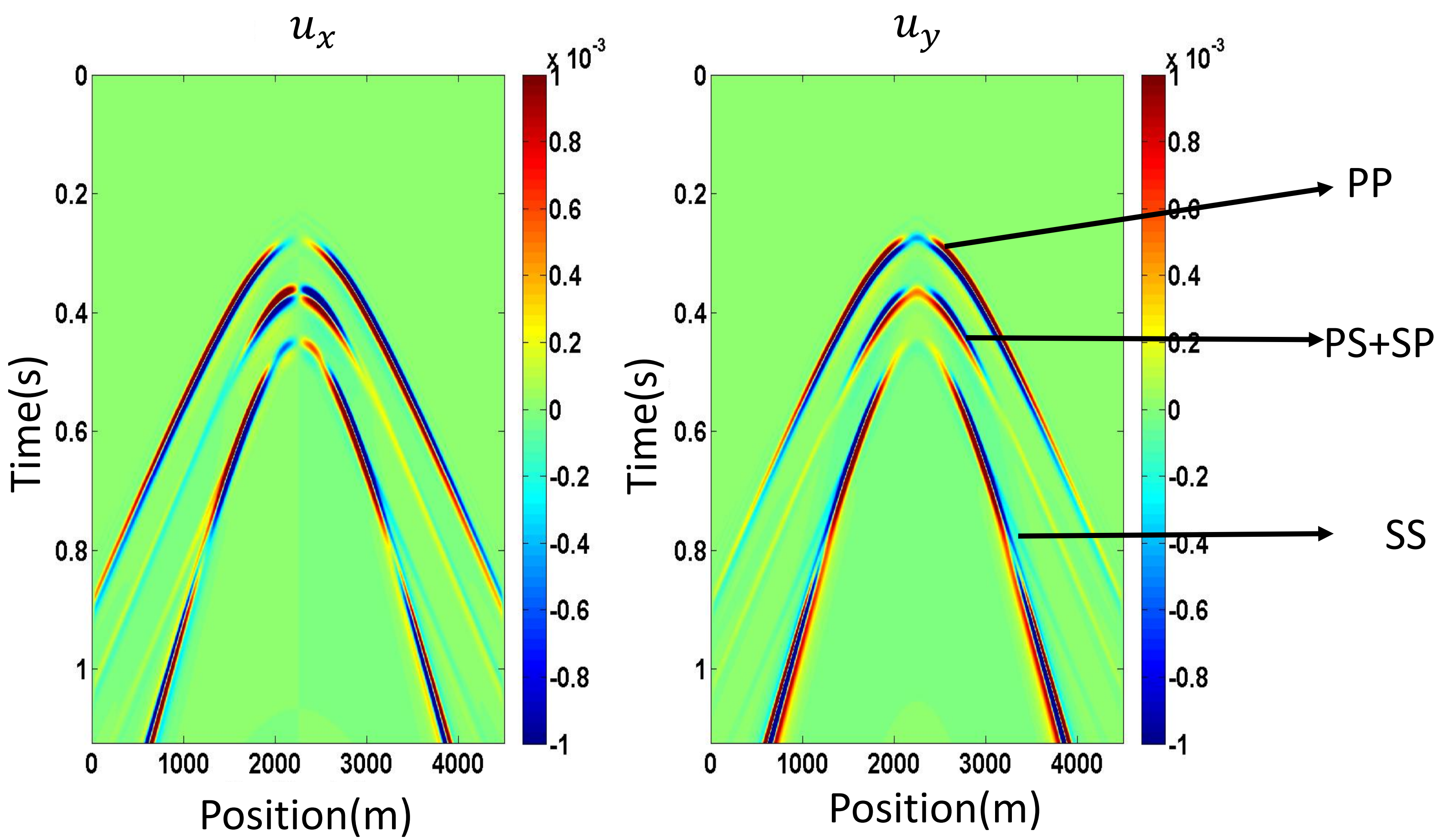
$$ {}^{P}_{SI}\mathbb{V}_{visco} = \left({}^{P}_{SI}\mathbb{V}^{\beta}_{e} + i\,{}^{P}_{SI}\mathbb{V}^{\beta}_{ane}\right) A_{\beta} $$

$$ {}^{SI}_{SI}\mathbb{V}_{visco} = \left({}^{SI}_{SI}\mathbb{V}^{\beta}_{e} + i\,{}^{SI}_{SI}\mathbb{V}^{\beta}_{ane}\right) A_{\beta} $$

Born approximation



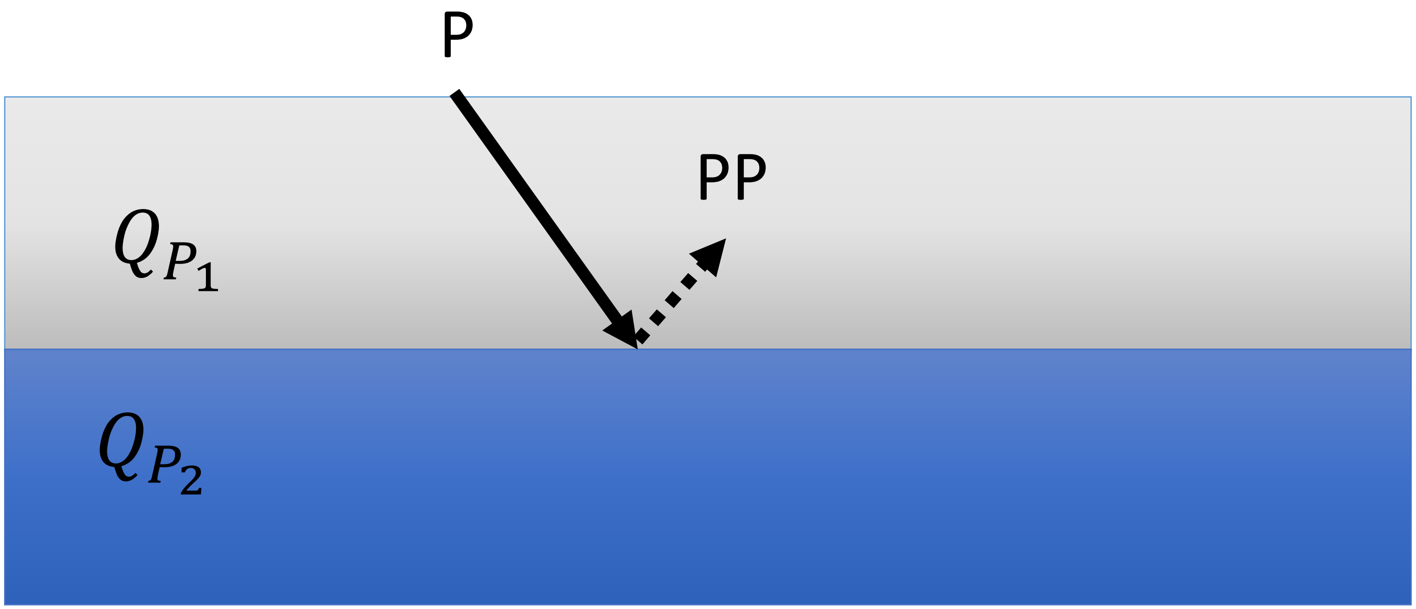
Contrast in S-wave velocity (numeric)



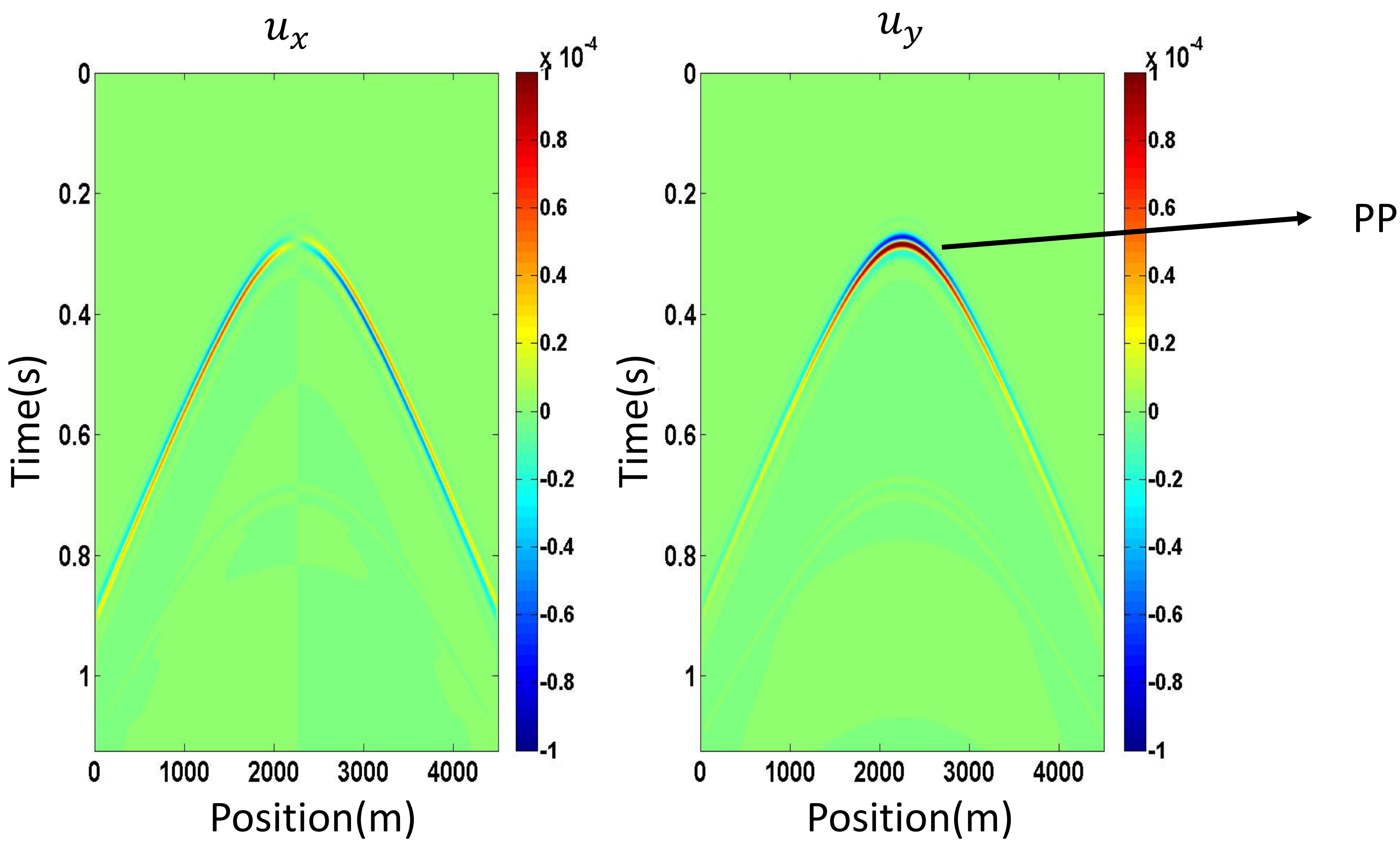
Contrast in P-wave quality factor

$$ {}^{P}_{P}\mathbb{V}_{visco} = i \left({}^{P}_{P}\mathbb{V}^{Q_p}_{ane} \right) A_{Q_p} $$

Born approximation



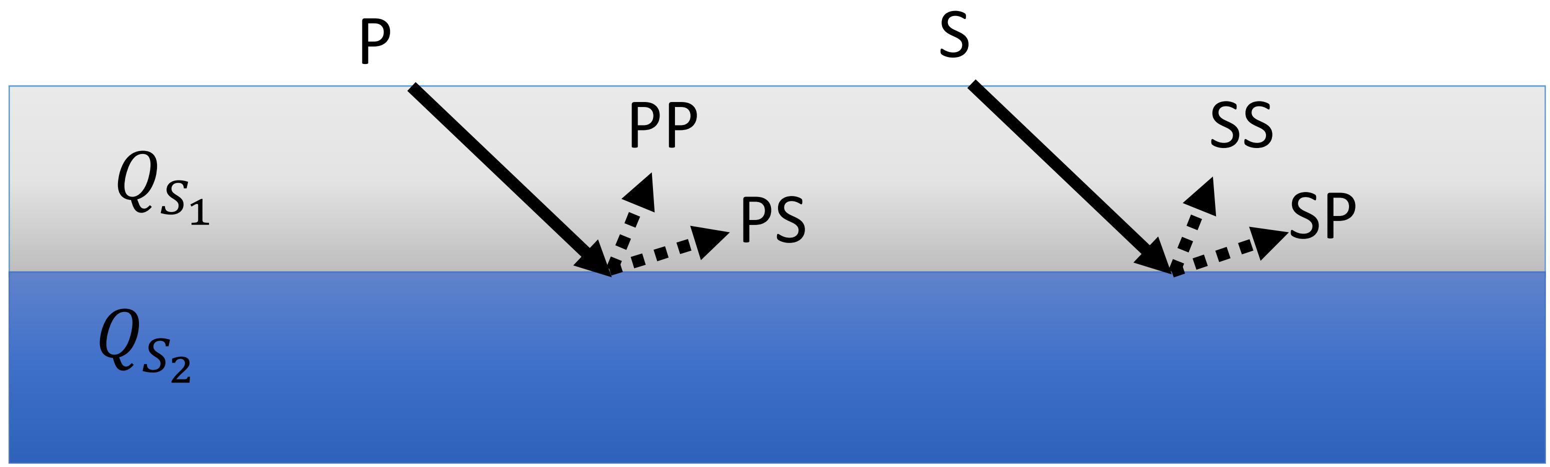
Contrast in P-wave quality factor



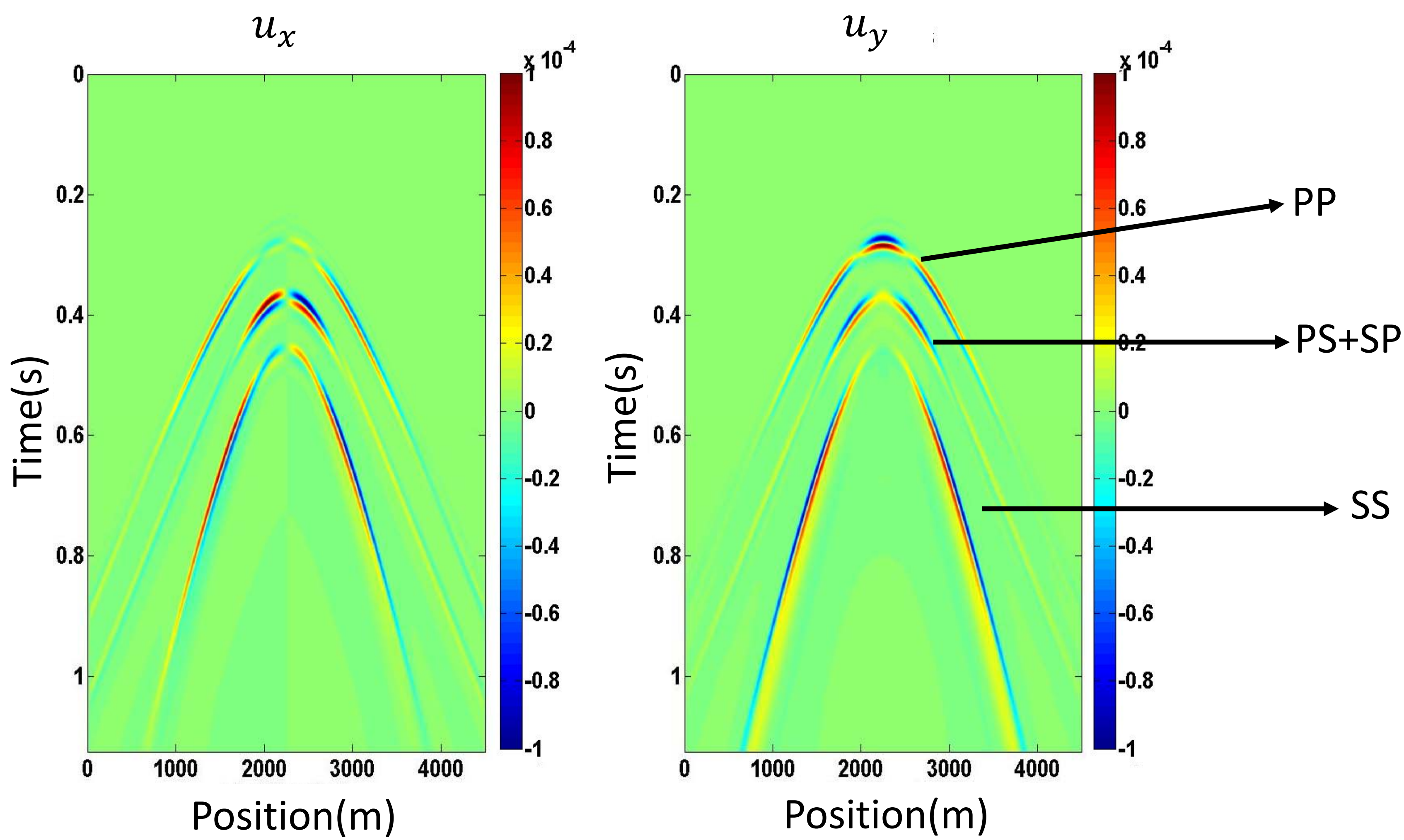
Contrast in S-wave quality factor

$$
\left.
\begin{aligned}
{}^{P}_{P}\mathbb{V}_{visco} &= i\left({}^{P}_{P}\mathbb{V}^{Q_{hs}}_{ane}\right) A_{Q_s} \\
{}^{SI}_{P}\mathbb{V}_{visco} &= i\left({}^{SI}_{P}\mathbb{V}^{Q_{hs}}_{ane}\right) A_{Q_s} \\
{}^{P}_{SI}\mathbb{V}_{visco} &= i\left({}^{P}_{SI}\mathbb{V}^{Q_{hs}}_{ane}\right) A_{Q_s} \\
{}^{SI}_{SI}\mathbb{V}_{visco} &= i\left({}^{SI}_{SI}\mathbb{V}^{Q_{hs}}_{ane}\right) A_{Q_s}
\end{aligned}
\right\}
$$

Born approximation



Contrast in S-wave quality factor



Summary and conclusion

- **Contributions of perturbations in elastic and anelastic properties to the scattered waves are numerically examined.**
- **Scattering potential is a complex function in which the real part is elastic scattering potential and imaginary part corresponds to anelasticity in medium.**
- **Perturbation in quality factor for P-wave generates only P-to-P reflection.**
- **Comparing to the elastic case we expect the changes not only in amplitude of scattered wave but also in the phase behaviour.**
- **This research will feed directly into ongoing CREWES efforts to formulate FWI for multicomponent land data and determine petrophysically important parameters". Qp and Qs are definitely in that class.**

Thank you

- All CREWES sponsors and NSERC