1.5D Internal Multiple Prediction in the Plane wave domain

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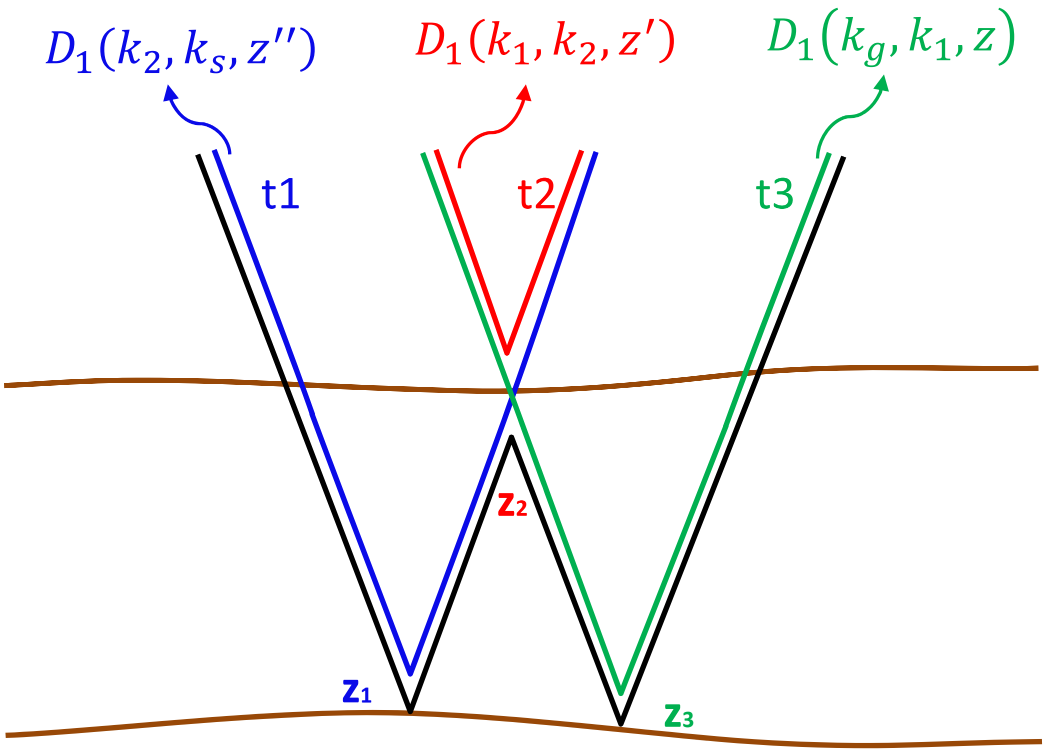
Outline

- Inverse scattering algorithm
- Pseudo-depth monotonicity condition
- IM prediction in the plane wave domain
- Conclusion and Future work
- Acknowledgements

Inverse scattering series algorithm



$t_{IM} = t1 + t3 - t2$

Actual depth relationship:

$z_1 > z_2$ and $z_2 < z_3$

Lower-Higher-Lower

(Weglein et al., 2003)

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2D IM prediction algorithm *(Weglein et al., 1997,2003)*

Primary ③

$$b_{3_{IM}}(k_g,k_s,\omega)=\frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} dk_1 e^{-iq_1(\varepsilon_g-\varepsilon_s)} dk_2 e^{-iq_2(\varepsilon_g-\varepsilon_s)} \int_{-\infty}^{+\infty} dz e^{i(q_g+q_1)z} b_1(k_g,k_1,z)$$

$$\times \int_{-\infty}^{z-\epsilon} dz' e^{-i(q_1+q_2)z'} b_1(k_1,k_2,z') \int_{z'+\epsilon}^{+\infty} dz'' e^{i(q_2+q_s)z''} b_1(k_2,k_s,z'')$$

Primary ②

Primary ①

$$where \qquad q_X=\frac{\omega}{c_0}\sqrt{1-\frac{k_X{}^2 c_0{}^2}{\omega^2}}; \qquad k_z=q_g+q_s; \qquad z=\frac{c_0 t}{2}$$

z *is the Pseudo – depth which satisfied:* $z>z'$ *and* $z'<z''$.

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$\boldsymbol{b_1(k_g, k_s, z)}$ Construction

The procedure for getting the input was given by Innanen (2012):

1. Start with the dataset $\mathrm{d}(x_g, x_s, t)$,
2. Fourier Transform from (x_g, x_s, t) to (k_g, k_s, ω),

$$\mathrm{d}(x_g, x_s, t) \rightarrow \mathrm{D}(k_g, k_s, \omega)$$

3. A change of variables from ω to k_z $(where\ k_z = q_g + q_s)$,

$$\mathrm{D}(k_g, k_s, \omega) \rightarrow \mathrm{D}(k_g, k_s, k_z)$$

4.Then scaled by $-i2q_s$,

$$b_1(k_g, k_s, k_z) = -i2q_s\mathrm{D}(k_g, k_s, k_z)$$

5. Inverse Fourier Transform

$$b_1(k_g, k_s, k_z) \rightarrow b_1(k_g, k_s, z\ )$$

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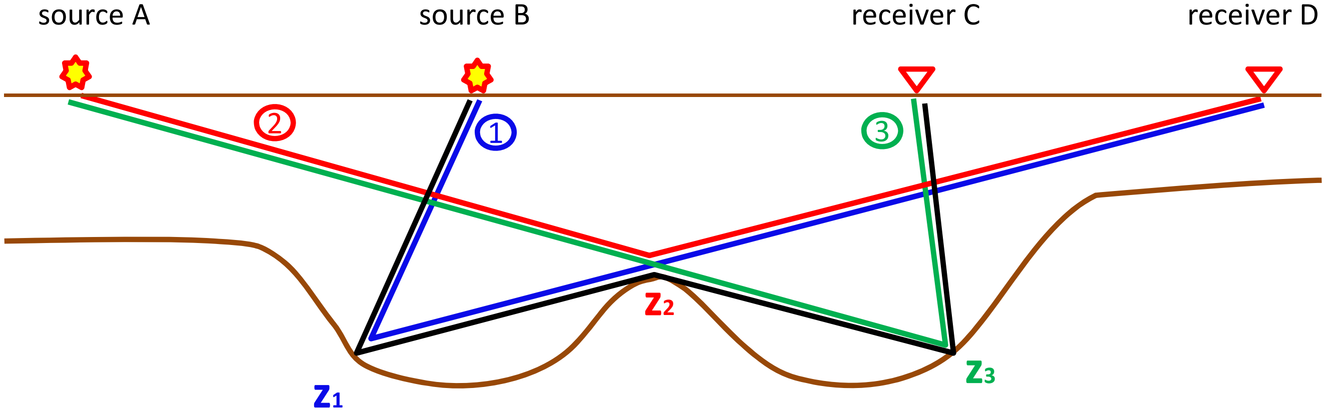
Pseudo-depth monotonicity condition



(Weglein and Nita et al., 2003,2009)

$$z_1^{actual} > z_2^{actual} \Leftrightarrow z_1^{pseudo} > z_2^{pseudo}$$

$$z_1^{actual} > z_2^{actual} \begin{matrix} \Leftarrow \\ \nRightarrow \end{matrix} t_1 > t_2$$

$$z_1^{actual} > z_2^{actual} \Leftrightarrow \tau_1 > \tau_2$$

$$z = \frac{c_0 \tau}{2} \ ?$$

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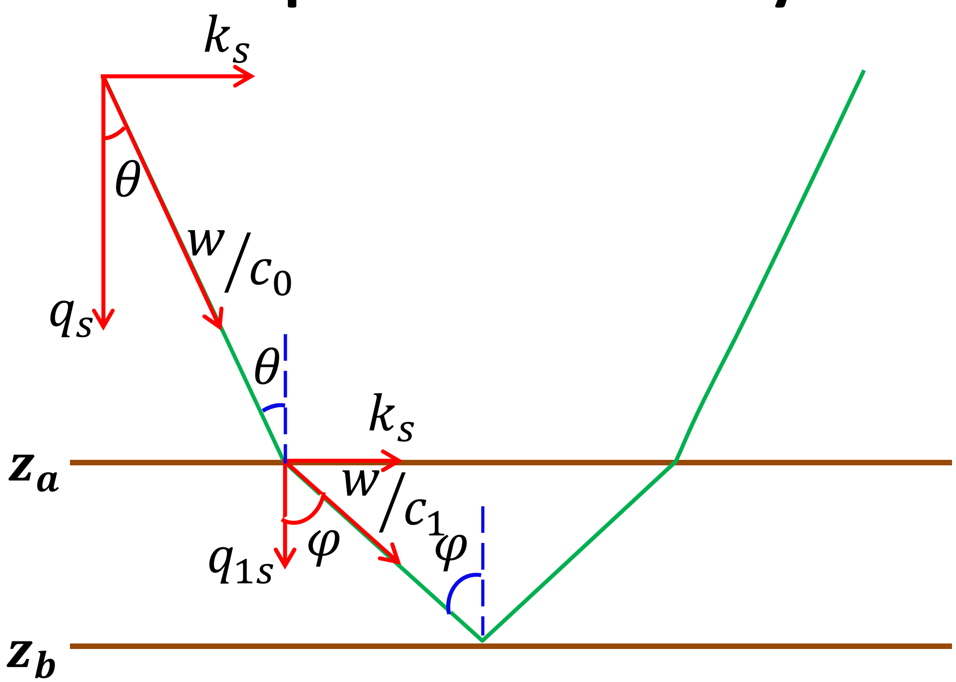
Pseudo-depth monotonicity condition



(Nita et al., 2009)

$$v_x = {}^{c_0}/_{sin\theta} \qquad v_z = {}^{c_0}/_{cos\theta}$$

Intercept time: $\tau = {}^{2z}/_{v_z} = \frac{2zcos\theta}{c_0}$

$$q_s z_a = \frac{wcos\theta}{c_0} * z_a = w\frac{\tau_a}{2} = q_g z_a$$

$$\boldsymbol{k_z z_a = w\tau_a} \qquad \textbf{(1)}$$

$$\boldsymbol{\gamma_1(z_b - z_a) = w(\tau_b - \tau_a)} \qquad \textbf{(2)}$$

$$where\ \ \gamma_1 = q_{1s} + q_{1g} = \frac{2wcos\varphi}{c_1}$$

$$\boldsymbol{k_z z_a + \gamma_1(z_b - z_a) = w\tau_b} \qquad \textbf{(3)}$$

$$\boldsymbol{k_z z_b{}' = w\tau_b} \qquad \textbf{(4)}$$

1.5D IM prediction in $(\boldsymbol{k_x}, \boldsymbol{z})$ domain *(Weglein et al.,1997,2003)*

Assume $k_z = 2q_g = 2q_s$; $k_g = k_s$;

$$b_{3_{IM}}(k_g, \omega) = \int_{-\infty}^{+\infty} dz e^{ik_z z} b_1(k_g, z) \int_{-\infty}^{z-\epsilon} dz' e^{-ik_z z'} b_1(k_g, z') \int_{z'+\epsilon}^{+\infty} dz'' e^{ik_z z''} b_1(k_g, z'')$$

$where \qquad q_X = \frac{\omega}{c_0}\sqrt{1 - \frac{{k_X}^2 {c_0}^2}{\omega^2}};$

$z \; is \; the \; Pseudo - depth: \quad z = \frac{c_0 \tau}{2}$

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1.5D IM prediction algorithm in plane wave domain

In (k_z, z) domain , $\mathrm{d}(x_g, t) \rightarrow \mathrm{D}(k_g, \omega) \rightarrow \mathrm{D}(k_g, k_z) \rightarrow b_1(k_g, k_z) \rightarrow b_1(k_g, k_s, z)$

$k_g = \frac{\omega}{c_0} sin\theta = p_g \omega \Rightarrow (p, z)$ domain: $\mathrm{D}(p_g, \omega) \rightarrow \mathrm{D}(p_g, k_z) \rightarrow b_1(p_g, k_z) \rightarrow b_1(p_g, z)$

(p, z) domain : *(Nita and Weglein, 2009)*

$$b_{3_{IM}}(p_g, \omega) = \int_{-\infty}^{+\infty} dz e^{i2q_g z} b_1(p_g, z) \int_{-\infty}^{z-\epsilon} dz' e^{-i2q_g z'} b_1(p_g, z') \int_{z'+\epsilon}^{+\infty} dz'' e^{i2q_g z''} b_1(p_g, z'')$$

$where \quad q_X = \frac{\omega}{c_0}\sqrt{1 - p_X{}^2 c_0{}^2}$

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1.5D IM prediction algorithm in plane wave domain

$$k_z z = \omega\tau \quad \Longrightarrow \quad \begin{cases} k_z z \longrightarrow \omega\tau \\ b_1(k_g, z) \longrightarrow b_1(p, \tau) \end{cases}$$

(p, τ) domain : *(Coates and Weglein, 1996)*

$$b_{3_{IM}}(p_g, \omega) = \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} b_1(p_g, \tau) \int_{-\infty}^{\tau-\epsilon} d\tau' e^{-i\omega\tau'} b_1(p_g, \tau') \int_{\tau'+\epsilon}^{+\infty} d\tau'' e^{i\omega\tau''} b_1(p_g, \tau'')$$

$where$ p_g is the horizontal slowness, or ray parameter,

τ is the intercept times for primaries.

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$b_1(p,z)$ Construction

Similar to the procedure given by Innanen (2012):

1. Start with the dataset $\mathrm{d}(x,t)$,
2. Tau-p Transform from (x,t) to (p,τ),

$$\mathrm{d}(x,t) \rightarrow D_1(p,\tau)$$

3. Fourier Transform from τ to ω,

$$D_1(p,\tau) \rightarrow \mathrm{D}(p,\omega)$$

4. A change of variables from ω to k_z $(where\ k_z = q_g + q_g)$,

$$\mathrm{D}(p,\omega) \rightarrow \mathrm{D}(p,k_z)$$

5. Then scaled by $-i2q_s$,

$$b_1(p,k_z) = -i2q_s\mathrm{D}(p,k_z)$$

6. Inverse Fourier Transform

$$b_1(p,k_z) \rightarrow b_1(p,z\ )$$

$\boldsymbol{b_1(p,\tau)}$ Construction

1. Start with the dataset $\mathrm{d}(x,t)$,
2. Tau-p Transform from (x,t) to (p,τ),

$$\mathrm{d}(x,t) \to D_1(p,\tau)$$

3. Fourier Transform from τ to ω,

$$D_1(p,\tau) \to \mathrm{D}(p,\omega)$$

4.Then scaled by $-i2q_s$,

$$b_1(p,\omega) = -i2q_s\mathrm{D}(p,\omega\ )$$

5. Inverse Fourier Transform

$$b_1(p,\omega) \to b_1(p,\tau\ )$$

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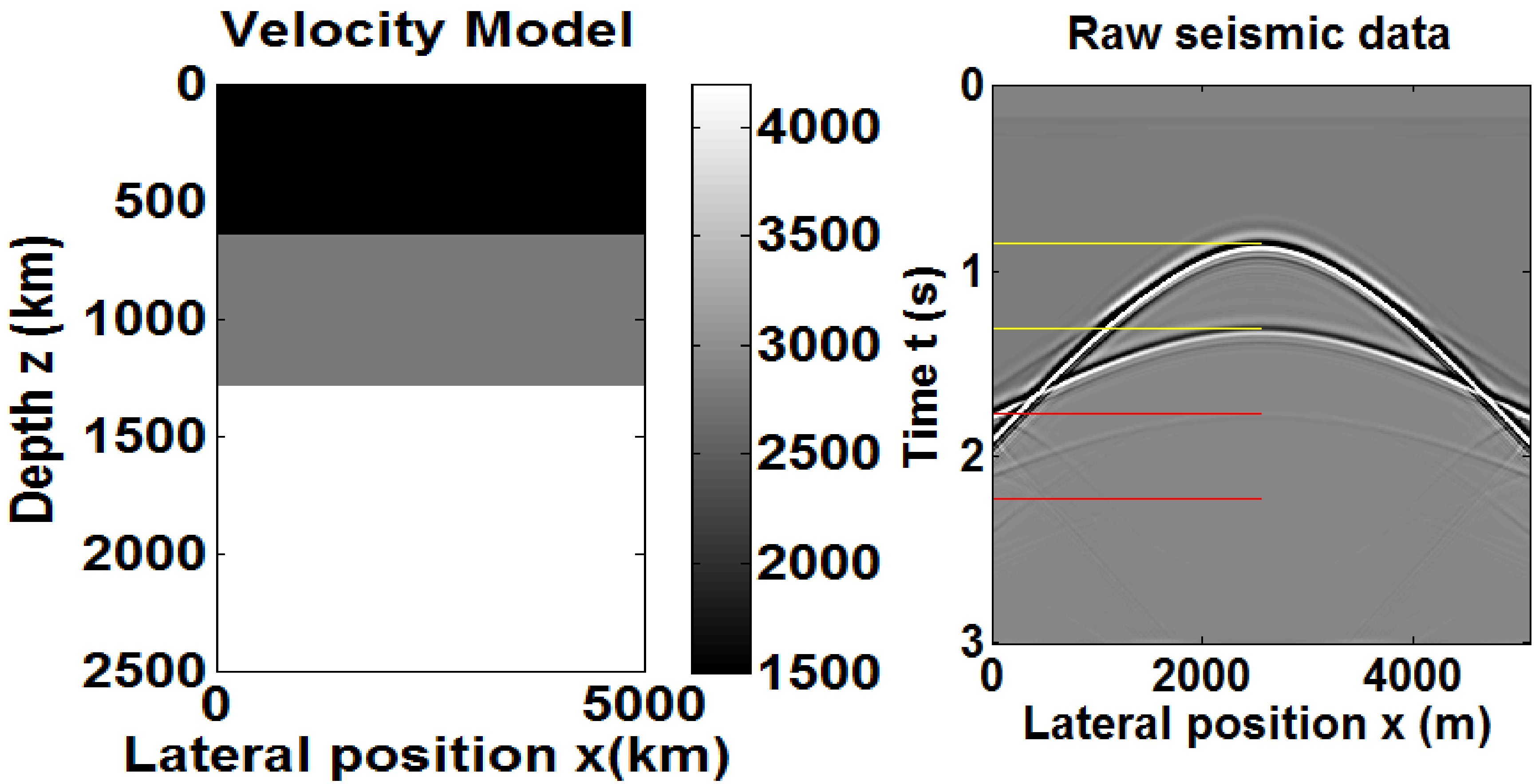
IM prediction on synthetic data



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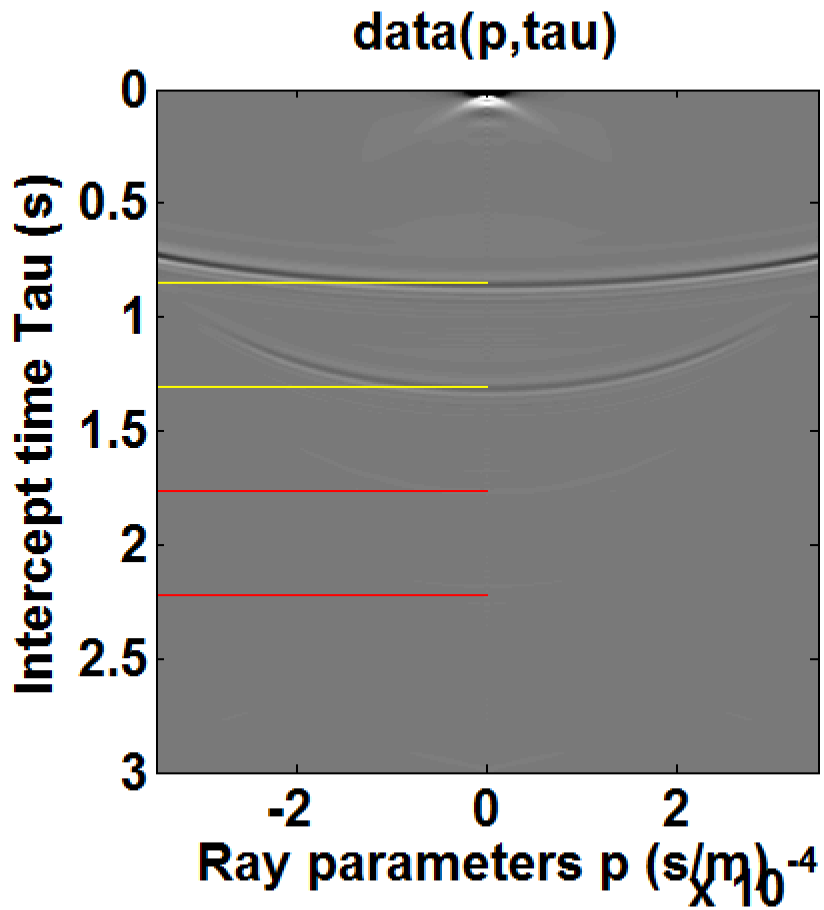
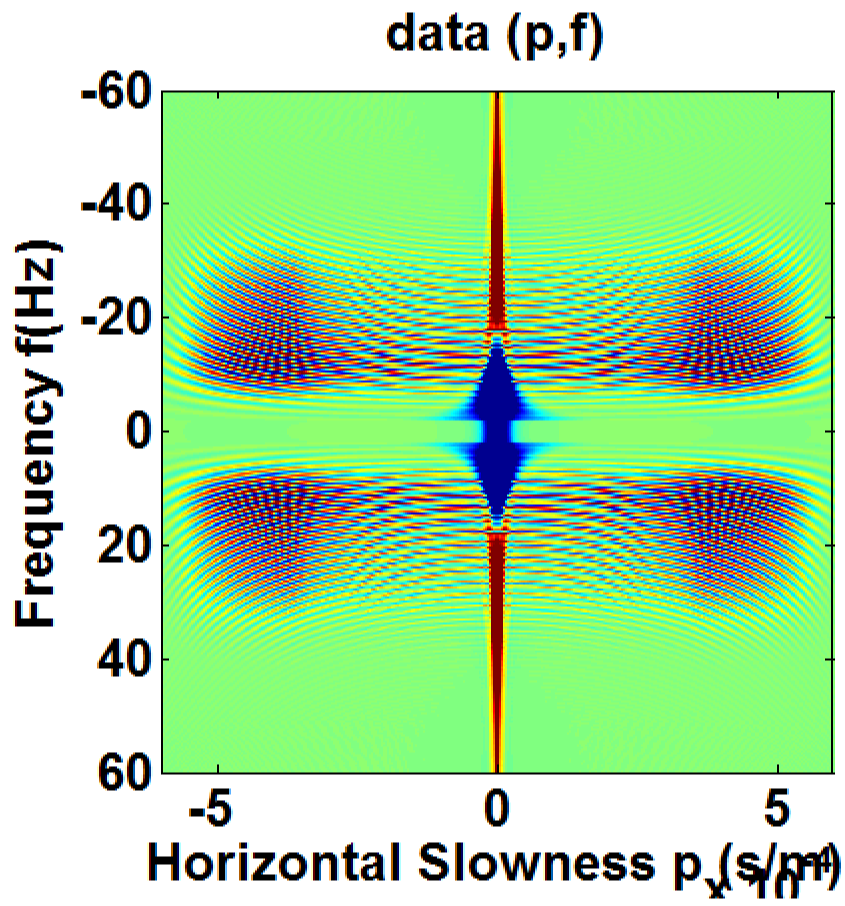
IM prediction on synthetic data





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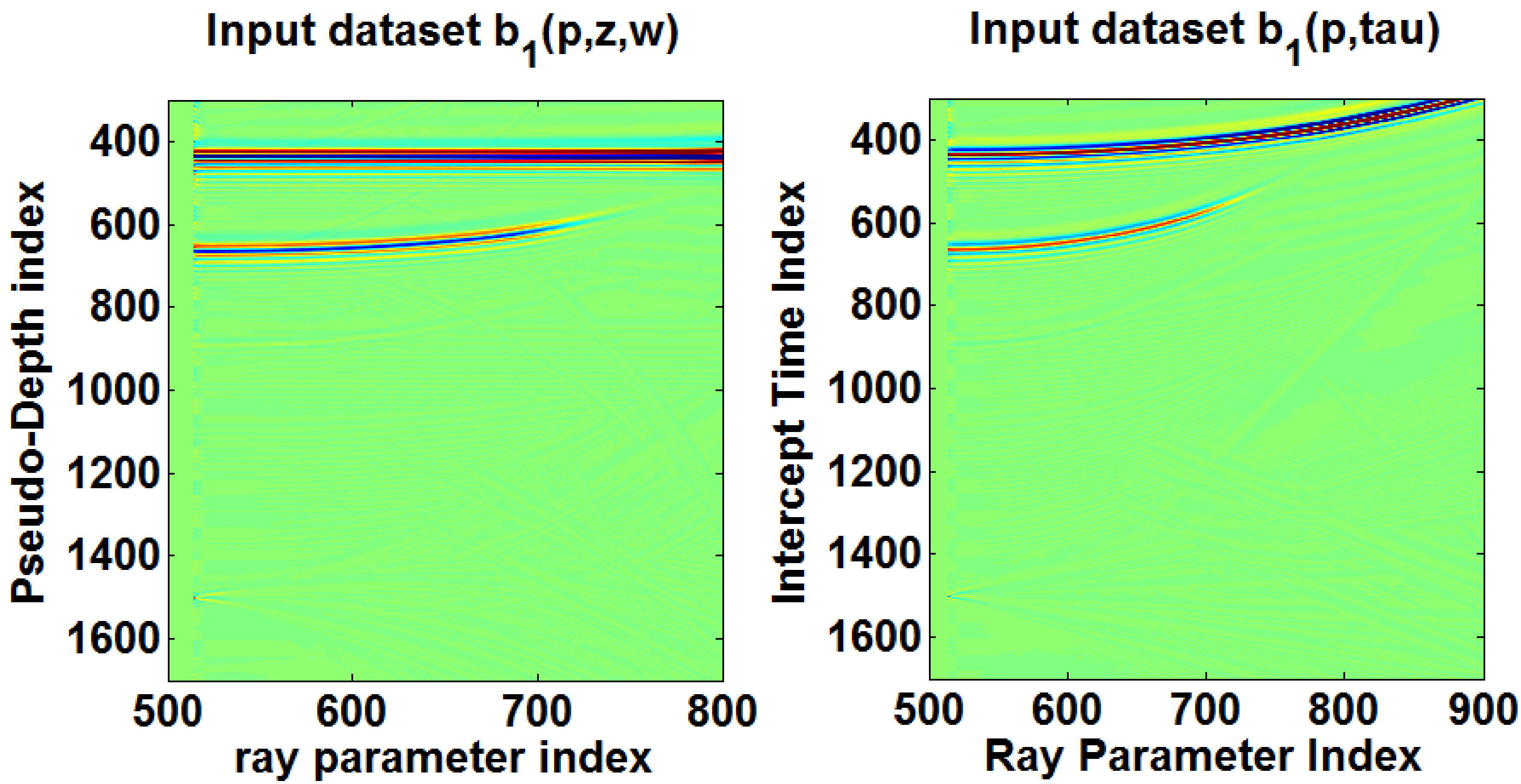
IM prediction on synthetic data



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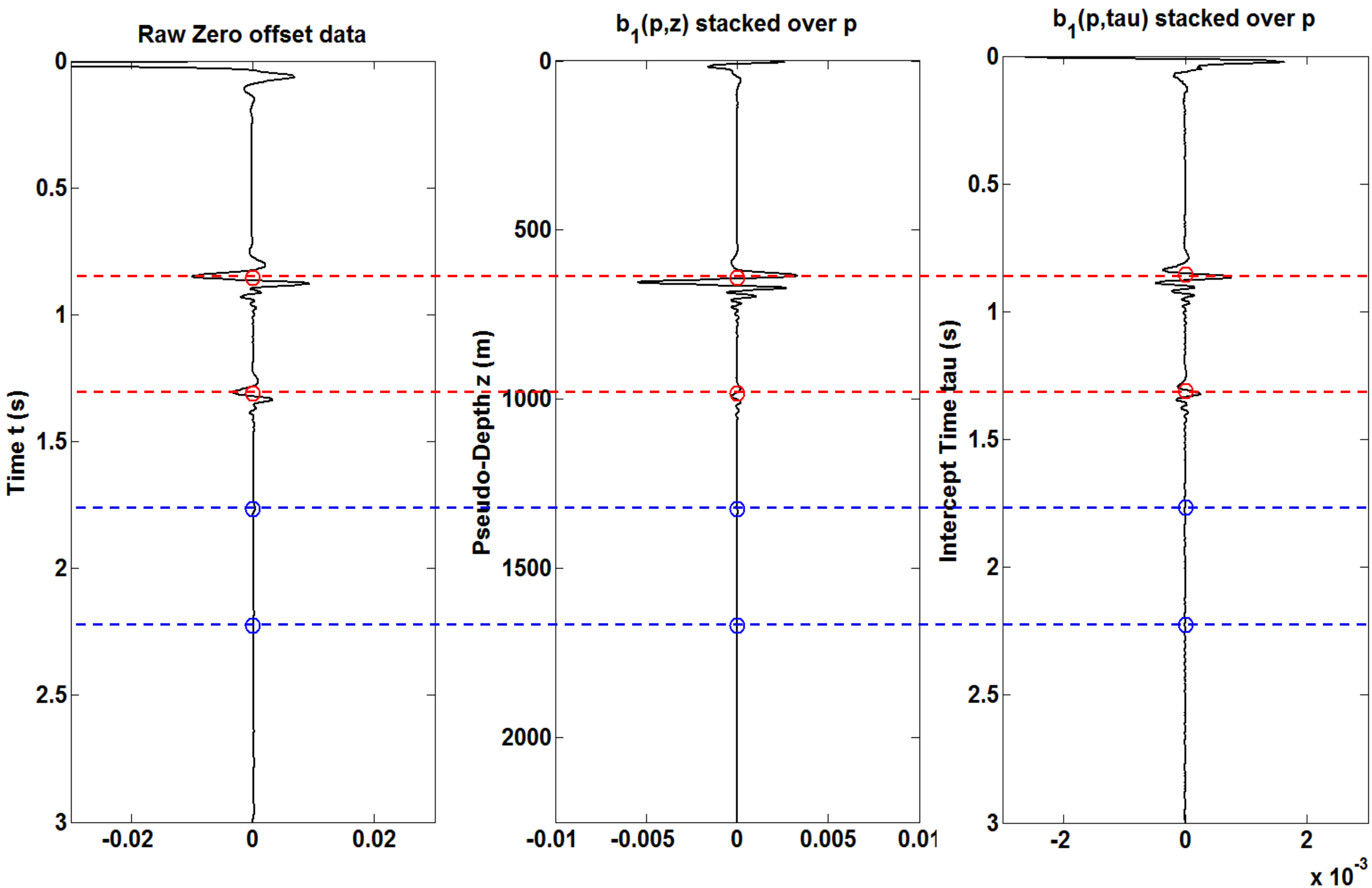
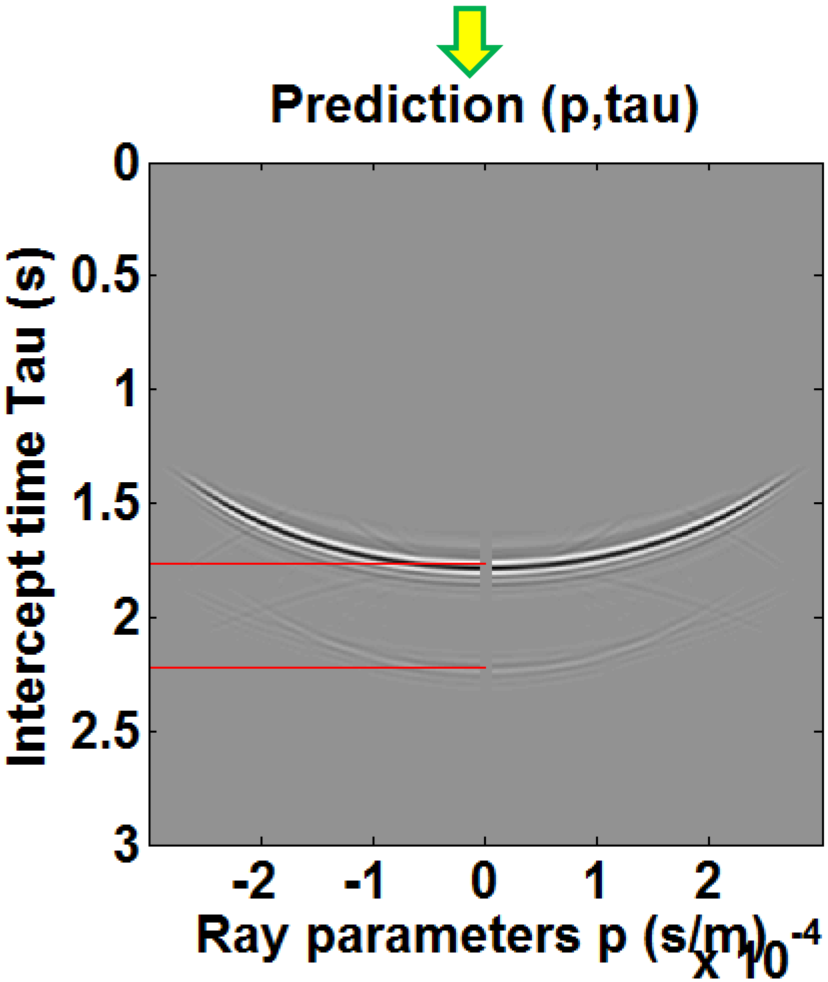
IM prediction on synthetic data

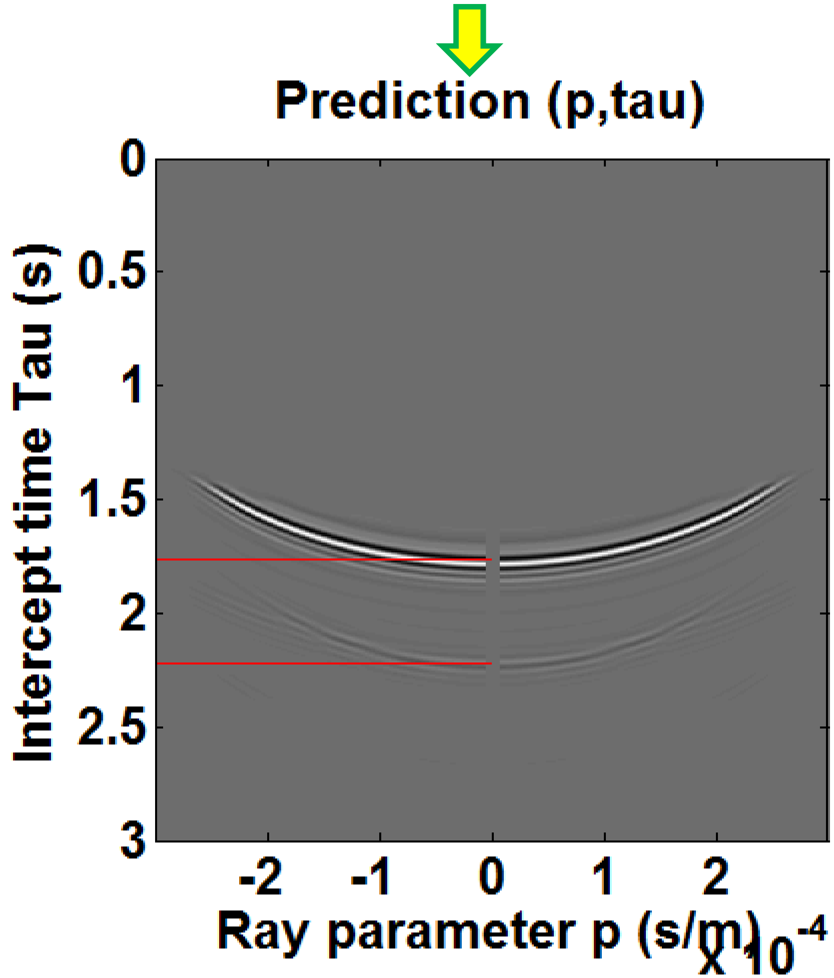


IM prediction on synthetic data

Input data: $b_1(p, z)$



Input data: $b_1(p, \tau)$



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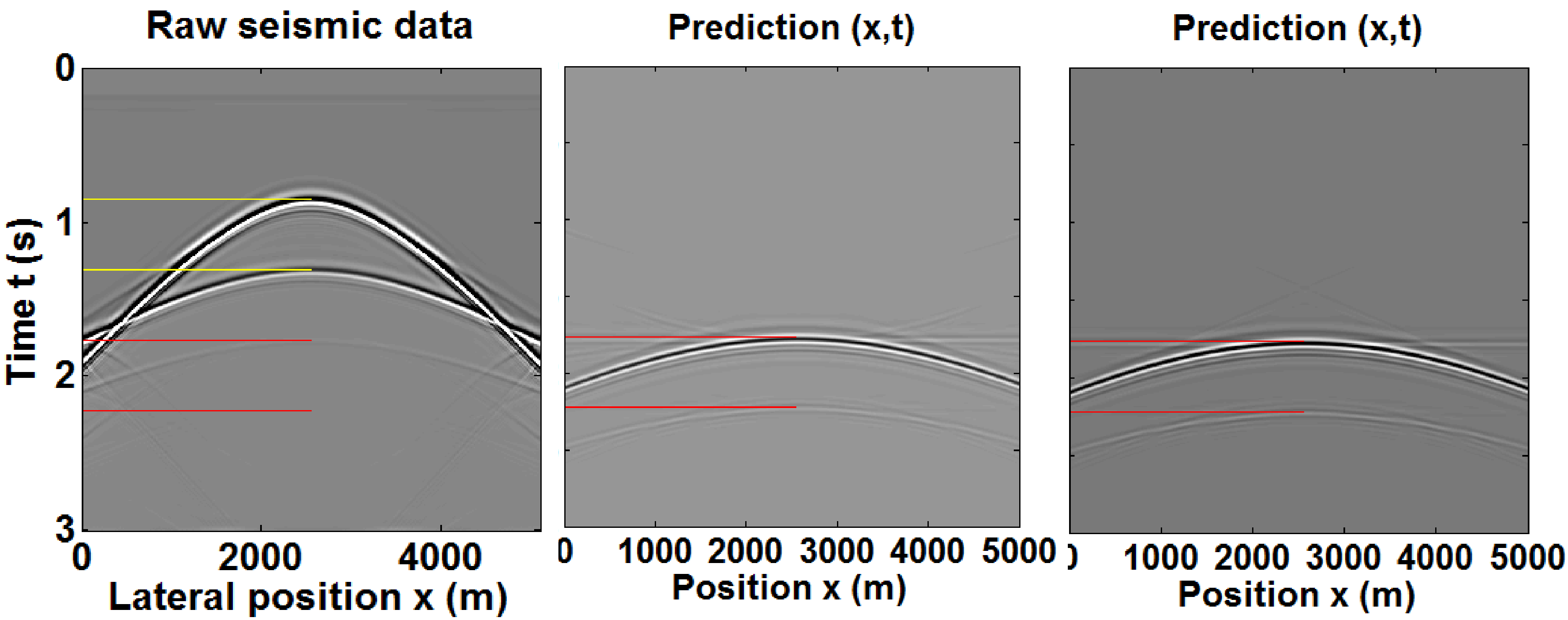
IM prediction on synthetic data

Raw seismic data | *Integral of* $b_1(p, z)$ | *Intergral of* $b_1(p, \tau)$



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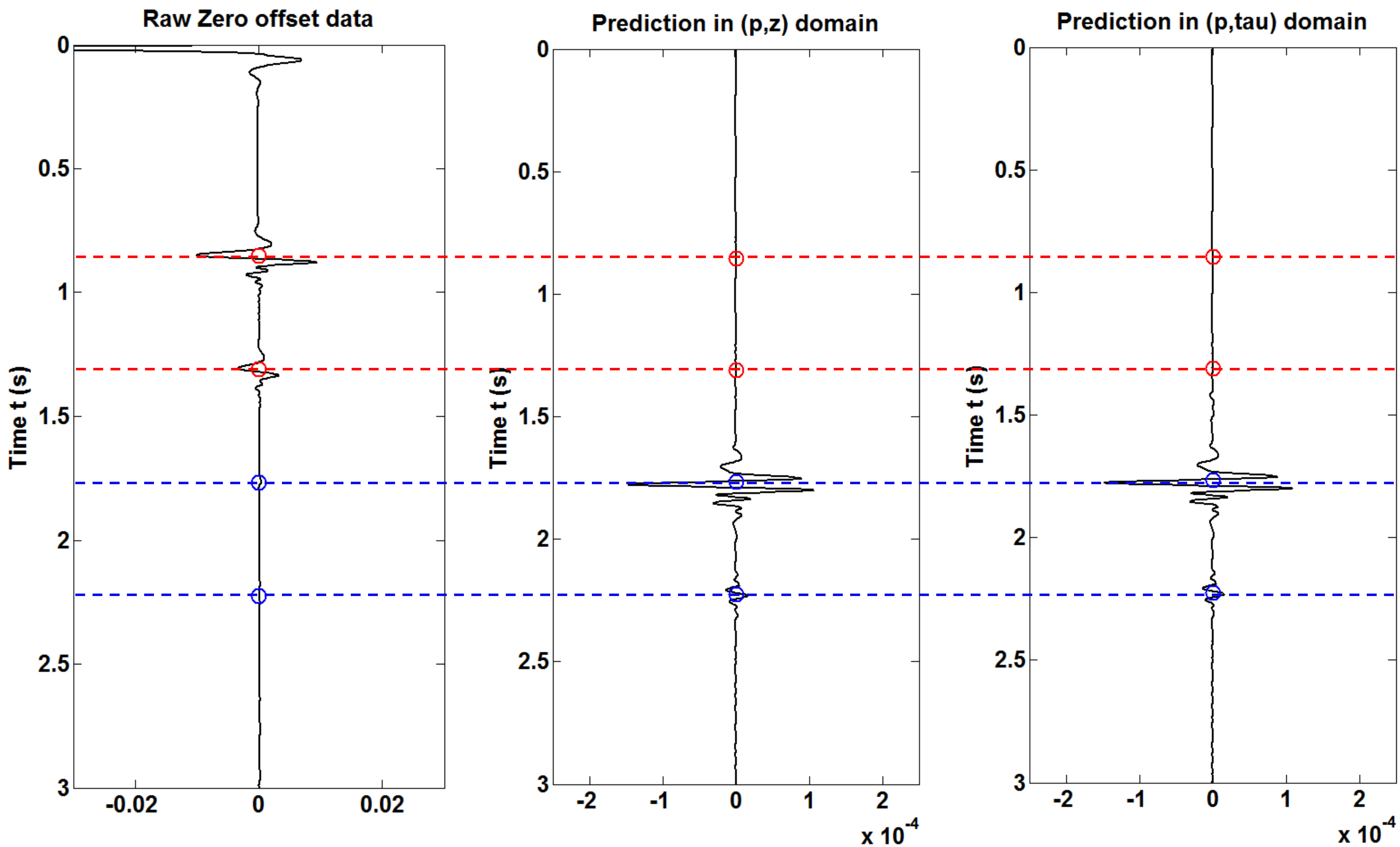
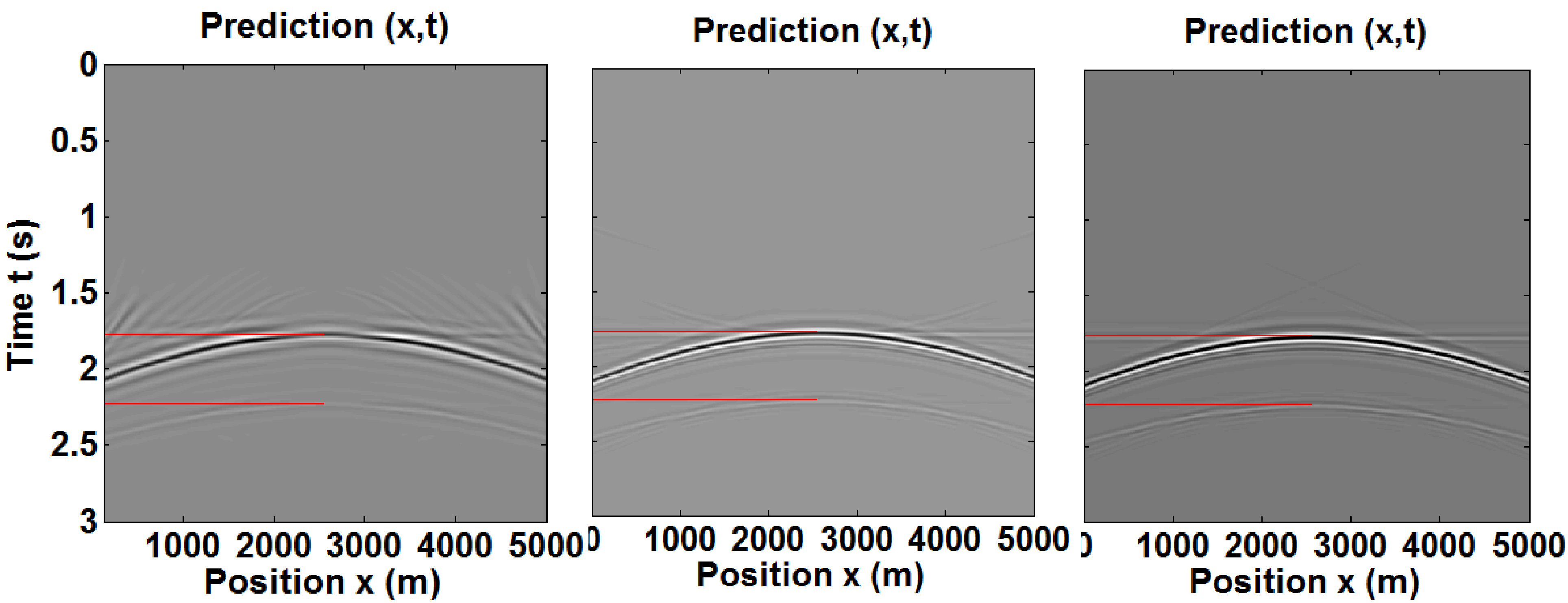
IM prediction on synthetic data



IM prediction on synthetic data

$Integral\ of\ b_1(k_x, z)$ | $Integral\ of\ b_1(p, z)$ | $Intergral\ of\ b_1(p, \tau)$



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IM prediction algorithm (2D) in the plane wave domain

$$b_{3_{IM}}(p_g,p_s,\omega)=\frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} dp_1 e^{-iq_1(\varepsilon_g-\varepsilon_s)}dp_2 e^{-iq_2(\varepsilon_g-\varepsilon_s)}\int_{-\infty}^{+\infty} dz e^{i(q_g+q_1)z}b_1(p_g,p_1,z)$$

$$\times\int_{-\infty}^{z-\epsilon} dz' e^{-i(q_1+q_2)z'}b_1(p_1,p_2,z')\int_{z'+\epsilon}^{+\infty} dz'' e^{i(q_2+q_s)z''}b_1(p_2,p_s,z'')$$

$$b_{3_{IM}}(p_g,p_s,\omega)=\frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} dp_1 e^{-i\omega(\tau_{1g}-\tau_{1s})}dp_2 e^{-i\omega(\tau_{2g}-\tau_{2s})}\int_{-\infty}^{+\infty} d\tau e^{i\omega\tau}b_1(p_g,p_1,\tau)$$

$$\times\int_{-\infty}^{\tau-\epsilon} d\tau' e^{-i\omega\tau'}b_1(p_1,p_2,\tau')\int_{\tau'+\epsilon}^{+\infty} d\tau'' e^{i\omega\tau''}b_1(p_2,p_s,\tau'')$$

Conclusion and Future Work

Inverse Scattering Series algorithm also works for predicting IM in the plane wave domain, both in (p, z) domain and in (p, τ) domain.

An improved numerical accuracy and reduced Fourier artifacts IM prediction can be generated by using inverse Scattering Series algorithm in the plane wave domain.

The procedure for generating the input data in the plane wave domain is greatly simplified.

Free-surface multiples suppression with a predictive deconvolution often works better in the plane wave domain.

We believe the plane wave domain is a promising way of posing full 2D versions of the algorithm.

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Thanks
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Questions ?