Five-dimensional interpolation: exploring different Fourier operators

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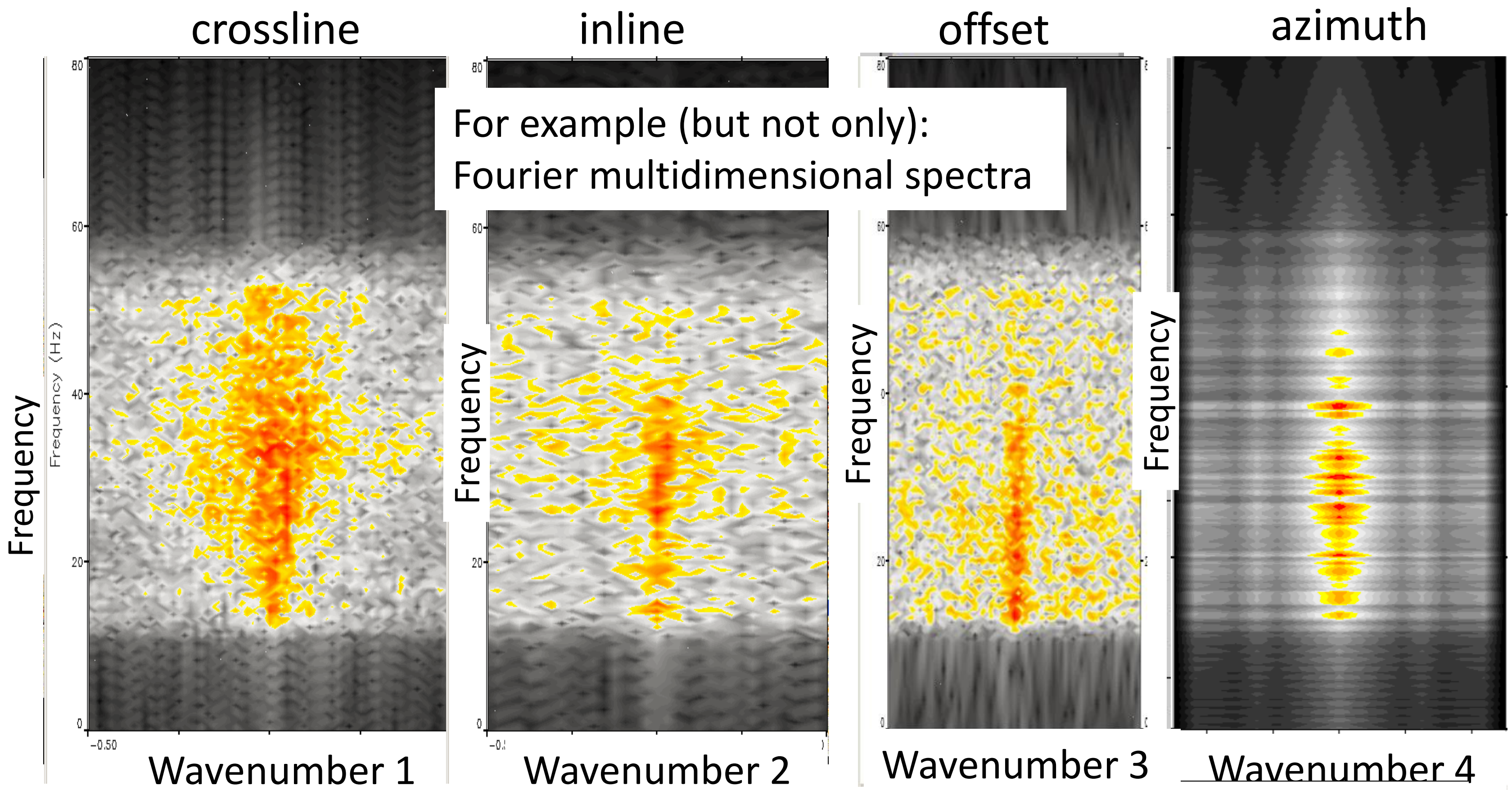
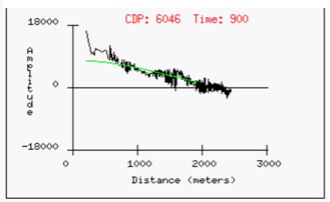
Overview

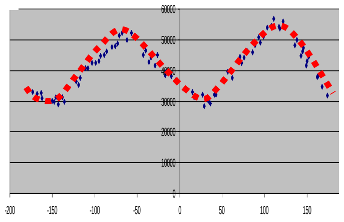
- 5D interpolation: the big picture.
- Least squares inversion: all is in the operator.
- Fourier operators:
  - FFT4 with binning
  - DFT no binning
  - NFFT interpolation + FFT
- Examples:
  - Survey coordinates with jittering
  - AVAz example with flooding
  - Orthogonal survey with real data.
- Conclusions

The big picture:

Interpolation is a modeling process that honors recorded data







By choosing meaningful spatial dimensions it can honor geology=structure + AVO + AVAz

Least squares formulation: all is in the operator.

Change of variables

$\tilde{L}$ modified operator (left and right preconditioned)

$\tilde{d}$ modified data (left weight data)

$\tilde{m}$ modified model (inverse left weighted model)

$$\tilde{\mathrm{d}} = \mathrm{W_d d}$$

$$\tilde{\mathrm{m}} = \mathrm{W_m m}$$

$$\tilde{\mathrm{L}} = \mathrm{W_d L W_m^{-1}}$$

Modeling equation in new variables

$$\tilde{\mathrm{d}} = \tilde{\mathrm{L}}\tilde{\mathrm{m}} \Leftrightarrow \mathrm{W_d d} = \mathrm{W_d L W_m^{-1} W_m m}$$

Cost function in new variables

$$J = \left\| \tilde{\mathbf{d}} - \tilde{\mathbf{L}}\tilde{\mathbf{m}} \right\|^2 + \lambda \left\| \tilde{\mathbf{m}} \right\|_{\mathbf{I}}$$

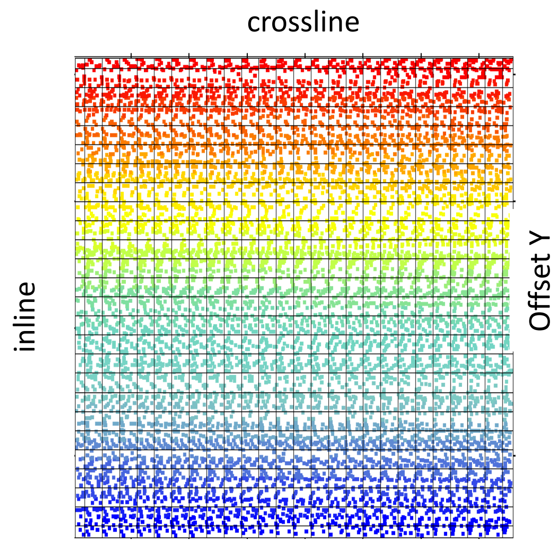
Data residuals in a particular norm choice

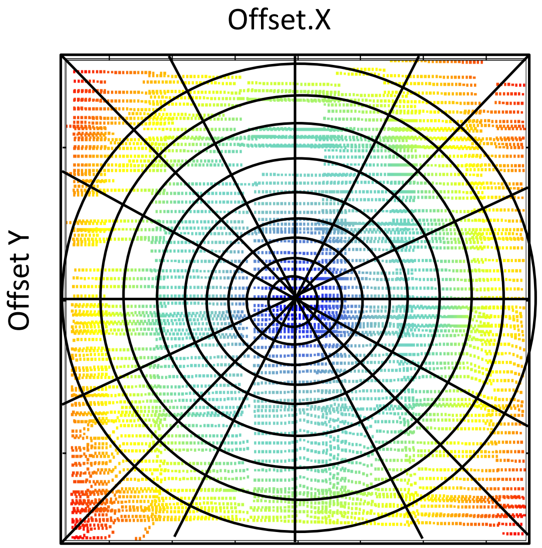
Model size in a particular norm choice

Inversion solution in new variables

$$\tilde{\mathbf{m}} = (\tilde{\mathbf{L}}^{\mathbf{H}}\tilde{\mathbf{L}} + \lambda\mathbf{I})^{-1}\tilde{\mathbf{L}}^{H}\tilde{\mathbf{d}}$$

To Bin or not to bin





Neighbor grid points contain redundant information

$$ratio = \frac{\text{original}}{\text{ngridpoints}} = 3\%$$

F is 4D FFT for regular data
Irregular data requires binning along spatial dimensions.
Also:

- **Traces in the same bins have to be deal with**
- **Cell sizes have to be adapted for each group.**
- **Different axis have different tolerance to binning.**

$$x = x_{\min} \ldots x_{\max}$$

$$k_x = k_{x\min} \ldots k_{x\max}$$

$$\mathbf{F} = \exp(-\mathrm{i}k_x \mathrm{x})$$

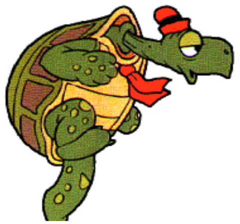
F is Discrete Fourier Transform
True positions are used (no binning).
Each iteration involves expensive matrix multiplications

$$U_j = \sum_{x_{min}}^{x_{max}} \exp\left(-ik_j x_i\right) \times u_i$$

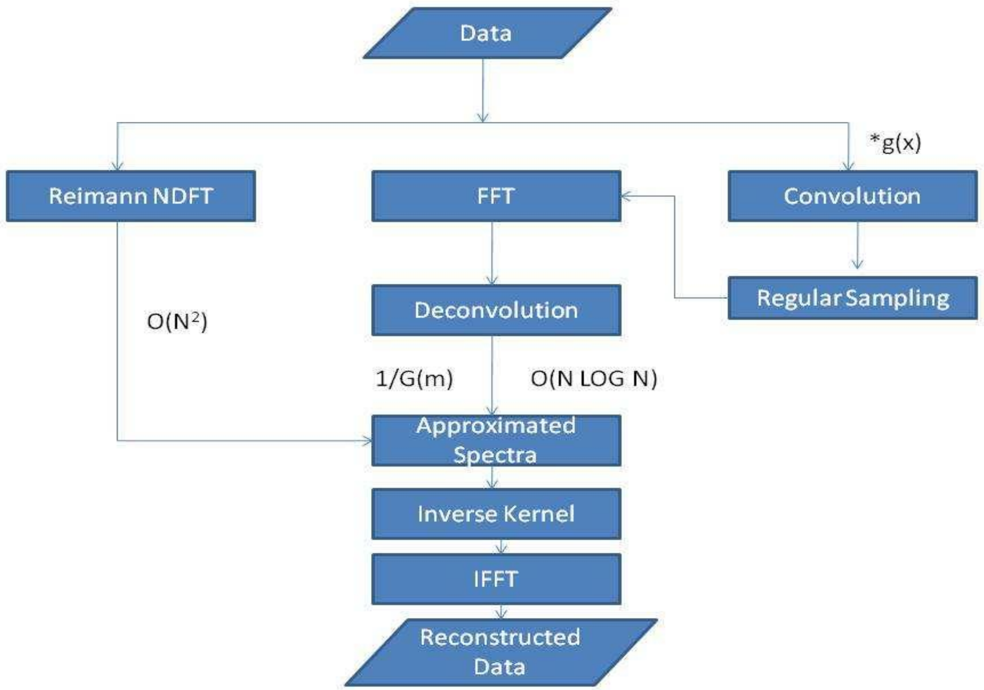
$$\mathbf{U_j} = \sum_{\mathbf{x1_{min}}}^{\mathbf{x1_{max}}} \sum_{\mathbf{x2_{min}}}^{\mathbf{x2_{max}}} \sum_{\mathbf{x3_{min}}}^{\mathbf{x3_{max}}} \sum_{\mathbf{x4_{min}}}^{\mathbf{x4_{max}}} \exp\left(-\mathbf{ik_j x_i}\right) \times \mathbf{u_i}$$

$$\mathbf{u_i} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix} \text{ spatial coordinates}$$

$$\mathbf{U_j} = \begin{bmatrix} U_1 & U_2 & U_3 & U_4 \end{bmatrix} \text{ wavenumber coordinates}$$

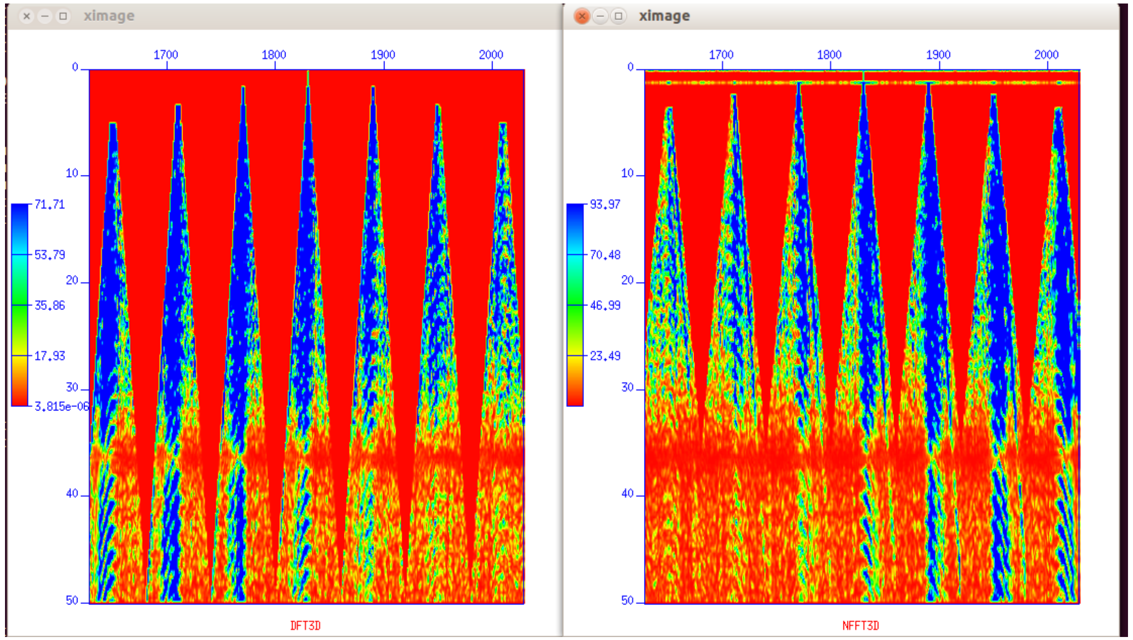


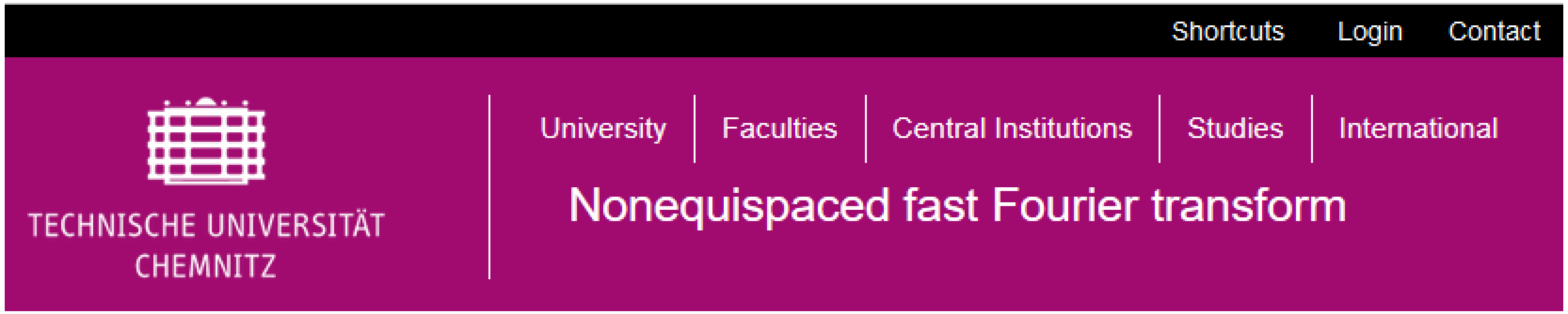
NFFT: interpolation + FFT



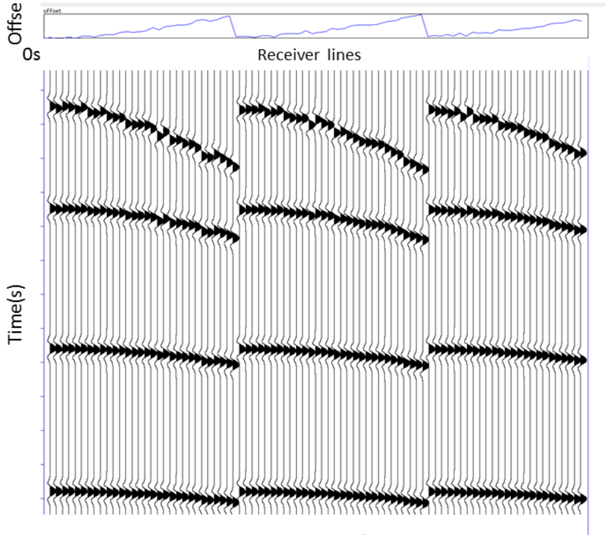
Flow diagram for NFFT (from Gulati and Fergusson, 2009)

DFT for a 3D gather NFFT for a 3D gather





Different Fourier Operators



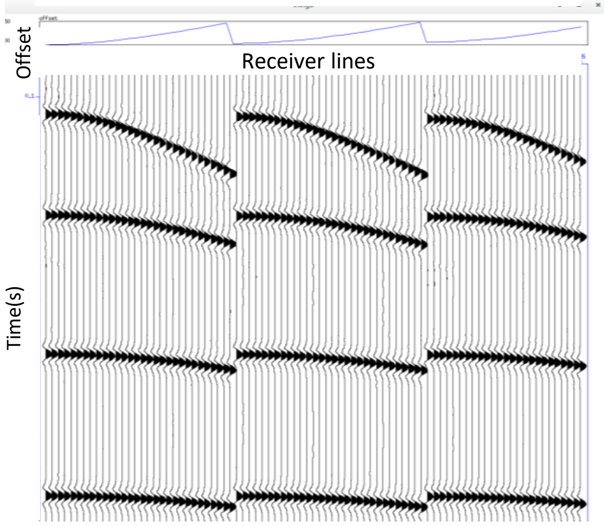
Irregular shots/receivers

Three different operators for calculate multidimensional Fourier spectra

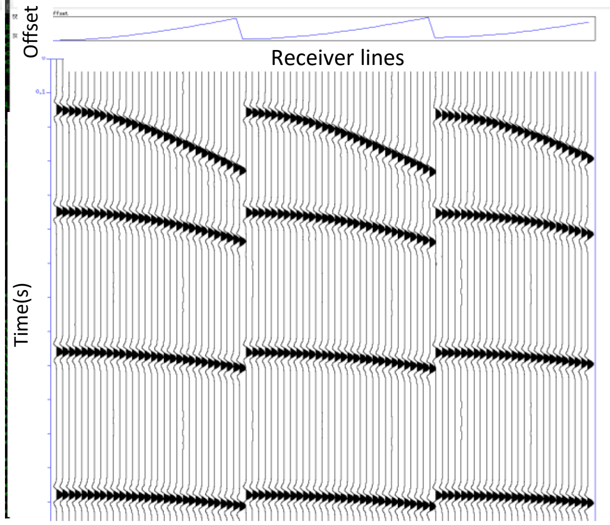
- DFT exact locations
- FFT with binning
- NFFT interpolation + binning

Given enough samples, the three operators give exactly the same spectra, Therefore, the three produce the same data after inverse Fourier transform.

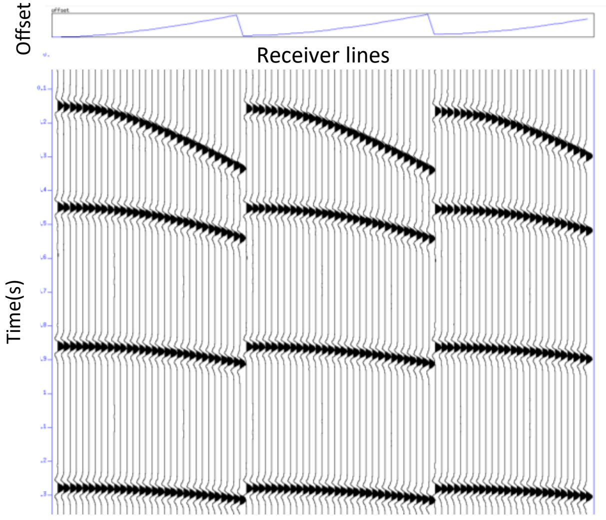
Note: if there is enough sampling, curvature is not a problem



DFT operator



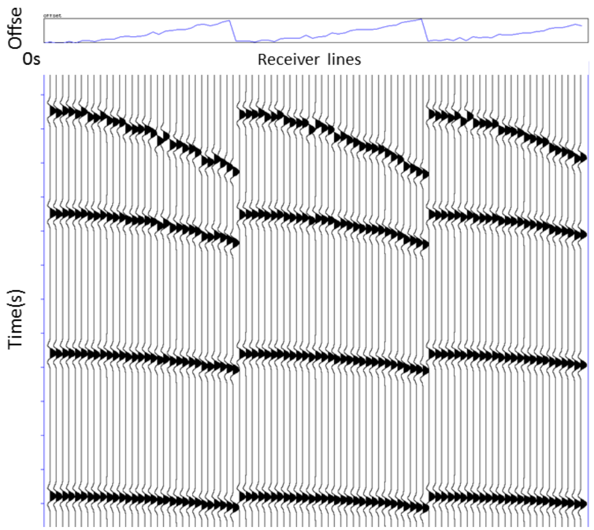
FFT operator



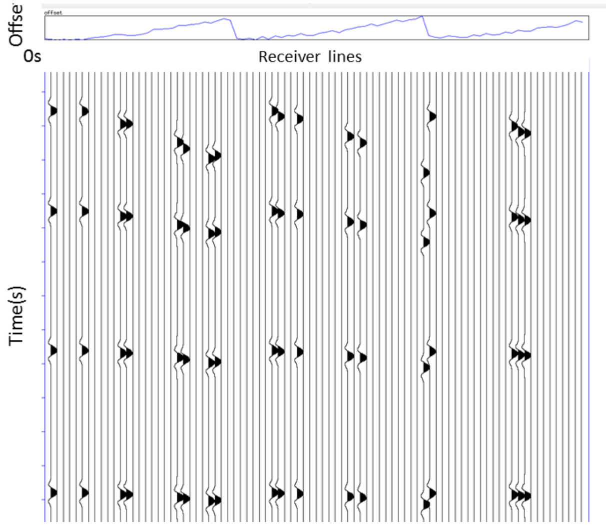
NFFT operator

Example 1: regularization to source coordinates.

Note: curvature handling depends on binning not Fourier transform.



Irregular full data

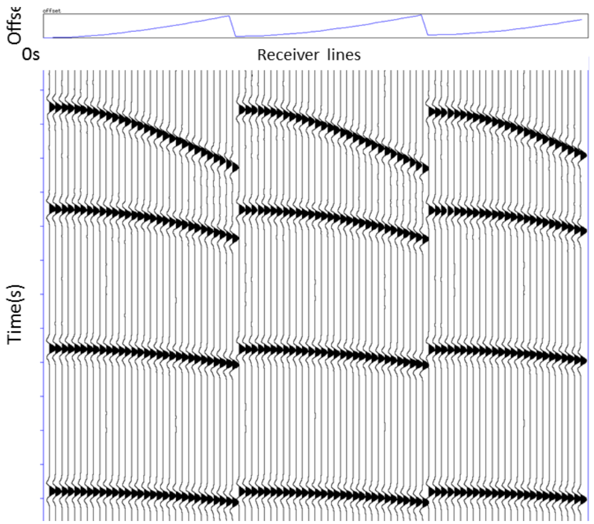


Irregular input data

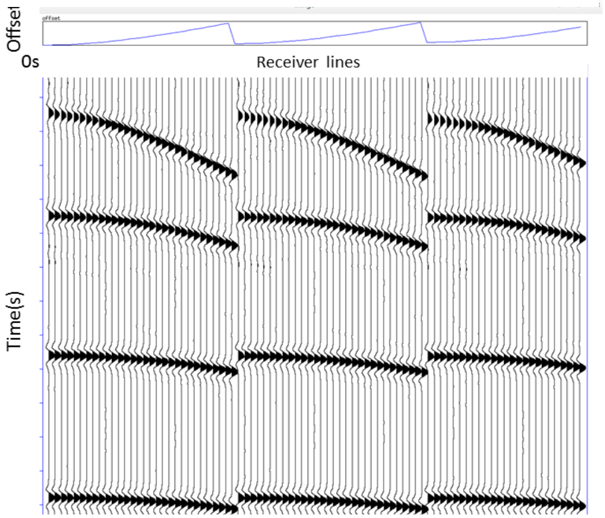
- Jitter due to irregular shot and receiver coordinates.
- Decimation by 75%.
- No NMO applied for stressing test.

Results

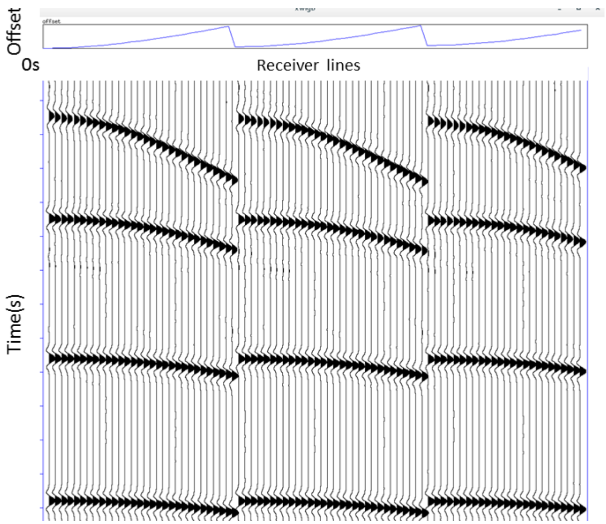
- Regularization with either method moves shots and receivers to regular locations.
- Jitter disappears.
- No binning errors if done properly.
- All can handle curvature but DFT the best.



DFT operator

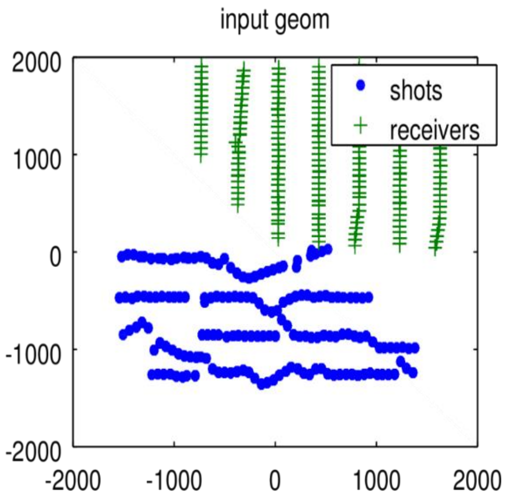


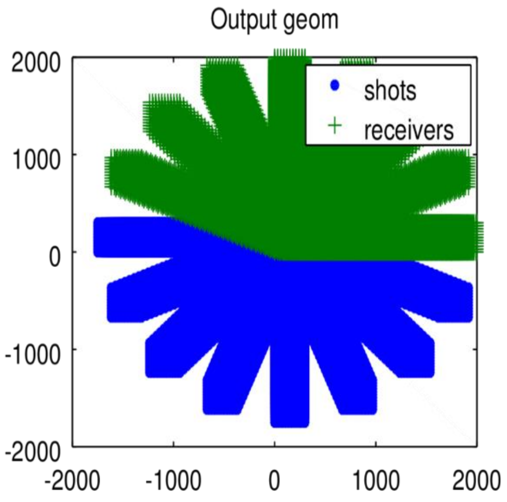
FFT4 operator

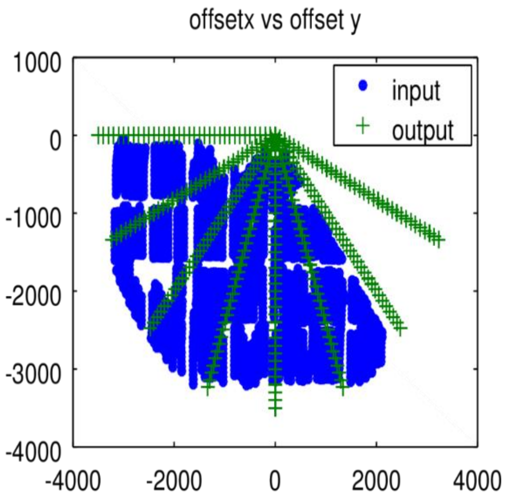


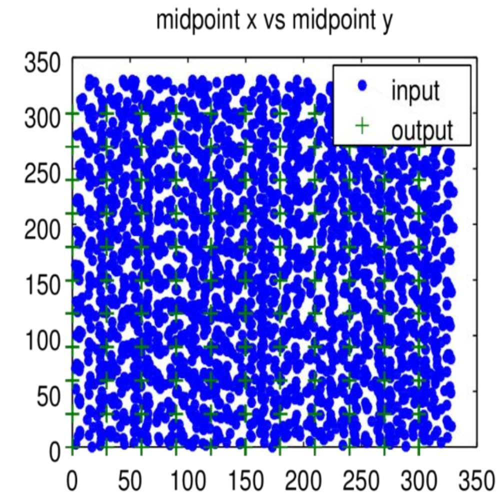
NFFT operator

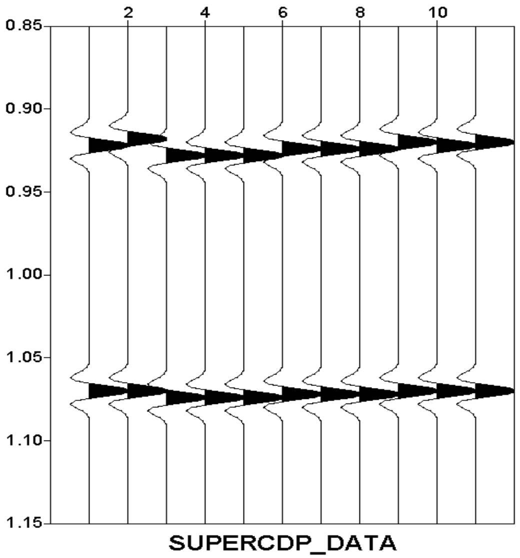
Example 2: AVAz in flooding regularization

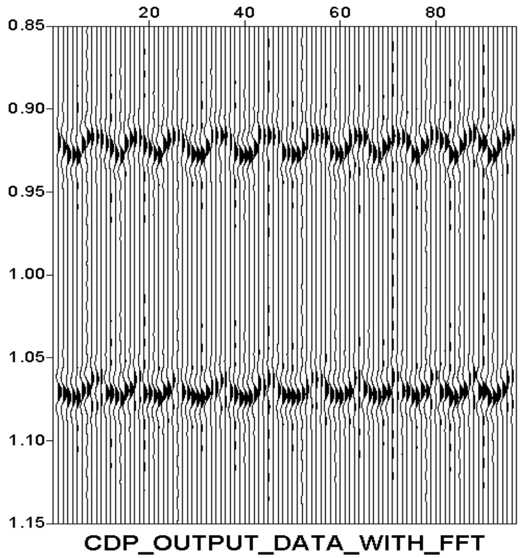


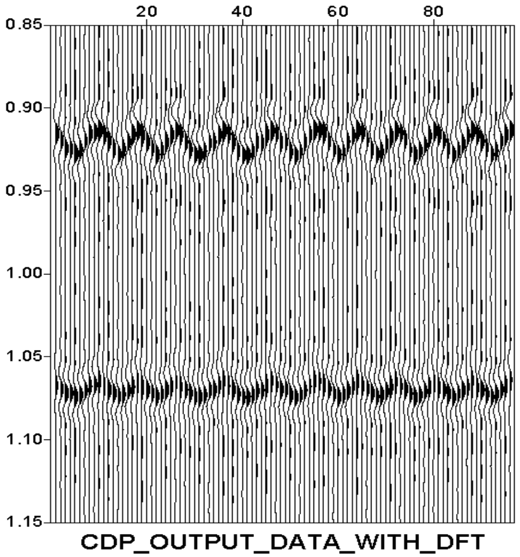


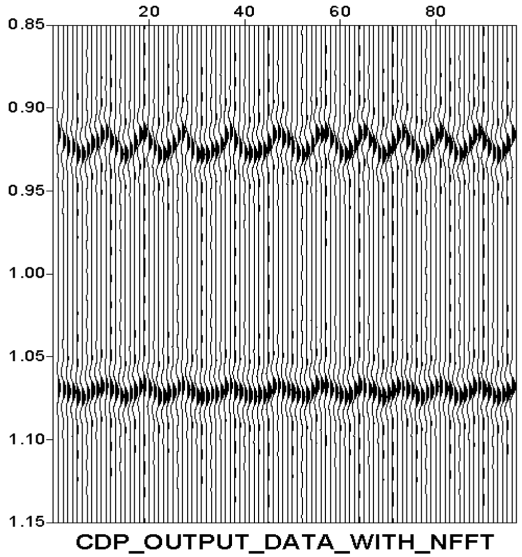




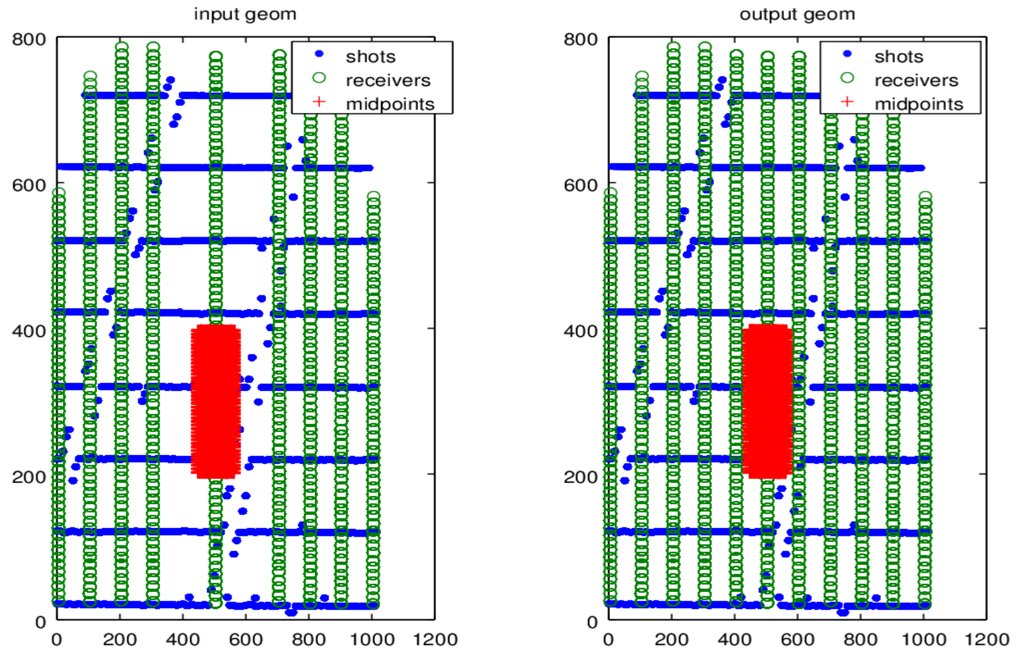


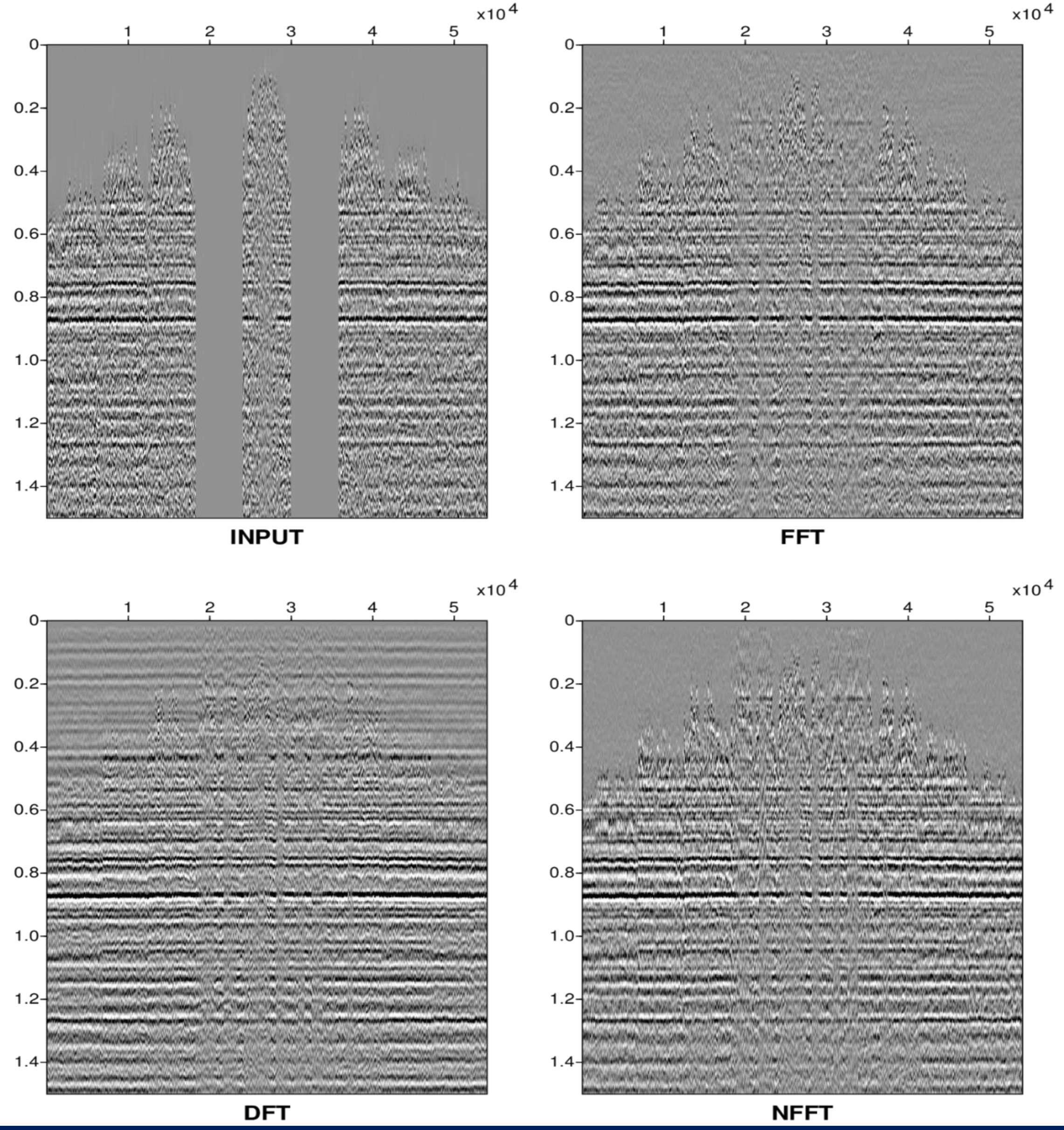






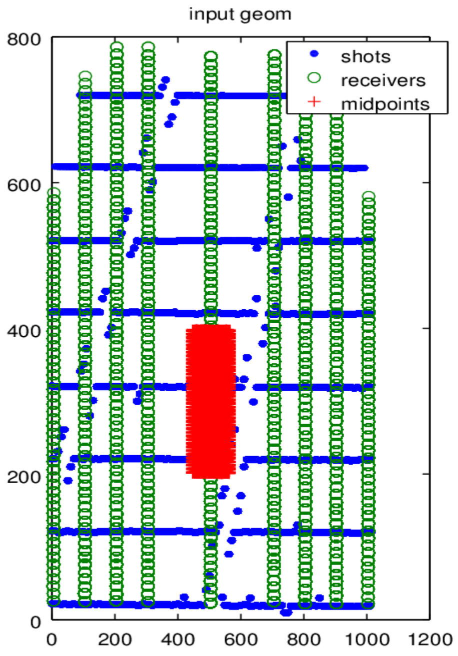
Example 3: Orthogonal geometry, decreasing line spacing

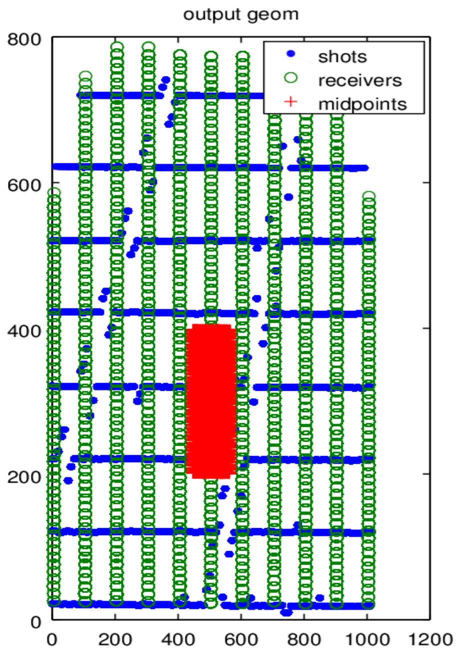




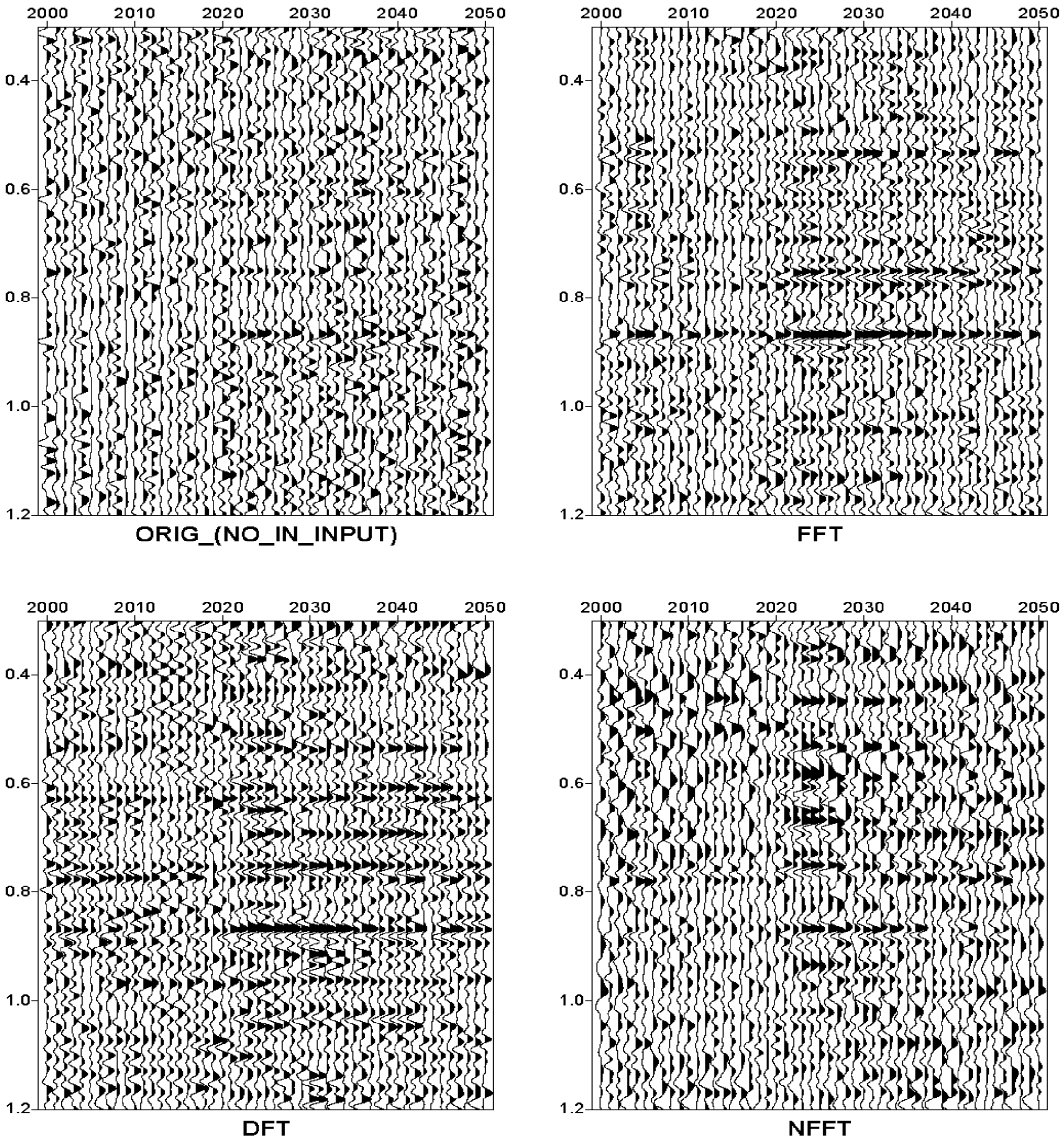
Midpoints for 1 zone (window in xline, inline, offset, azimuth) with size 200mx200mx1000x360º and contributing shots and receivers (before and after)

Example 3: Orthogonal geometry, decreasing line spacing





Midpoints for 1 zone (window in xline, inline, offset, azimuth) with size 200mx200mx1000x360^{o} and contributing shots and receivers (before and after)



Conclusions

- Fourier is widely used because it adapts well to amplitude and phase variations
- FFT serves well for most land scenarios but fails in coarse binning for far offsets
- DFT is the most flexible, and adapts well to any input/output but is very slow
- DFT requires small windows or 4D, mostly used for marine streamer data
- NFFT is a good compromise, but requires more memory and is slower than FFT.

Acknowledgements

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