Fluid/porosity term and fracture weaknesses inversion

from AVAZ using azimuthal elastic impedance (EI)

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Outline

- Introduction
- Fluid substitution and approximation in HTI media
- Reflection coefficient and azimuthal EI
- Bayesian Markov Chain Monte Carlo (MCMC) inversion
- Examples
- Discussions and conclusions

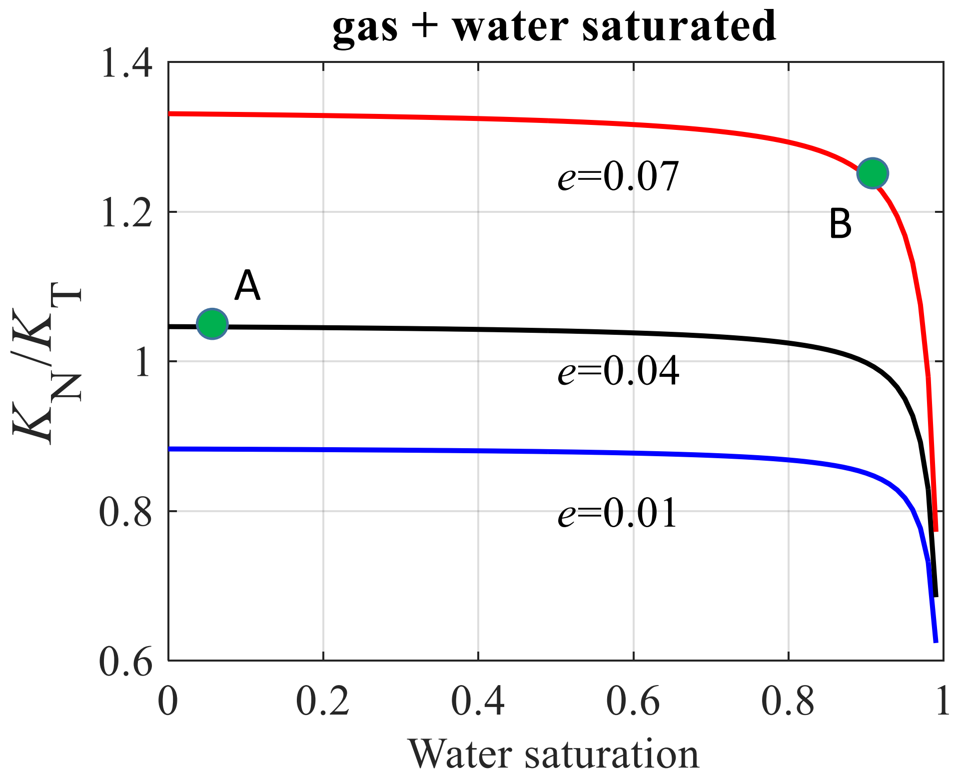
Introduction

- Fluid content indicator (Schoenberg and Sayers, 1995; Bakulin et al., 2000)

$$\frac{K_N}{K_T} = g\frac{\Delta_N(1-\Delta_T)}{\Delta_T(1-\Delta_N)}$$

$$\Delta_N = \frac{4e}{3g(1-g)\left[1+\frac{1}{\pi(1-g)}\left(\frac{k'+4/3\mu'}{\mu\alpha}\right)\right]}$$

$$\Delta_T = \frac{16e}{3(3-2g)\left[1+\frac{4}{\pi(3-2g)}\left(\frac{\mu'}{\mu\alpha}\right)\right]}$$



Introduction

- Gassmann's equation is expressed in terms of the Biot coefficient

$$K_{sat} = K_{dry} + \beta^2 M = K_{dry} + f$$

- The parameter f is the fluid/porosity term (Russell et al., 2003).
- Under the Voigt medium assumption (all constituents have the same strain), Gassmann's equation is re-expressed as

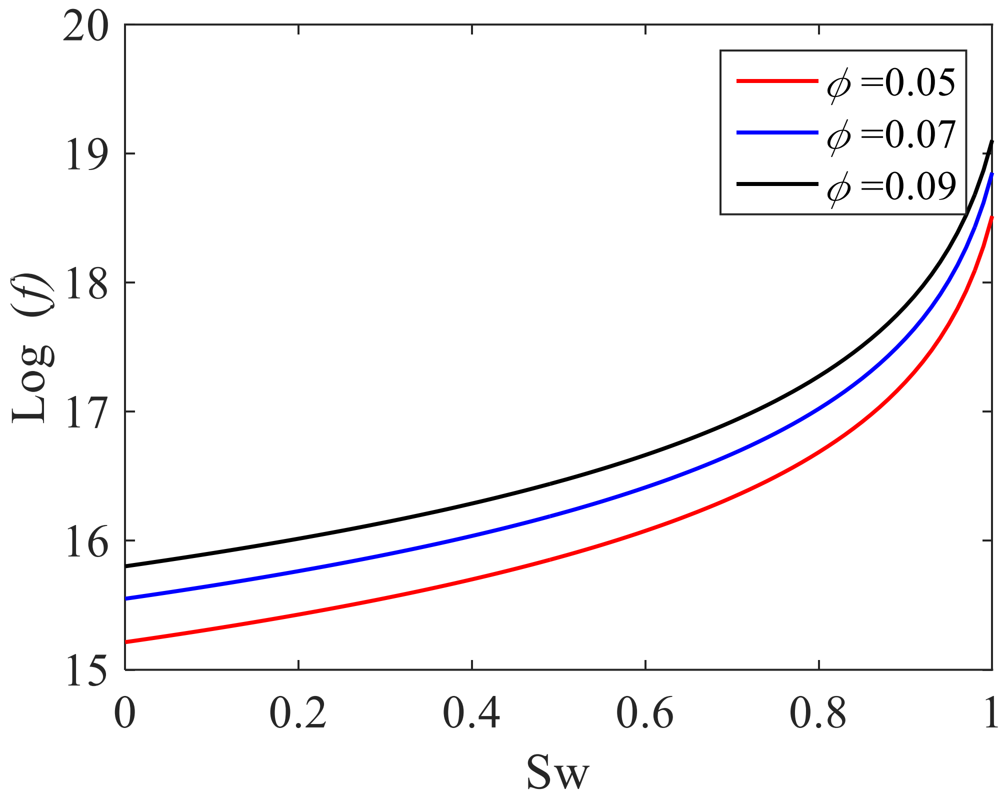
$$K_{sat} = K_{dry} + \phi K_f$$

- Hence, the parameter f is given by

$$f = \phi K_f$$

Introduction

- The fluid/porosity term variation



Fluid substitution and approximation in HTI media

- Fluid substitution in HTI media(Huang et al., 2015)

$$C_{11}^{\text{sat}} = (\lambda+2\mu)(1-\Delta_{\text{N}}) + \frac{\left[K_0 - K_d(1-\Delta_{\text{N}})\right]^2}{(K_0/K_f)\phi(K_0-K_f)+(K_0-A)}$$

$$C_{12}^{\text{sat}} = \lambda(1-\Delta_{\text{N}}) + \frac{\left[K_0 - K_d(1-\Delta_{\text{N}})\right]\left[K_0 - K_d(1-\chi\Delta_{\text{N}})\right]}{(K_0/K_f)\phi(K_0-K_f)+(K_0-A)}$$

$$C_{23}^{\text{sat}} = \lambda(1-\chi\Delta_{\text{N}}) + \frac{\left[K_0 - K_d(1-\chi\Delta_{\text{N}})\right]^2}{(K_0/K_f)\phi(K_0-K_f)+(K_0-A)}$$

$$C_{33}^{\text{sat}} = (\lambda+2\mu)(1-\chi^2\Delta_{\text{N}}) + \frac{\left[K_0 - K_d(1-\chi\Delta_{\text{N}})\right]^2}{(K_0/K_f)\phi(K_0-K_f)+(K_0-A)}$$

$$C_{44}^{\text{sat}} = \mu$$

$$C_{55}^{\text{sat}} = \mu(1-\Delta_{\text{T}})$$

$$A = K_d(1-\Delta_{\text{N}} K_d/M)$$

- Lamé parameters λ and μ, and P-wave modulus M are elastic properties of isotropic dry background.

Fluid substitution and approximation in HTI media

- Taking C_{11}^{sat} as an example

$$C_{11}^{\text{sat}} = (\lambda+2\mu)(1-\Delta_N) + \frac{K_0\left[\left(1-\frac{K_d}{K_0}\right)^2 + 2\left(1-\frac{K_d}{K_0}\right)\frac{K_d}{K_0}\Delta_N + \left(\frac{K_d}{K_0}\Delta_N\right)^2\right]}{\frac{K_0}{K_f}\phi + \left(1-\frac{A}{K_0}-\phi\right)}$$

- Usually K_f is much smaller than K_0

$$0 \le 1-\frac{A}{K_0}-\phi \ll \frac{K_0}{K_f}\phi$$

- Under the assumption of a Voigt medium

$$K_d = K_0(1-\phi)$$

- For small fracture weaknesses, we ignore the high order term of fracture weakness, $(\Delta_N)^2$

Fluid substitution and approximation in HTI media

- Finally, we obtain an approximate expression of C_{11}^{sat}

$$C_{11}^{\text{sat}} \approx (\lambda+2\mu)(1-\Delta_{\text{N}}) + f + 2\Delta_{\text{N}}K_f - 2f\Delta_{\text{N}}$$

- For other stiffness parameters

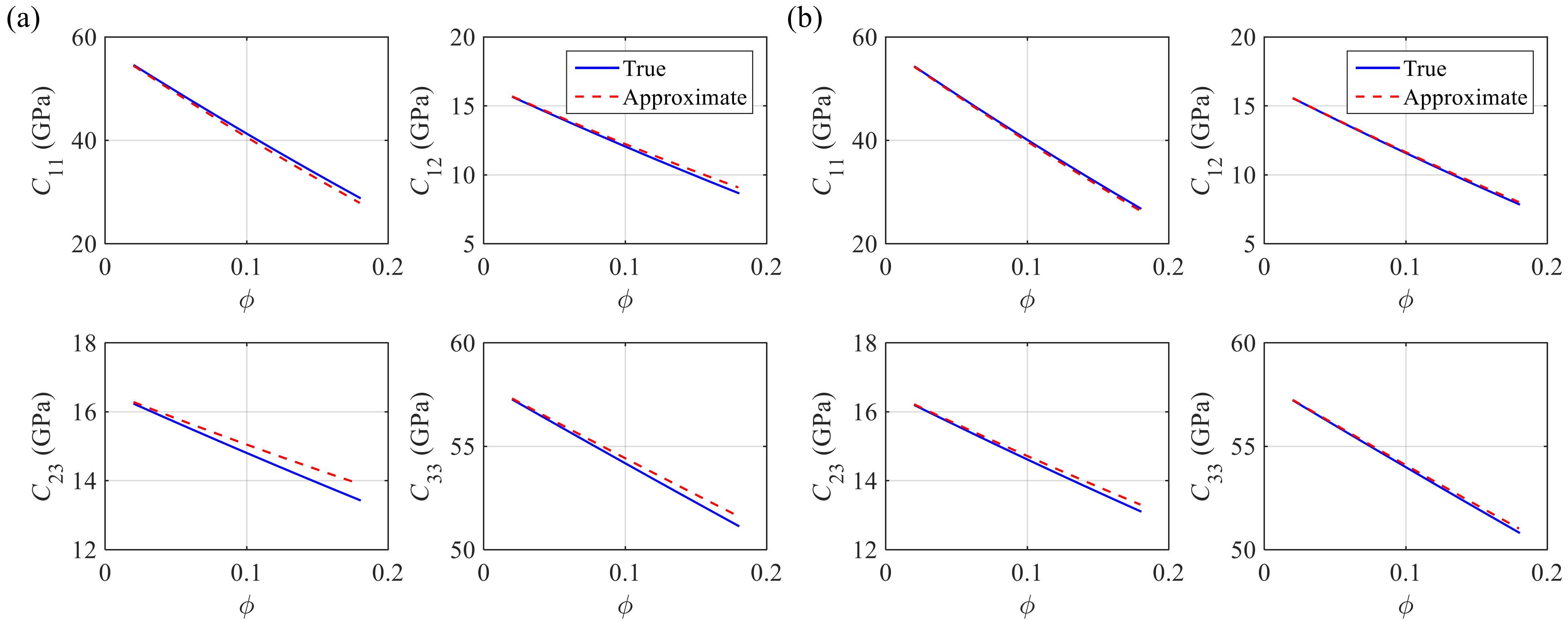
$$C_{12}^{\text{sat}} \approx \lambda(1-\Delta_{\text{N}}) + f + (\chi+1)\Delta_{\text{N}}K_f - (\chi+1)f\Delta_{\text{N}}$$

$$C_{23}^{\text{sat}} \approx \lambda(1-\chi\Delta_{\text{N}}) + f + 2\chi\Delta_{\text{N}}K_f - 2\chi f\Delta_{\text{N}}$$

$$C_{33}^{\text{sat}} \approx (\lambda+2\mu)(1-\chi^2\Delta_{\text{N}}) + f + 2\chi\Delta_{\text{N}}K_f - 2\chi f\Delta_{\text{N}}$$

Fluid substitution and approximation in HTI media

- Accuracy test (Mineral and volume: quartz 0.5 and clay 0.5)



- Water saturation is 1
- Water saturation is 0.95

$e = 0.5\phi$

Reflection coefficient and azimuthal EI

- Linearized Rpp for weakly anisotropic media (Shaw and Sen, 2004)

$$R_{PP} = \frac{1}{4\rho\cos^2\theta} S$$

- S is the scattering function, ρ is density, and θ is the angle of incidence.

$$S = \Delta\rho\xi + \Delta C_{IJ}\eta$$

- Here, ξ and η are related to slowness and polarization of the seismic wave.
- For PP-waves, the slowness and polarization are given by

$$\mathrm{p_P} = \frac{1}{V_P}\left[\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\right], \mathrm{g_P} = \left[\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\right],$$

$$\mathrm{p_P'} = \frac{1}{V_P}\left[-\sin\theta\cos\varphi, -\sin\theta\sin\varphi, \cos\theta\right], \mathrm{g_P'} = \left[-\sin\theta\cos\varphi, -\sin\theta\sin\varphi, \cos\theta\right]$$

Reflection coefficient and azimuthal EI

- Perturbations in stiffness parameters

$$\Delta C_{11}^{\mathrm{sat}} \approx \Delta M - M\delta_{\Delta_\mathrm{N}} + \Delta f + 2\left(\Delta_\mathrm{N}\Delta K_f + \delta_{\Delta_\mathrm{N}} K_f\right)$$

$$\Delta C_{12}^{\mathrm{sat}} \approx \Delta\lambda - \lambda\delta_{\Delta_\mathrm{N}} + \Delta f + (\chi+1)\Delta_\mathrm{N}\Delta K_f + (\chi+1)K_f\delta_{\Delta_\mathrm{N}}$$

$$\Delta C_{23}^{\mathrm{sat}} \approx \Delta\lambda - \lambda\chi\delta_{\Delta_\mathrm{N}} + \Delta f + 2\chi\Delta_\mathrm{N}\Delta K_f + 2\chi K_f\delta_{\Delta_\mathrm{N}}$$

$$\Delta C_{33}^{\mathrm{sat}} \approx \Delta M - M\chi^2\delta_{\Delta_\mathrm{N}} + \Delta f + 2\chi\Delta_\mathrm{N}\Delta K_f + 2\chi K_f\delta_{\Delta_\mathrm{N}}$$

$$\Delta C_{44}^{\mathrm{sat}} \approx \Delta\mu$$

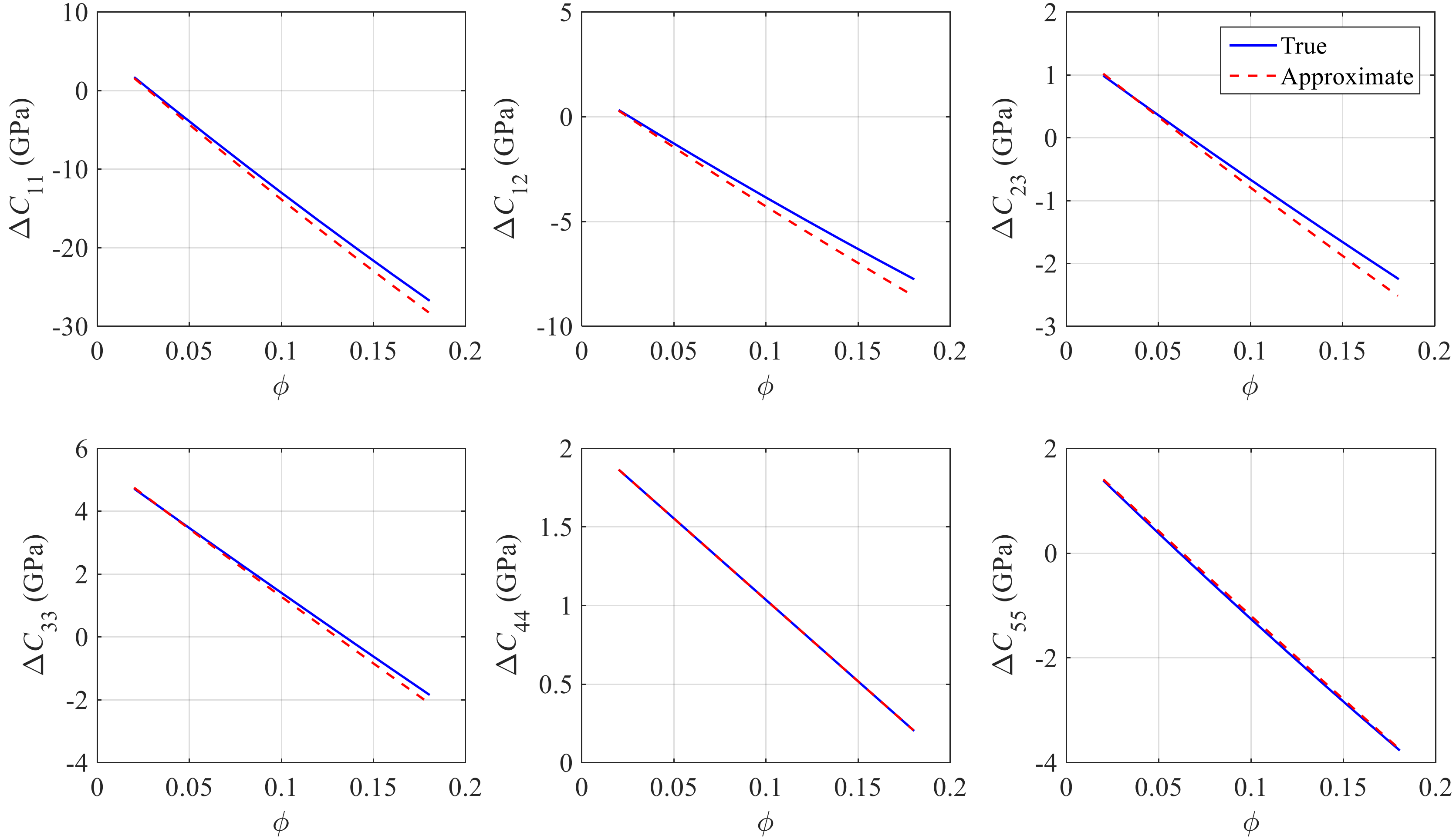
$$\Delta C_{55}^{\mathrm{sat}} \approx \Delta\mu - \mu\delta_{\Delta_\mathrm{T}}$$

Reflection coefficient and azimuthal EI

- Accuracy test (Mineral and volume: Quartz 0.5 and Clay 0.5)

No fractures
ϕ = 0.1, Sw = 1

ϕ = 0.02~0.18,
e = 0.5 ϕ
Sw = 0.8



Reflection coefficient and azimuthal EI

- Linearized PP –wave reflection coefficient in terms of fluid/porosity term and fracture weaknesses

$$R_{PP}(\theta,\varphi) = \frac{1}{4\cos^2\theta}\frac{\Delta\lambda_d}{\lambda_d} + \left(\frac{1}{4\cos^2\theta} - 2g_s\sin^2\theta\right)\frac{\Delta\mu}{\mu} + \frac{\cos 2\theta}{4\cos^2\theta}\frac{\Delta\rho}{\rho}$$

$$+\frac{1}{4\cos^2\theta}\left(1-\frac{g_s}{g_d}\right)\frac{\Delta f}{f}$$

$$-\frac{1}{4\cos^2\theta}\frac{g_s}{g_d}\left[1-2g_d\left(\sin^2\theta\sin^2\varphi+\cos^2\theta\right)\right]^2\delta_{\Delta_{\mathrm{N\_dry}}}$$

$$-g_s\tan^2\theta\cos^2\varphi\left(\sin^2\theta\sin^2\varphi-\cos^2\theta\right)\delta_{\Delta_{\mathrm{T\_dry}}}$$

Reflection coefficient and azimuthal EI

- Following Buland and Omre (2003), we express the derived PP-wave reflection coefficient as a time- continuous function

$$R_{PP}(t,\theta,\varphi)=\frac{1}{2}\frac{\partial}{\partial t}\ln \mathrm{EI}(t,\theta,\varphi)$$

$$=a_{\lambda_d}(t,\theta)\frac{\partial}{\partial t}\ln\lambda_d(t)+a_{\mu}(t,\theta)\frac{\partial}{\partial t}\ln\mu(t)+a_{\rho}(t,\theta)\frac{\partial}{\partial t}\ln\rho(t)$$

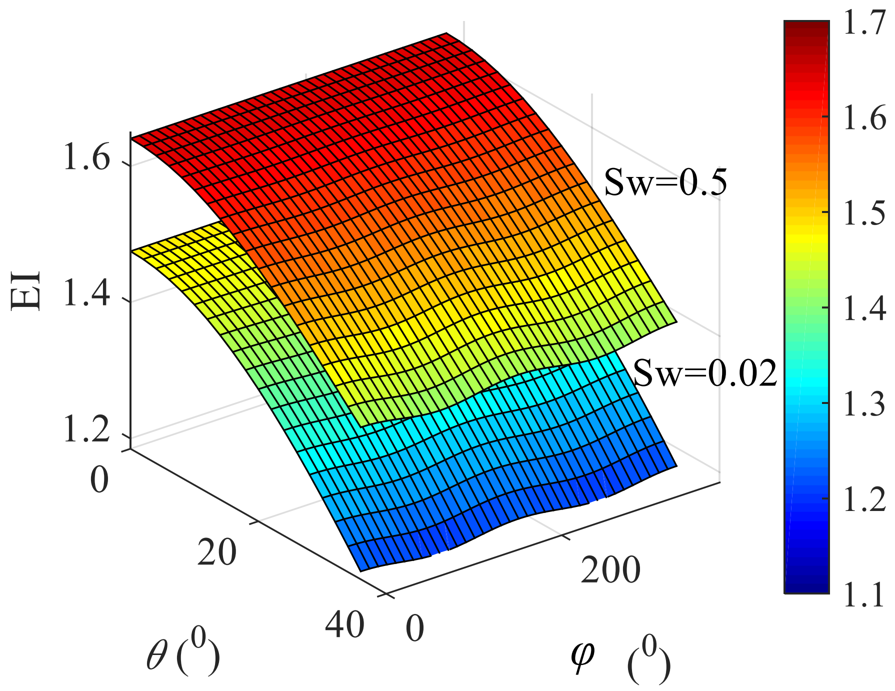
$$+a_f(t,\theta)\frac{\partial}{\partial t}\ln f(t)+a_{\Delta_N}(t,\theta,\varphi)\frac{\partial}{\partial t}\Delta_{\mathrm{N\_dry}}(t)+a_{\Delta_T}(t,\theta,\varphi)\frac{\partial}{\partial t}\Delta_{\mathrm{T\_dry}}(t)$$

- Azimuthal EI is obtained after taking an integral operation

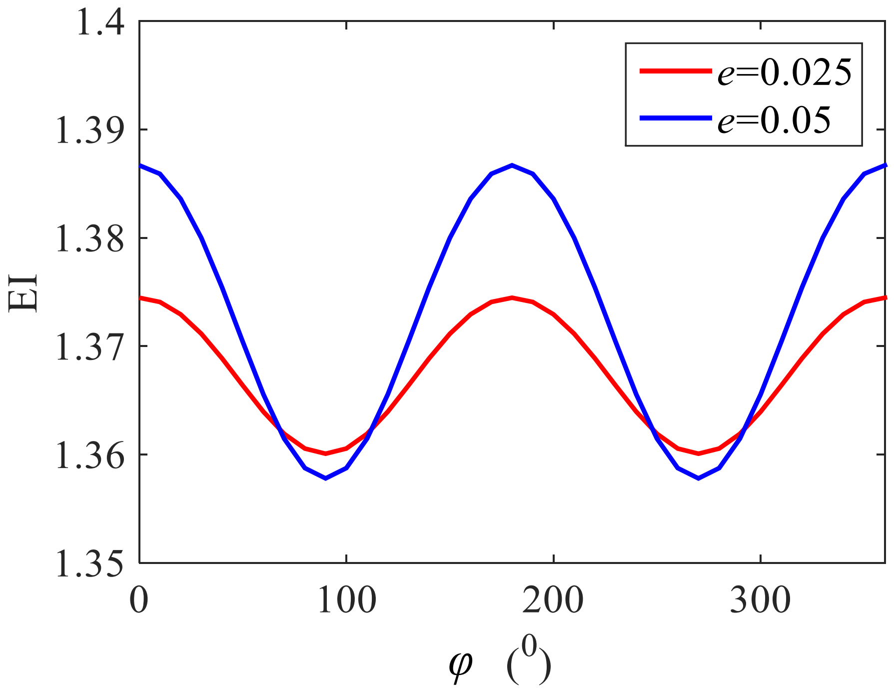
$$\mathrm{EI}(t,\theta,\varphi)=\left[\lambda_d(t)\right]^{a_{\lambda_d}(t,\theta)}\left[\mu(t)\right]^{a_{\mu}(t,\theta)}\left[\rho(t)\right]^{a_{\rho}(t,\theta)}\left[f(t)\right]^{a_f(t,\theta)}$$

$$\exp\left[a_{\Delta_N}(t,\theta,\varphi)\Delta_{\mathrm{N\_dry}}(t)+a_{\Delta_T}(t,\theta,\varphi)\Delta_{\mathrm{T\_dry}}(t)\right]$$

Reflection coefficient and azimuthal EI



- Fracture density is a constant, and water saturation is 0.5 and 0.02, respectively.



- Water saturation is a constant, and fracture density is 0.025 and 0.05, respectively.

Bayesian Markov Chain Monte Carlo (MCMC) inversion

- Relationship between reflection coefficient and azimuthal EI

$$R_{PP}(t,\theta,\varphi)=\frac{1}{2}\frac{\Delta \mathrm{EI}(t,\theta,\varphi)}{\overline{\mathrm{EI}}(t,\theta,\varphi)}\approx\frac{1}{2}d\ln\left[\mathrm{EI}(t,\theta,\varphi)\right]$$

- Convolution model

$$\begin{bmatrix} S(t_1,\theta,\varphi) \\ S(t_2,\theta,\varphi) \\ \vdots \\ S(t_i,\theta,\varphi) \\ S(t_{i+1},\theta,\varphi) \\ \vdots \\ S(t_{N-1},\theta,\varphi) \\ S(t_N,\theta,\varphi) \end{bmatrix} = \begin{bmatrix} w_1 & 0 & 0 & \cdots \\ w_2 & w_1 & 0 & \ddots \\ w_3 & w_2 & w_1 & \ddots \\ \vdots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & \ddots & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \ln \mathrm{EI}(t_1,\theta,\varphi) \\ \ln \mathrm{EI}(t_2,\theta,\varphi) \\ \vdots \\ \ln \mathrm{EI}(t_i,\theta,\varphi) \\ \ln \mathrm{EI}(t_{i+1},\theta,\varphi) \\ \vdots \\ \ln \mathrm{EI}(t_{N-1},\theta,\varphi) \\ \ln \mathrm{EI}(t_N,\theta,\varphi) \end{bmatrix}$$

$$\mathrm{B} = \mathrm{AX}$$

- The Least- square method is used to solve the inversion for azimuthal EI.

Bayesian Markov Chain Monte Carlo (MCMC) inversion

- Bayesian inference

$$P(m|d)=\frac{P(d|m)P(m)}{P(d)}\propto P(d|m)P(m)$$

- Likelihood function

$$P(d|m)=\frac{1}{\left(2\pi\sigma_{noise}^{2}\right)^{\frac{N}{2}}}\exp\left\{-\sum\frac{\left[d-G(m)\right]^{2}}{2\sigma_{noise}^{2}}\right\}$$

- Prior probability distribution function (PDF)

$$P(m)=P(\ln\lambda_d)P(\ln\mu)P(\ln\rho)P(\ln f)P(\Delta_{\mathrm{N}})P(\Delta_{\mathrm{T}})$$

$$=\frac{1}{\left(2\pi\sigma_{\ln\lambda_d}^{2}\right)^{\frac{N}{2}}}\exp\left[-\sum\frac{\left(\ln\lambda_d-m_{\ln\lambda_d}\right)^{2}}{2\sigma_{\ln\lambda_d}^{2}}\right]\frac{1}{\left(2\pi\sigma_{\ln\mu}^{2}\right)^{\frac{N}{2}}}\exp\left[-\sum\frac{\left(\ln\mu-m_{\ln\mu}\right)^{2}}{2\sigma_{\ln\mu}^{2}}\right]\frac{1}{\left(2\pi\sigma_{\ln\rho}^{2}\right)^{\frac{N}{2}}}\exp\left[-\sum\frac{\left(\ln\rho-m_{\ln\rho}\right)^{2}}{2\sigma_{\ln\rho}^{2}}\right]$$

$$\frac{1}{\left(2\pi\sigma_{\ln f}^{2}\right)^{\frac{N}{2}}}\exp\left[-\sum\frac{\left(\ln f-m_{\ln f}\right)^{2}}{2\sigma_{\ln f}^{2}}\right]\frac{1}{\left(2\pi\sigma_{\Delta_{\mathrm{N}}}^{2}\right)^{\frac{N}{2}}}\exp\left[-\sum\frac{\left(\Delta_{\mathrm{N}}-m_{\Delta_{\mathrm{N}}}\right)^{2}}{2\sigma_{\Delta_{\mathrm{N}}}^{2}}\right]\frac{1}{\left(2\pi\sigma_{\Delta_{\mathrm{T}}}^{2}\right)^{\frac{N}{2}}}\exp\left[-\sum\frac{\left(\Delta_{\mathrm{T}}-m_{\Delta_{\mathrm{T}}}\right)^{2}}{2\sigma_{\Delta_{\mathrm{T}}}^{2}}\right]$$

Bayesian Markov Chain Monte Carlo (MCMC) inversion

- Posterior PDF

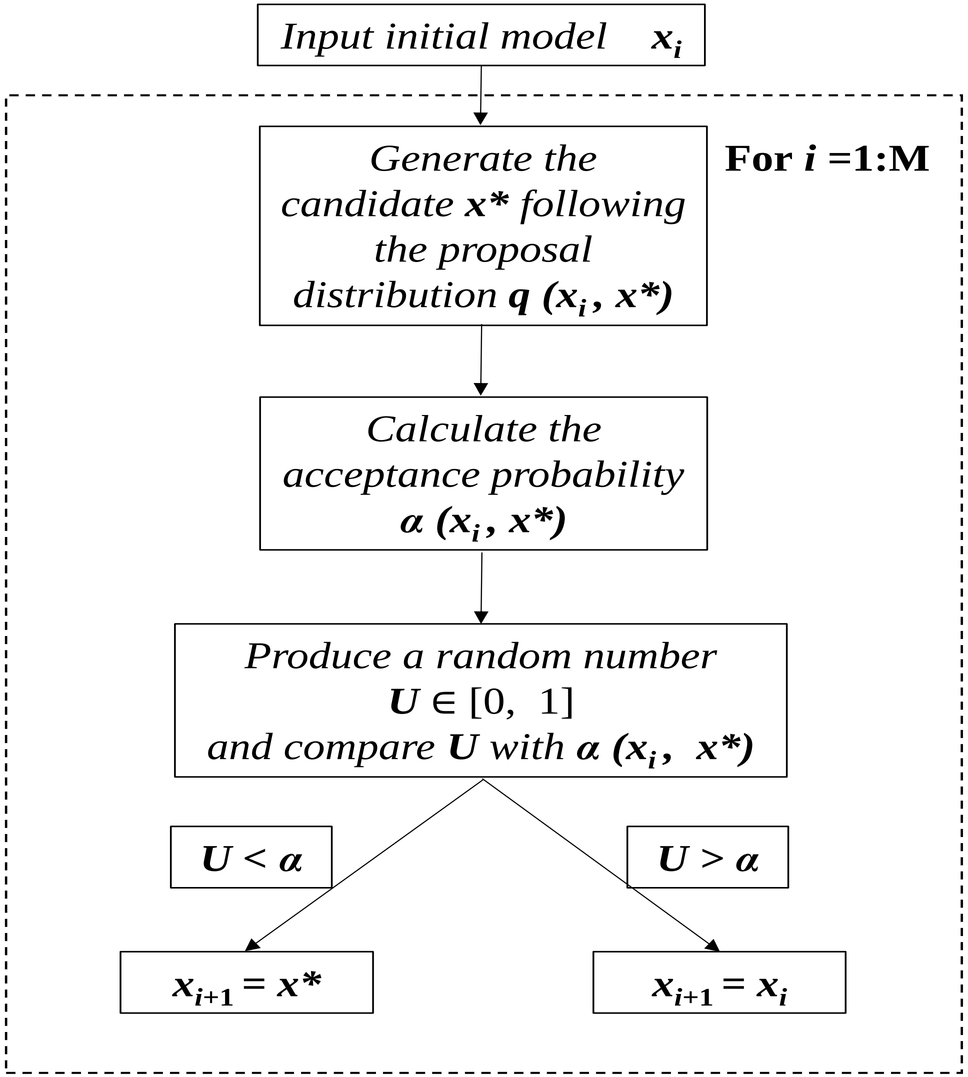
$$P(m|d)=\frac{1}{\left(2\pi\sigma_{noise}^2\right)^{\frac{N}{2}}}\frac{1}{\left(2\pi\sigma_{\ln\lambda_d}^2\right)^{\frac{N}{2}}}\frac{1}{\left(2\pi\sigma_{\ln\mu}^2\right)^{\frac{N}{2}}}\frac{1}{\left(2\pi\sigma_{\ln\rho}^2\right)^{\frac{N}{2}}}\frac{1}{\left(2\pi\sigma_{\ln f}^2\right)^{\frac{N}{2}}}\frac{1}{\left(2\pi\sigma_{\Delta_N}^2\right)^{\frac{N}{2}}}\frac{1}{\left(2\pi\sigma_{\Delta_T}^2\right)^{\frac{N}{2}}}\exp\left[\psi(x)\right]$$

- where

$$\psi(x)=-\sum\frac{\left(\ln\lambda_d-m_{\ln\lambda_d}\right)^2}{2\sigma_{\ln\lambda_d}^2}-\sum\frac{\left(\ln\mu-m_{\ln\mu}\right)^2}{2\sigma_{\ln\mu}^2}-\sum\frac{\left(\ln\rho-m_{\ln\rho}\right)^2}{2\sigma_{\ln\rho}^2}-\sum\frac{\left(\ln f-m_{\ln f}\right)^2}{2\sigma_{\ln f}^2}-\sum\frac{\left(\Delta_N-m_{\Delta_N}\right)^2}{2\sigma_{\Delta_N}^2}-\sum\frac{\left(\Delta_T-m_{\Delta_T}\right)^2}{2\sigma_{\Delta_T}^2}-\sum\frac{\left[d-G(m)\right]^2}{2\sigma_{noise}^2}$$

Bayesian Markov Chain Monte Carlo (MCMC) inversion

- Metropolis-Hasting algorithm to construct transition kernel



- Step 1: obtain a candidate value x^* from a proposal distribution $q\ (x, x^*)$
- Step 2: find the candidate value that meets the acceptance probability $\alpha\ (x, x^*)$

Bayesian Markov Chain Monte Carlo (MCMC) inversion

- The proposal distribution: a symmetric distribution

$$q\left(x,x^{*}\right)=q\left(x^{*},x\right)$$

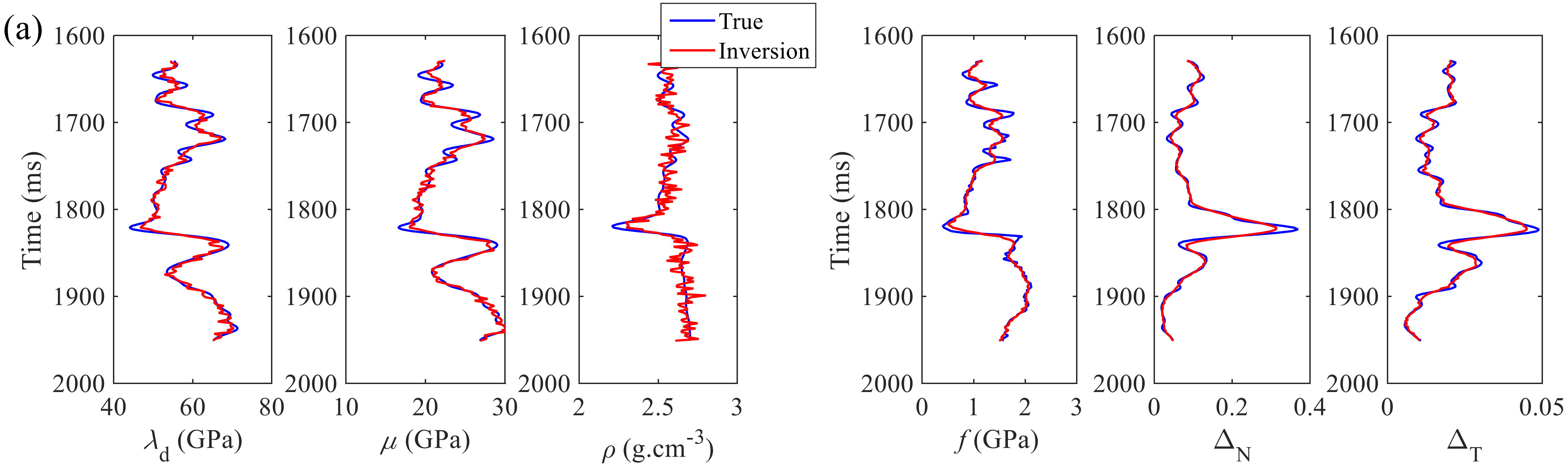
- The acceptance probability

$$\alpha\left(x,x^{*}\right)=\min\left[1,\frac{\pi\left(x^{*}\right)q\left(x^{*},x\right)}{\pi\left(x\right)q\left(x,x^{*}\right)}\right]=\min\left[1,\frac{\pi\left(x^{*}\right)}{\pi\left(x\right)}\right]$$

- The stationary distribution, $\pi\,(x*)$, should be equal to the posterior probability, and the acceptance probability

$$\alpha\left(x,x^{*}\right)=\exp\left\{\min\left[0,g\left(x^{*}\right)-g\left(x\right)\right]\right\}$$

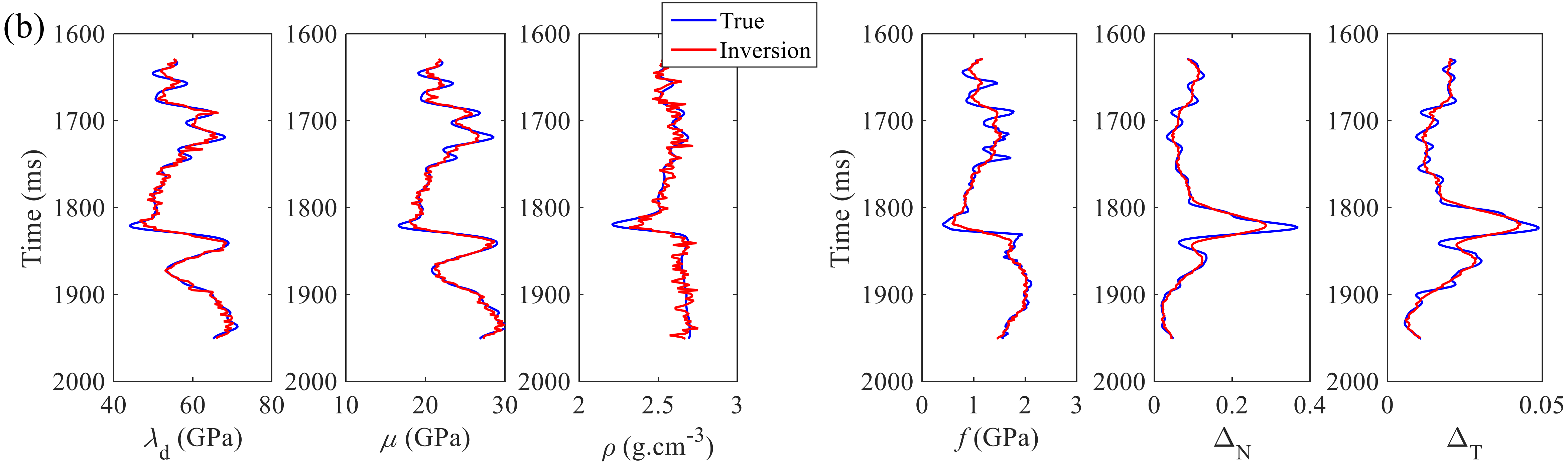
- Synthetic tests



- S/N=5

Examples

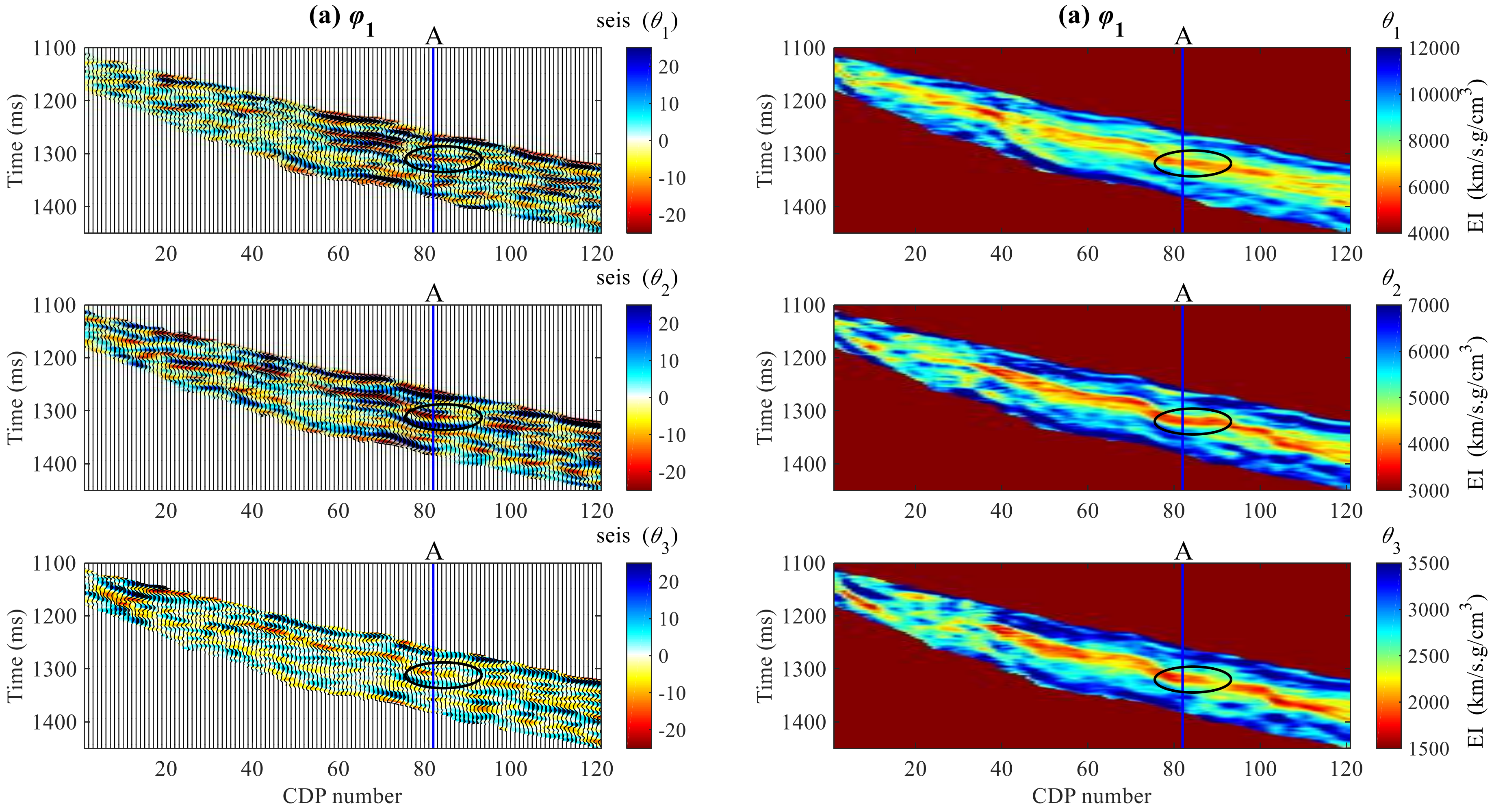
- Synthetic tests



- S/N=2

Examples

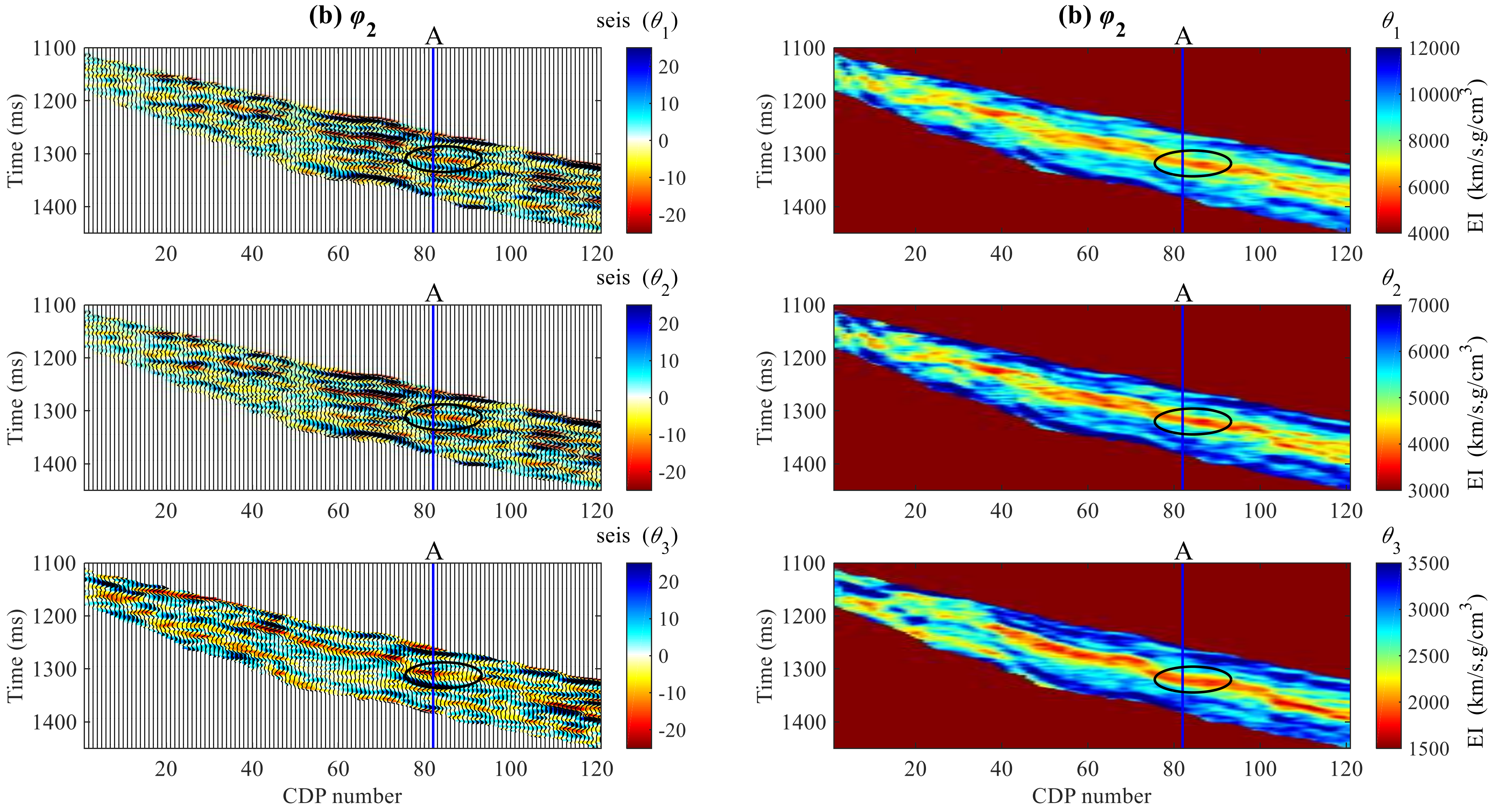
- Real data



$\varphi_1 = 0^0$
$\theta 1 = 8^0$
$\theta 2 = 16^0$
$\theta 3 = 24^0$

Examples

- Real data



$\varphi_2 = 30^0$

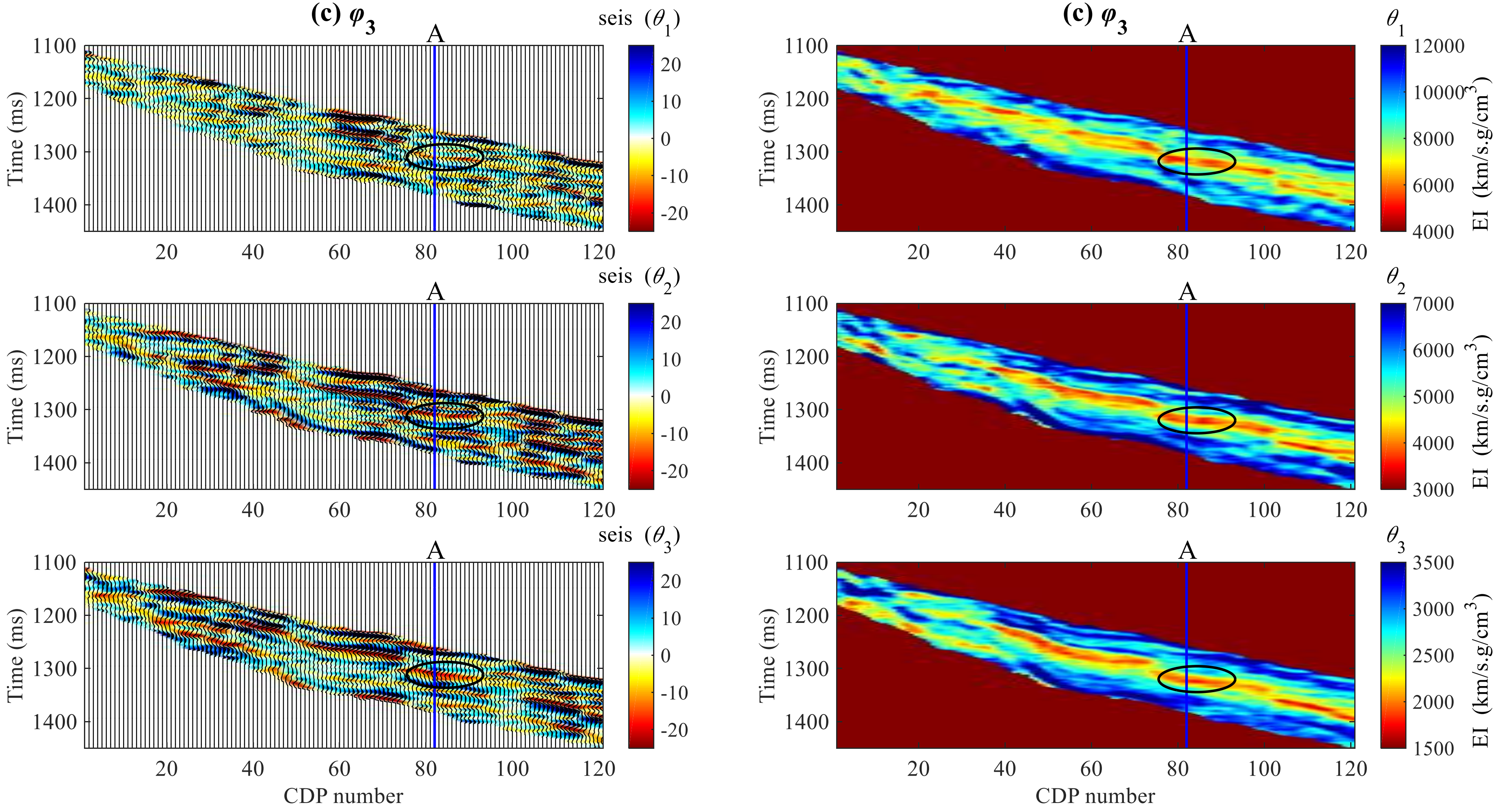
$\theta 1 = 8^0$

$\theta 2 = 16^0$

$\theta 3 = 24^0$

Examples

- Real data



$\varphi_3 = 60^0$

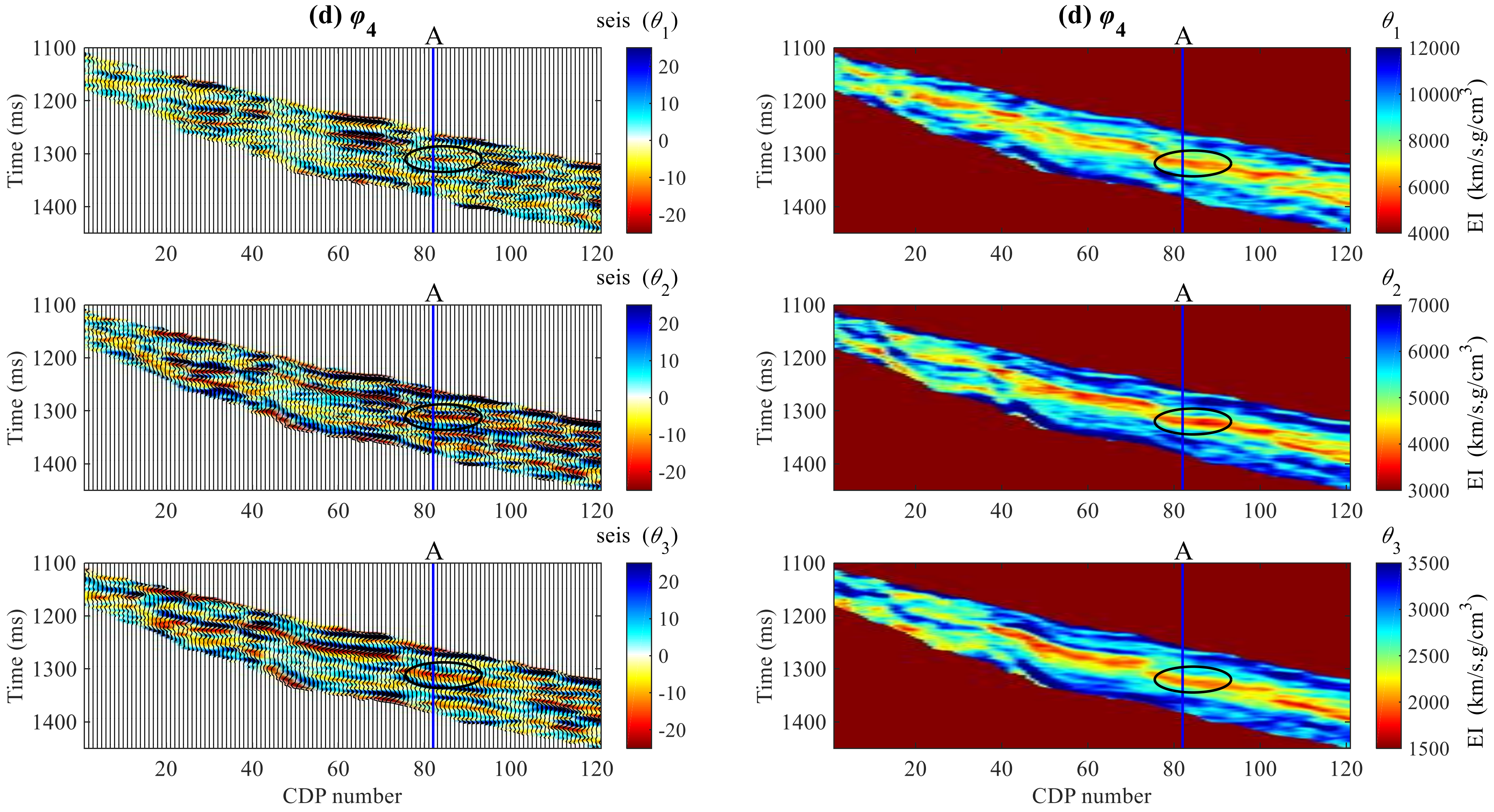
$\theta1 = 8^0$

$\theta2 = 16^0$

$\theta3 = 24^0$

Examples

- Real data



$\varphi_4 = 90^0$

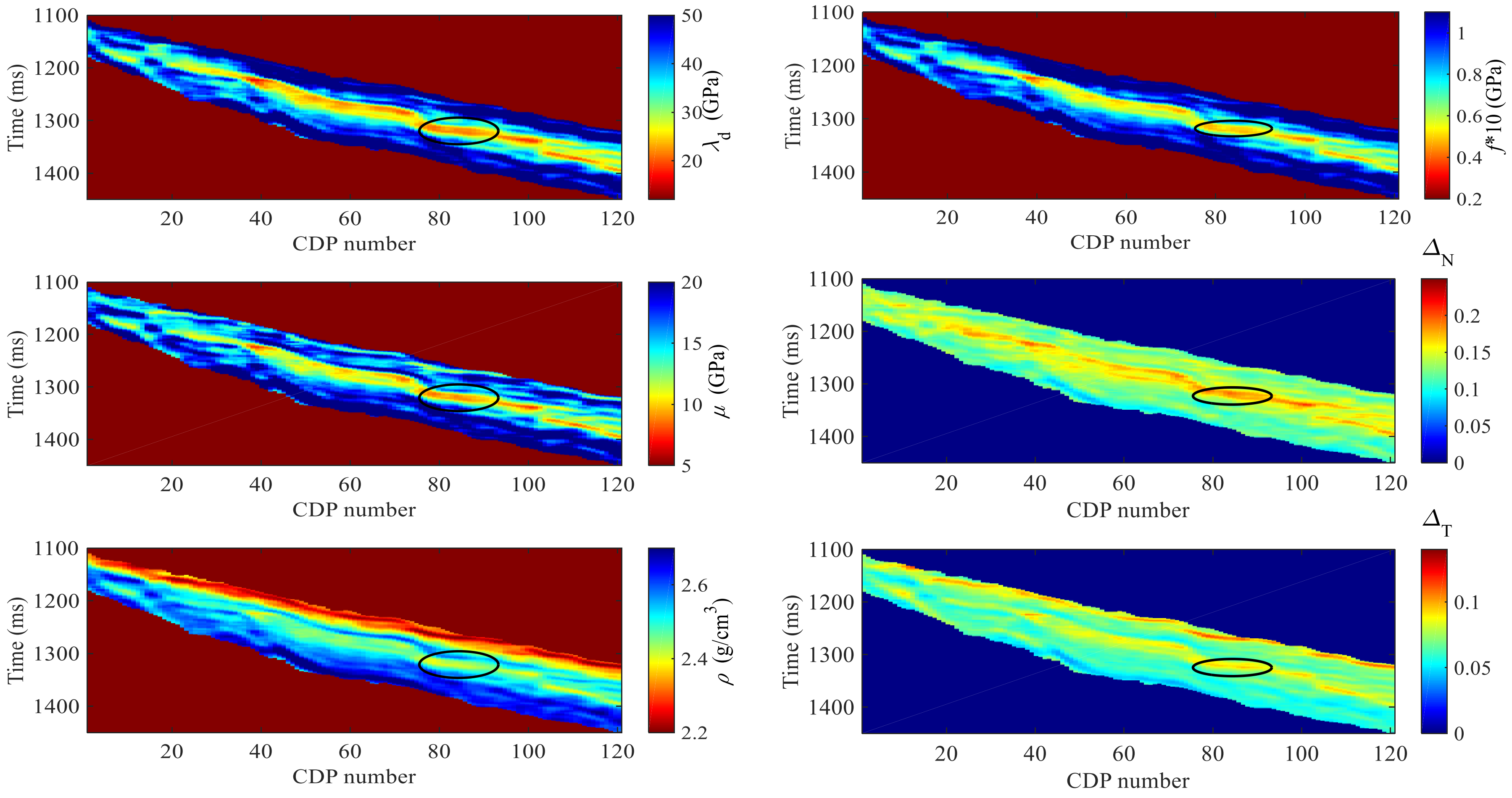
$\theta 1 = 8^0$

$\theta 2 = 16^0$

$\theta 3 = 24^0$

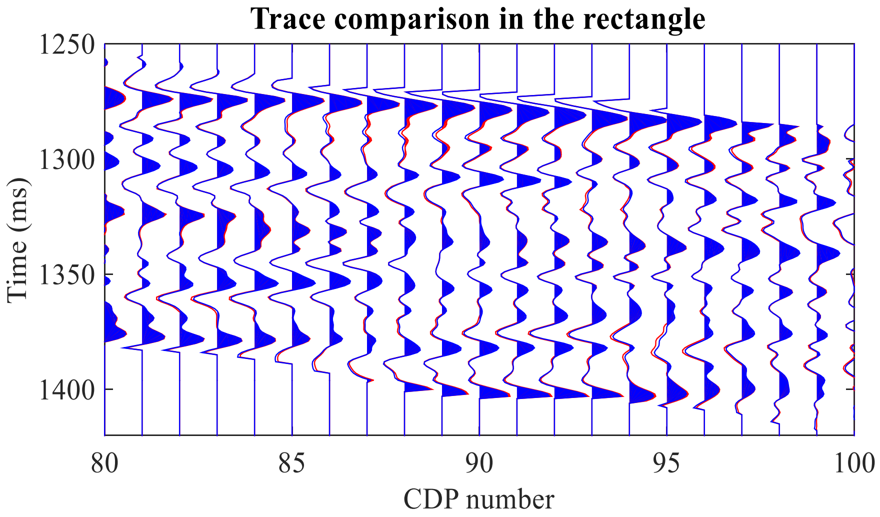
Examples

- Real data

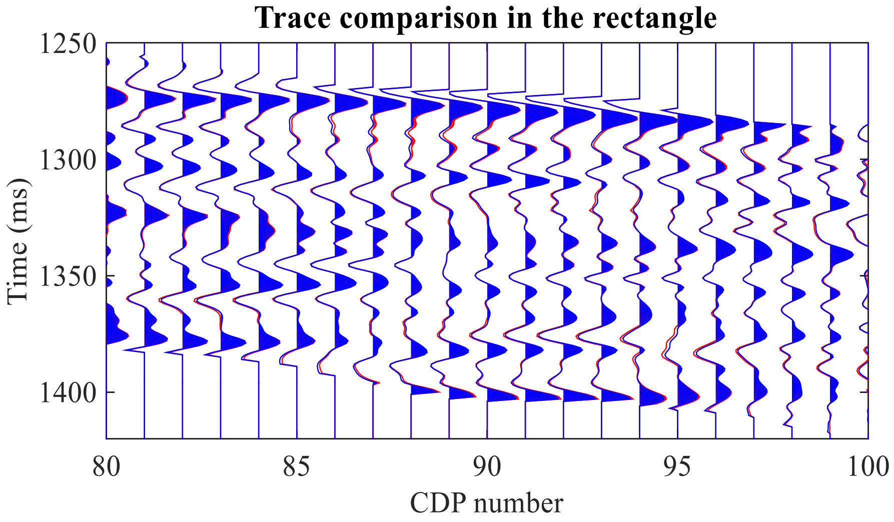


Examples

$\varphi = 0^0$



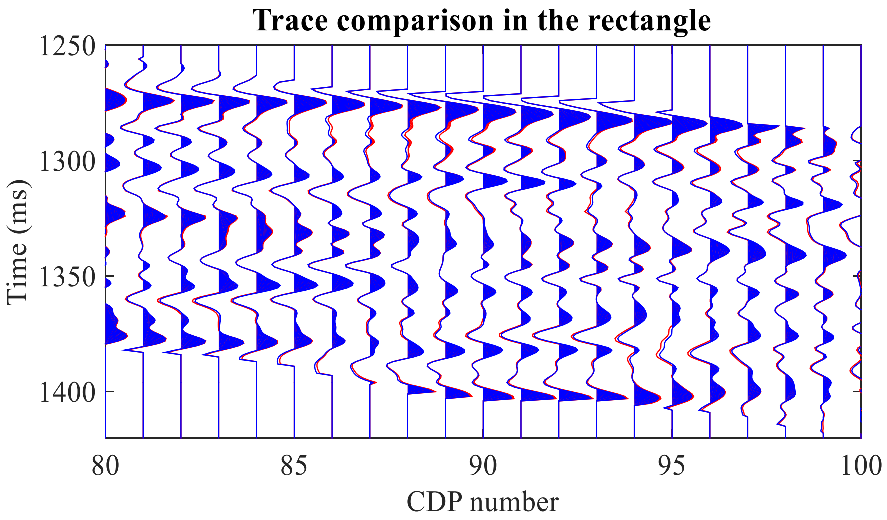
$\varphi = 60^0$



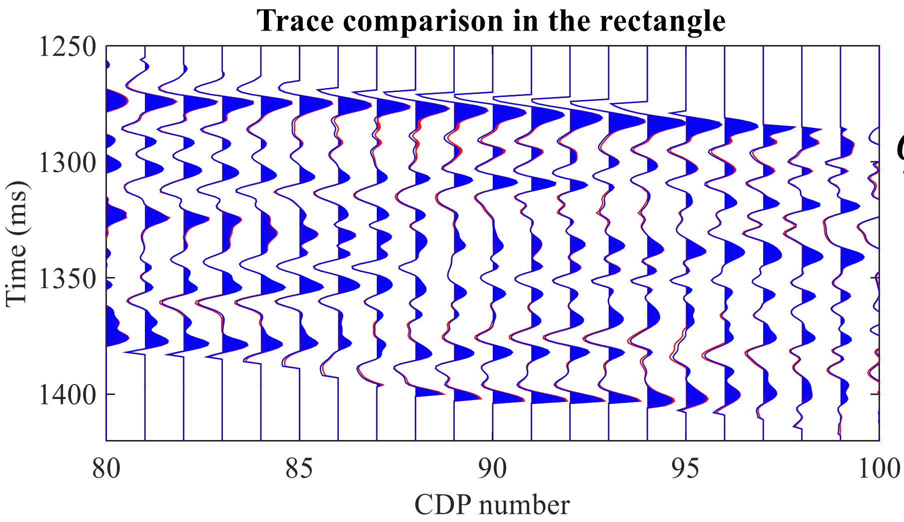
Real

Synthetic

$\varphi = 30^0$



$\varphi = 90^0$



Discussions and conclusions

- Assumptions: HTI model, Voigt material, gas-bearing fractured rock, and small porosity and fracture weakness.

- The derived reflection coefficient can be used to analyze the effects of fluid/porosity term and dry fractures, separately.

- Based on azimuthal EI, Bayesian MCMC inversion approach can make a stable and reliable estimation of elastic parameters, fluid/porosity term, and fracture weaknesses.

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