Nuts and bolts of least squares Kirchhoff migration

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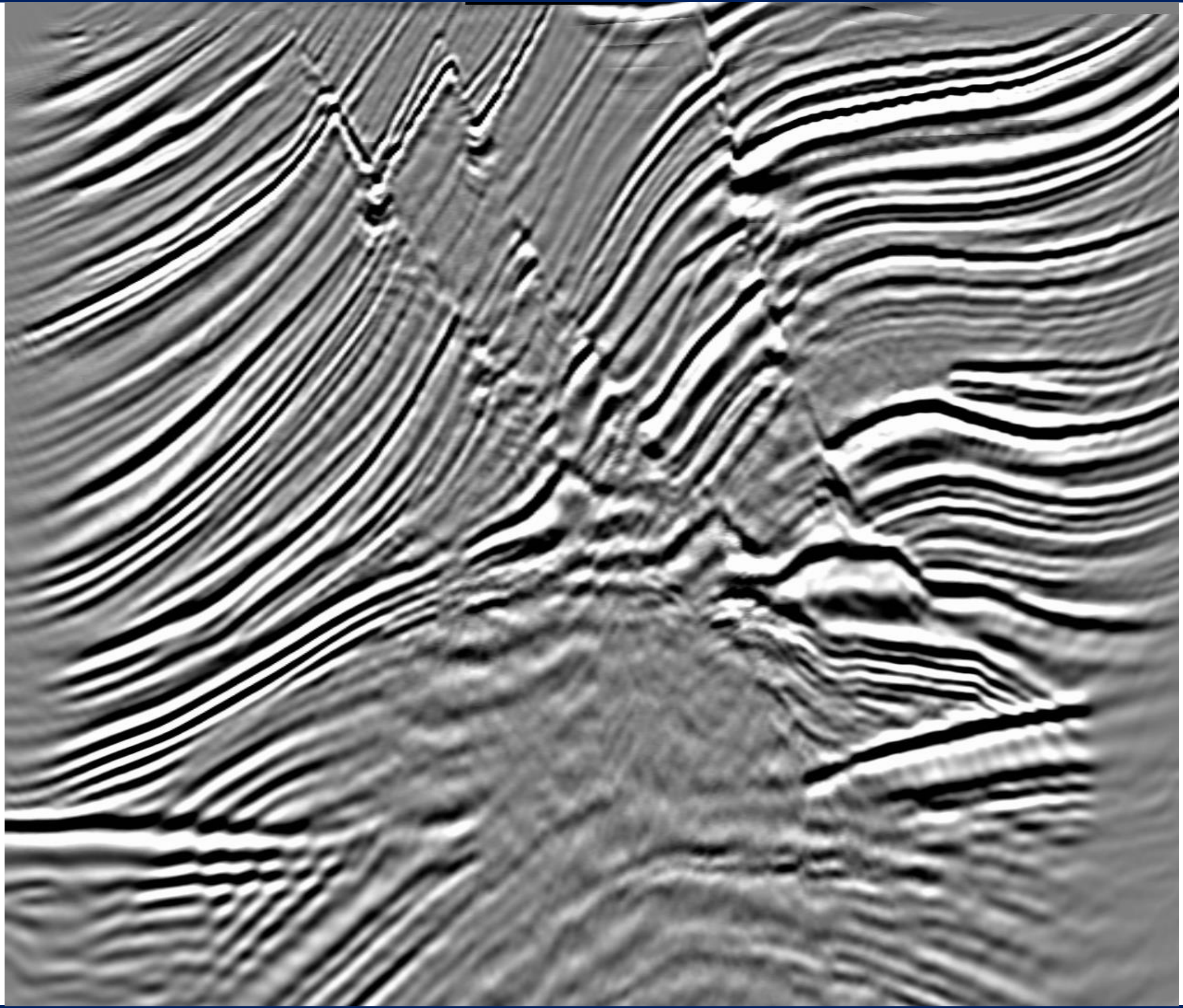
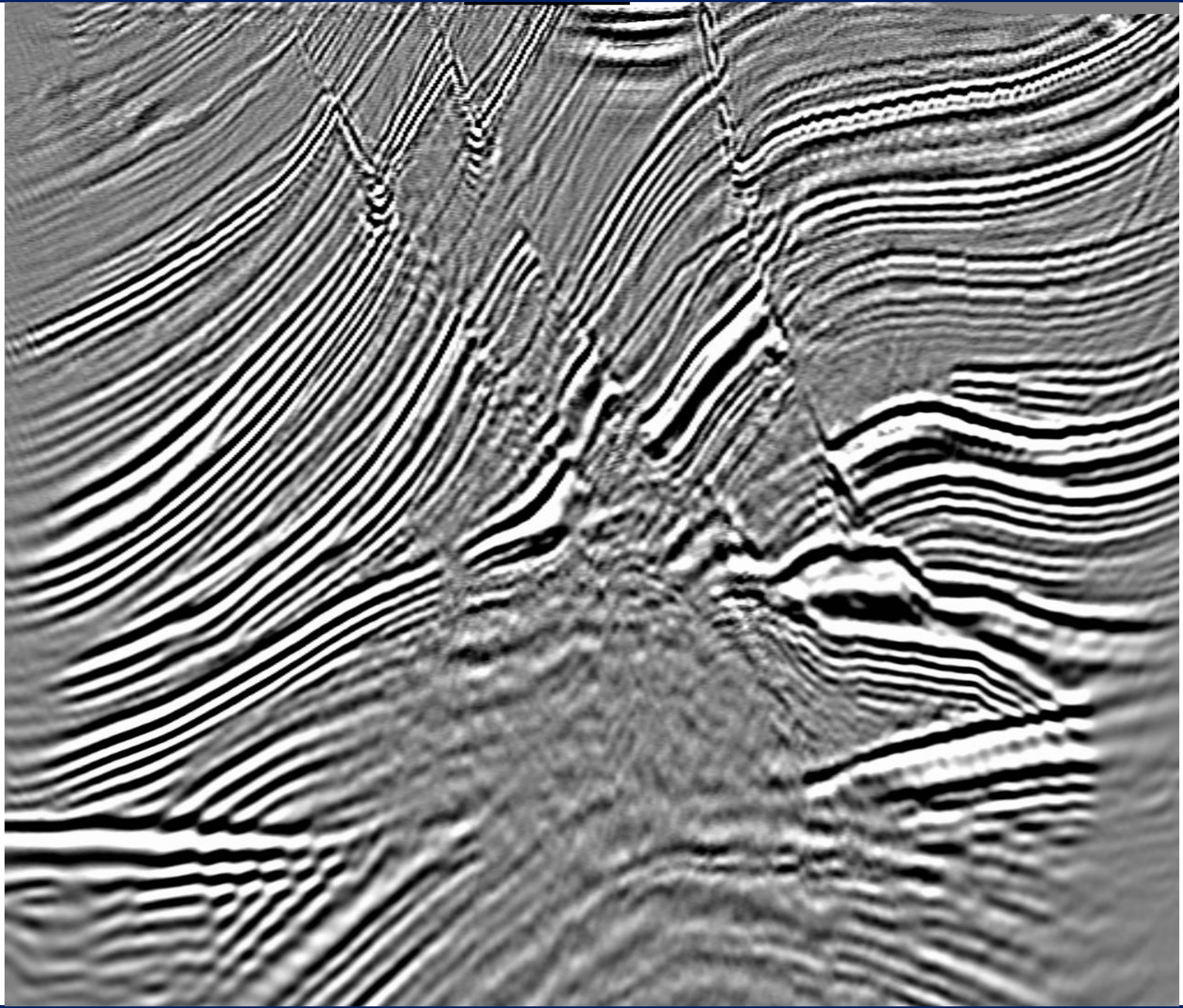
Overview

- Least squares inversion: modeling vs migration
- Finding the weights for the correct operator
- Convergence
- Noise control: sources of noise
- Conclusions

Migration Marmousi



LSMIG Marmousi



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Least squares formulation: modeling vs migration

Undesired features = discrepancy between prediction and data + size of model

L the operator (L modeling, L^H adjoint).
d acquired data
m model

Least Squares inversion

$$J = \|\mathbf{d} - \mathbf{L}\mathbf{m}\|^2 + \lambda \|\mathbf{m}\|_{\mathbf{W}}$$

W weights to inforce a particular solution

Data residuals in a particular norm choice

Model size in a particular norm choice

L, L^H: choose one, calculate the other
Do we start from modeling or migration?

Iteratively solve…

$$\mathbf{m} = (\mathbf{L}^{\mathbf{H}}\mathbf{L} + \mathbf{W}^{\mathbf{H}}\mathbf{W})^{-1}\mathbf{L}^{H}\mathbf{d}$$

Inverse of Hessian

mapping

$$\mathbf{d} = \mathbf{L}\mathbf{m}$$

$$d(\xi, \tau_s + \tau_r, \omega) = \int (-i\omega) W_a e^{i\omega(\tau_s + \tau_r)} m(x,z)\, dx dz$$

$$W_a = f(\tau_s, \tau_r, v_s, v_r)$$

Kirchhoff migration weights

$$m(\mathbf{x}) = W(z, t_s, t_r, v_s, v_r) \int_D d(\xi, \tau(\mathbf{x}, \xi)) d\xi$$ **Kirchhoff summation**

$$W = |\omega| \frac{z}{v_r^2} \frac{1}{\sqrt{t_s t_r}} \frac{t_s}{t_r}$$ **2D shot migration deconvolution IC**

$$W = (i\omega) \frac{v_s z}{v_r^2} \frac{t_s}{t_r^2}$$ **3D shot migration deconvolution IC**

$$W = |\omega| \left(\frac{z}{v_s^2 t_s^{3/2}}\right)\left(\frac{z}{v_r^2 t_r^{3/2}}\right)$$ **2D shot migration cross-correlation IC**

$$W = (i\omega) \left(\frac{z}{v_s^3 t_s^2}\right)\left(\frac{z}{v_r^3 t_r^2}\right)$$ **3D shot migration cross-correlation IC**

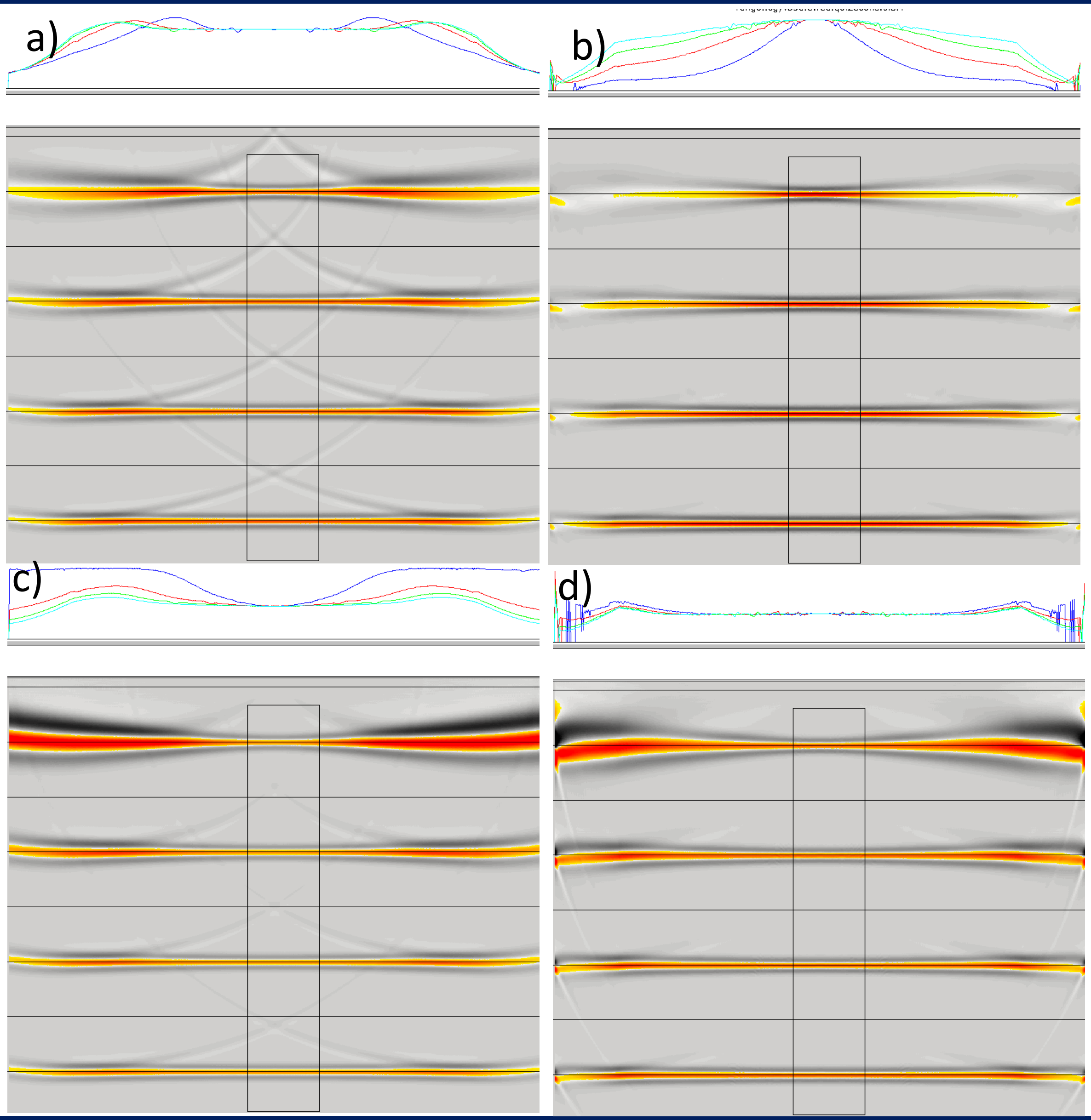
Zhang, Y., Gray, S. H., and Young, J., 2000, Exact and approximate weights for Kirchhoff migration: 70th Ann. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1036-1039.

Lorenzo Casasanta, personal communication

Amplitude Weights: Deconvolution vs Cross correlation

$$W = \sqrt{(i\omega)}\frac{z}{v_r^2}\frac{1}{\sqrt{t_s t_r}}\frac{t_s}{t_r}$$

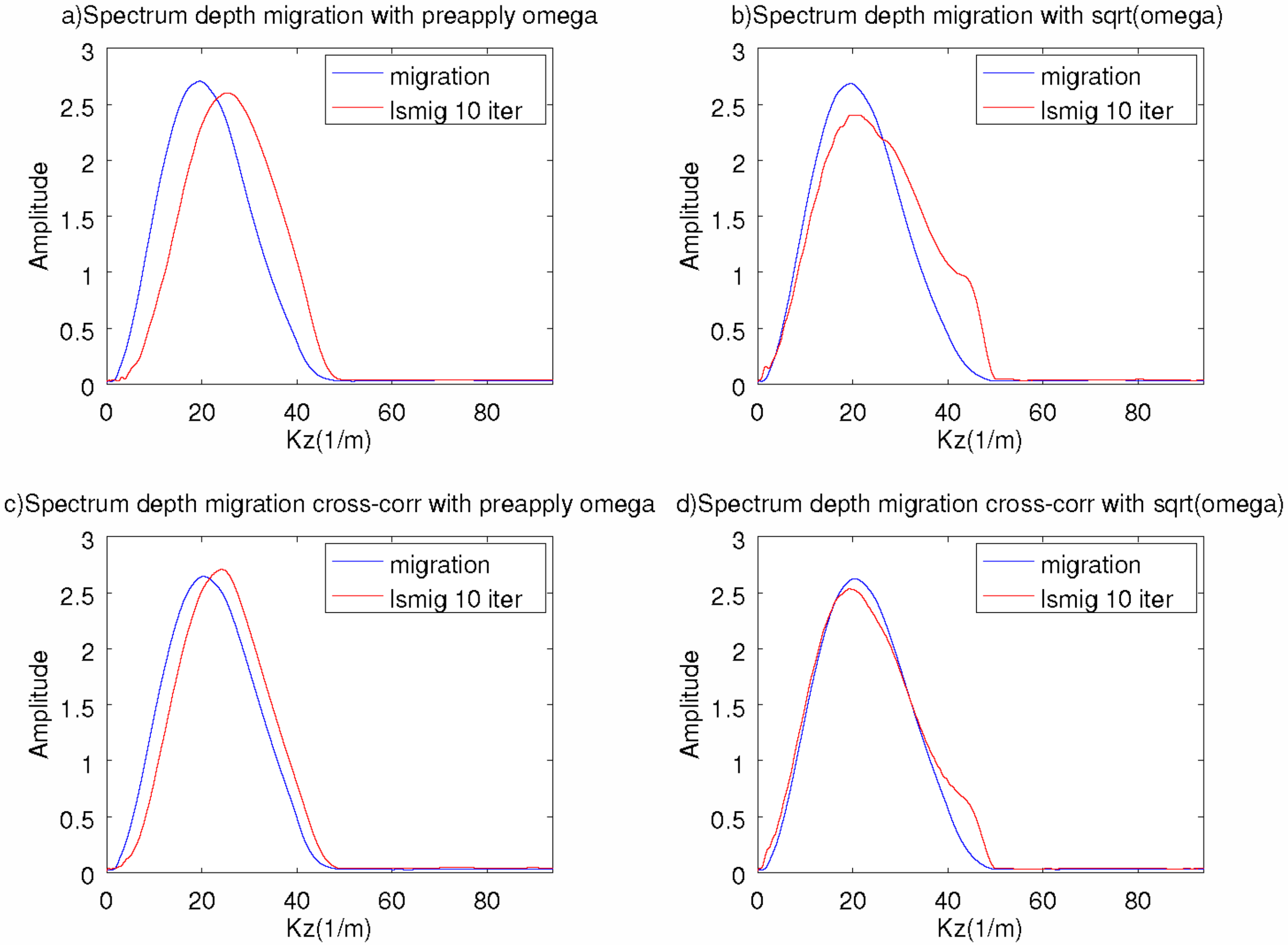
$$W = \sqrt{(i\omega)}\left(\frac{z}{v_s^2 t_s^{3/2}}\right)\left(\frac{z}{v_r^2 t_r^{3/2}}\right)$$



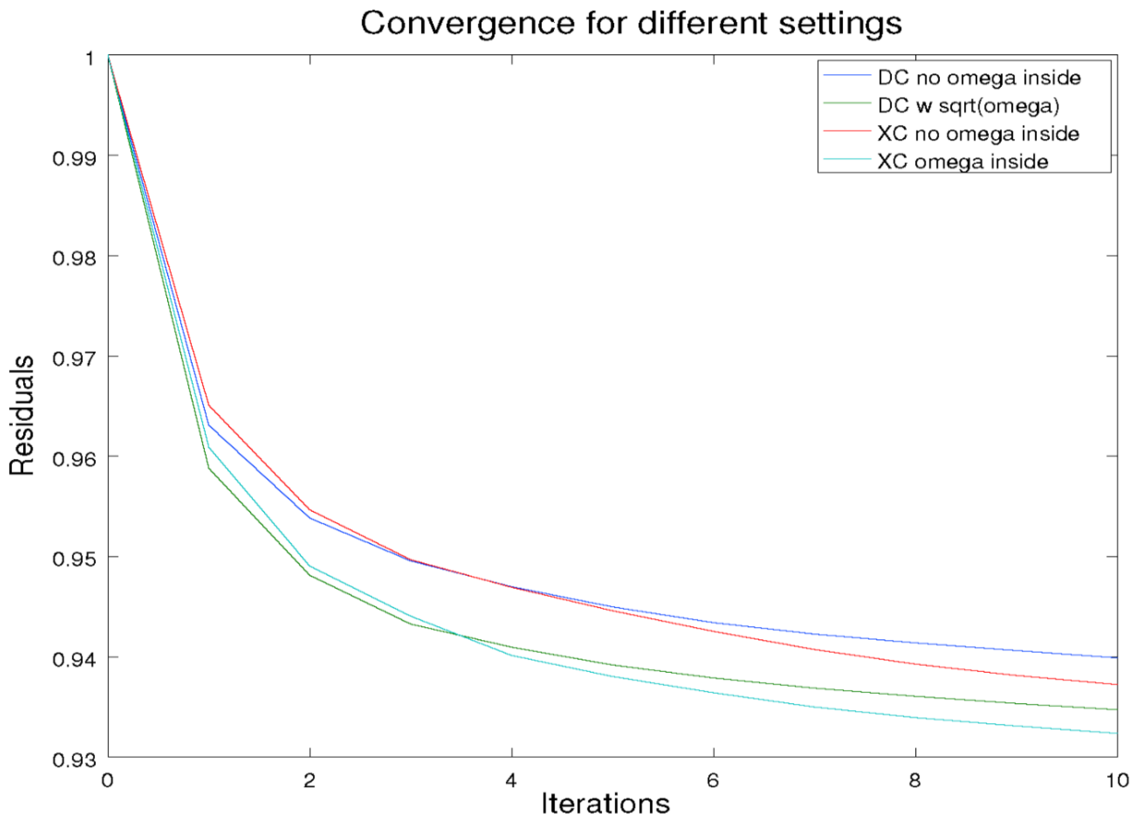
Phase shift filter

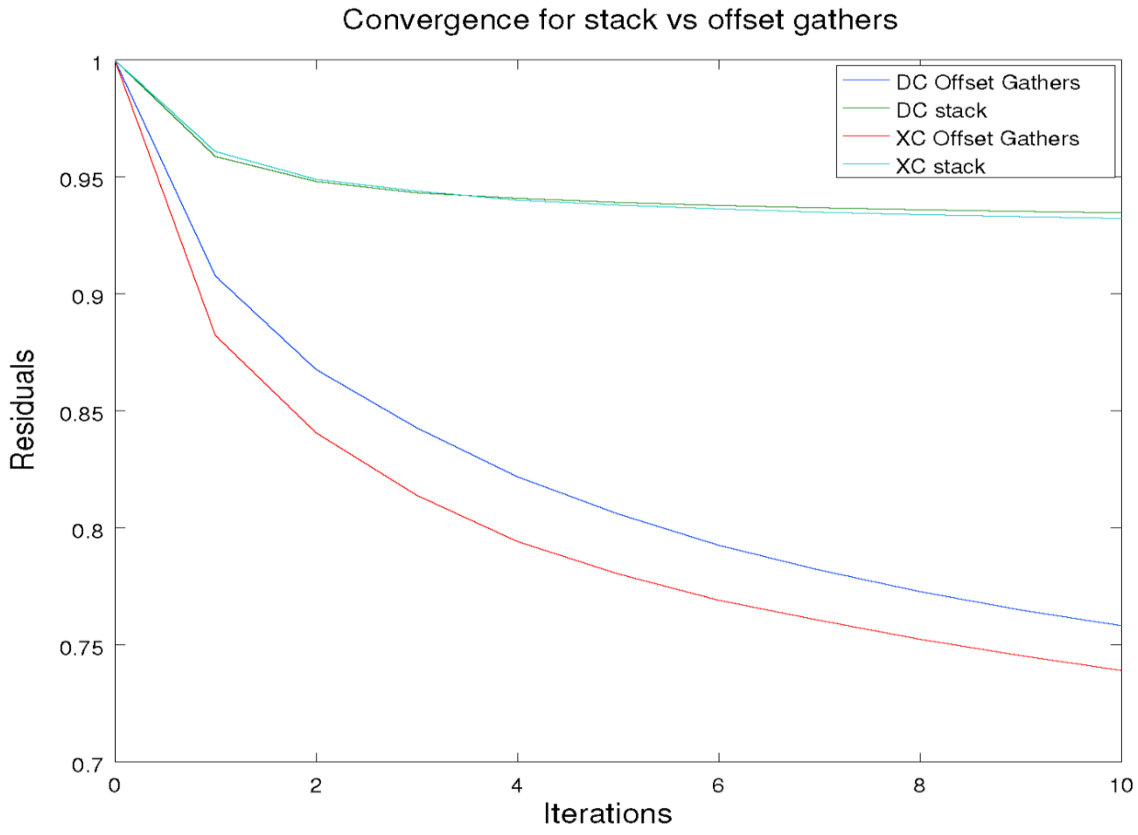
$$d(\xi,\tau_s+\tau_r,\omega)=\int(\cancel{-i\omega})W_a e^{i\omega(\tau_s+\tau_r)}m(x,z)dxdz$$

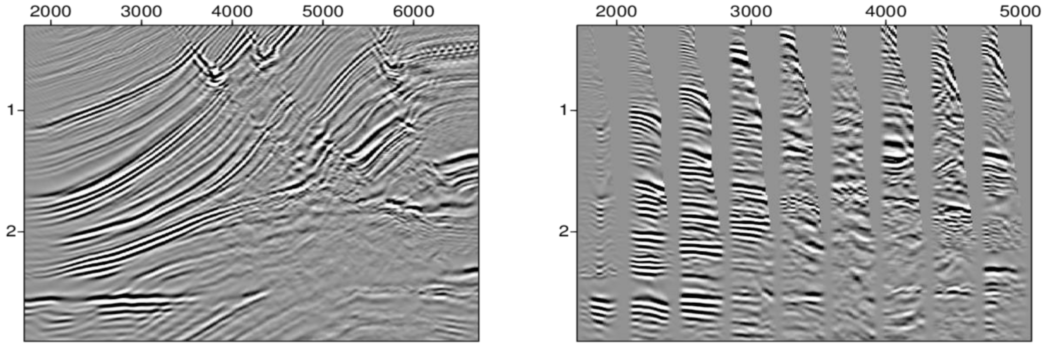
$$d(\xi,\tau_s+\tau_r,\omega)=\int(-i\omega)W_a e^{i\omega(\tau_s+\tau_r)}m(x,z)dxdz$$



Convergence



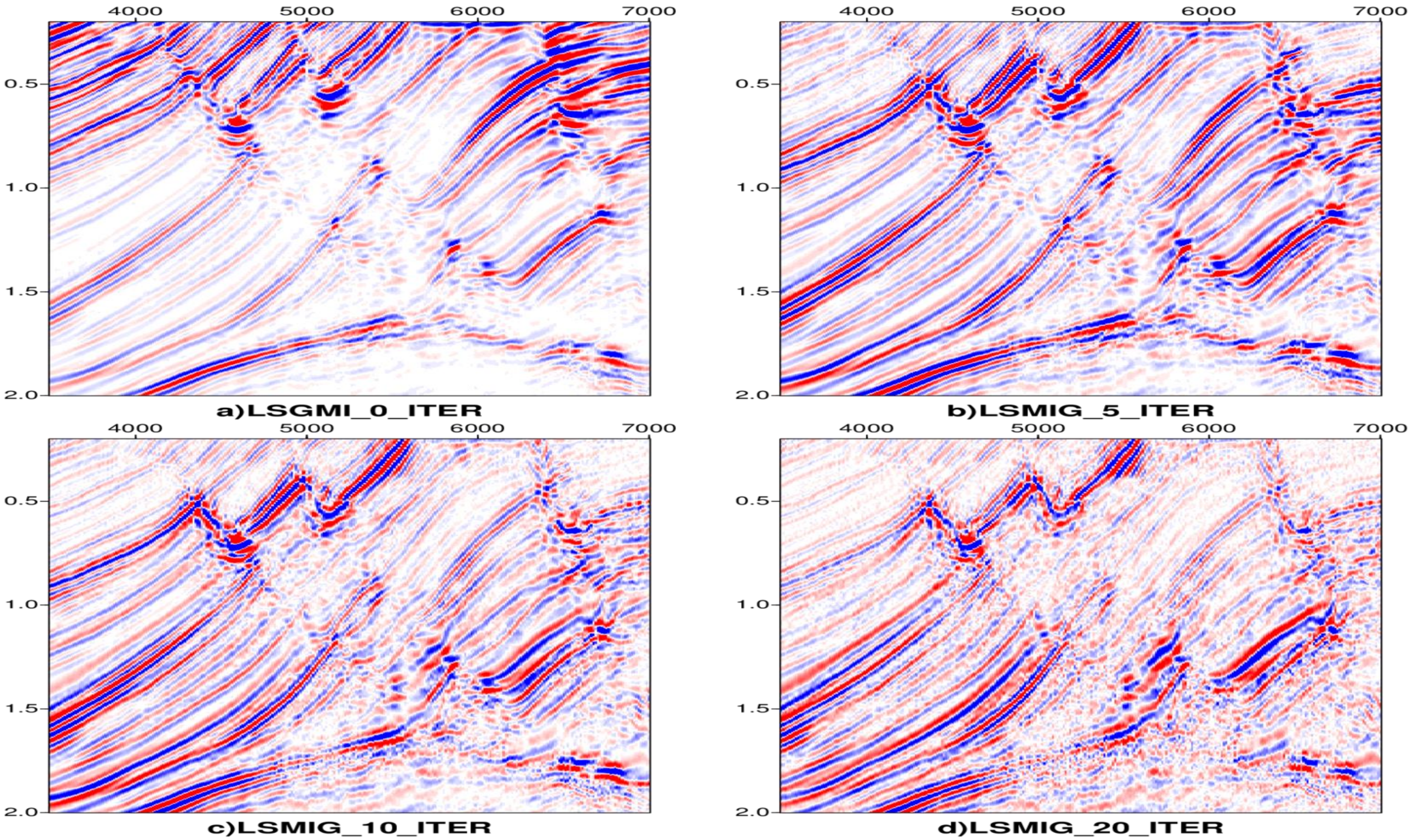




a)MODEL_TO_STACK

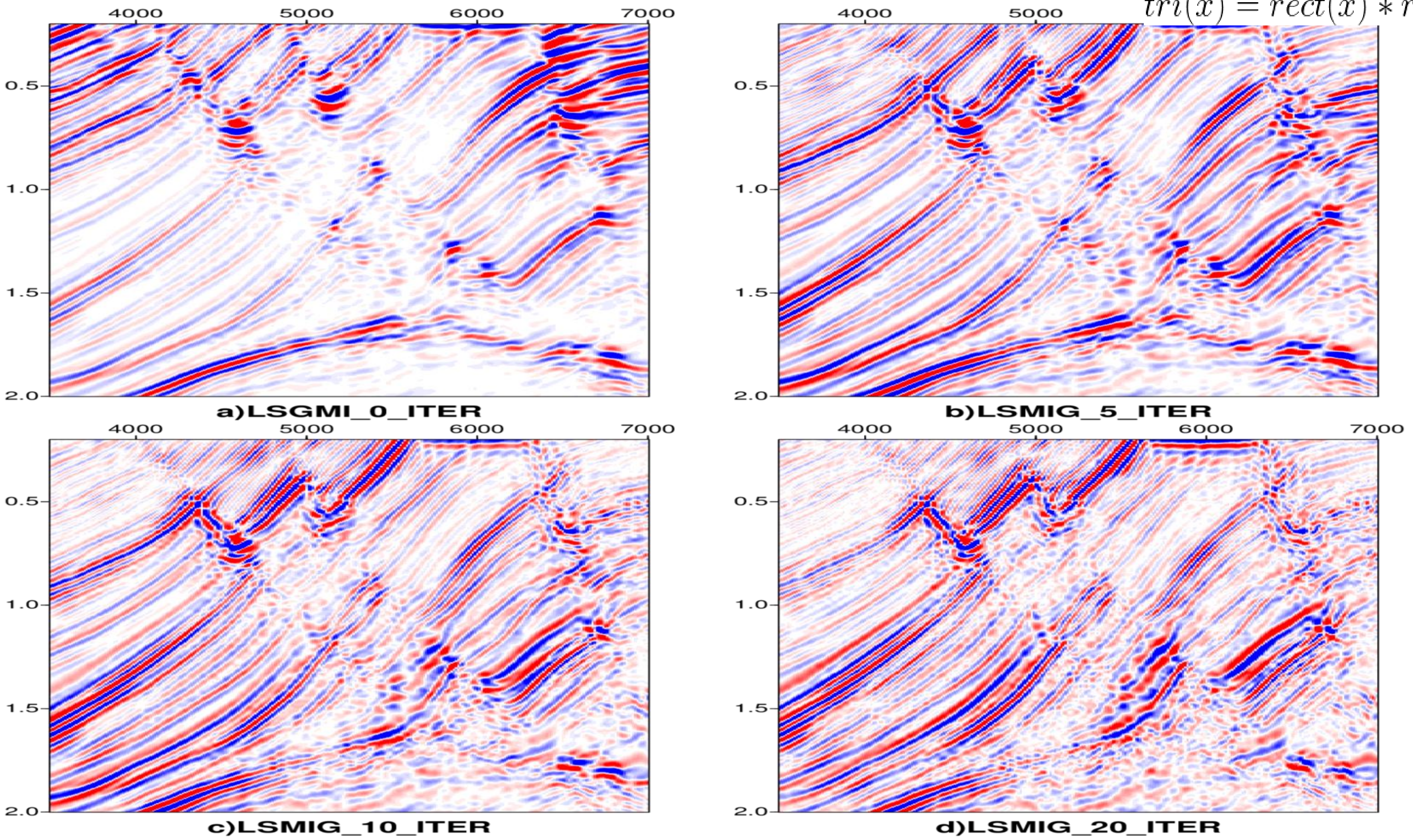
b)MODEL_WITH_OFFSETS

Noise control



Noise control

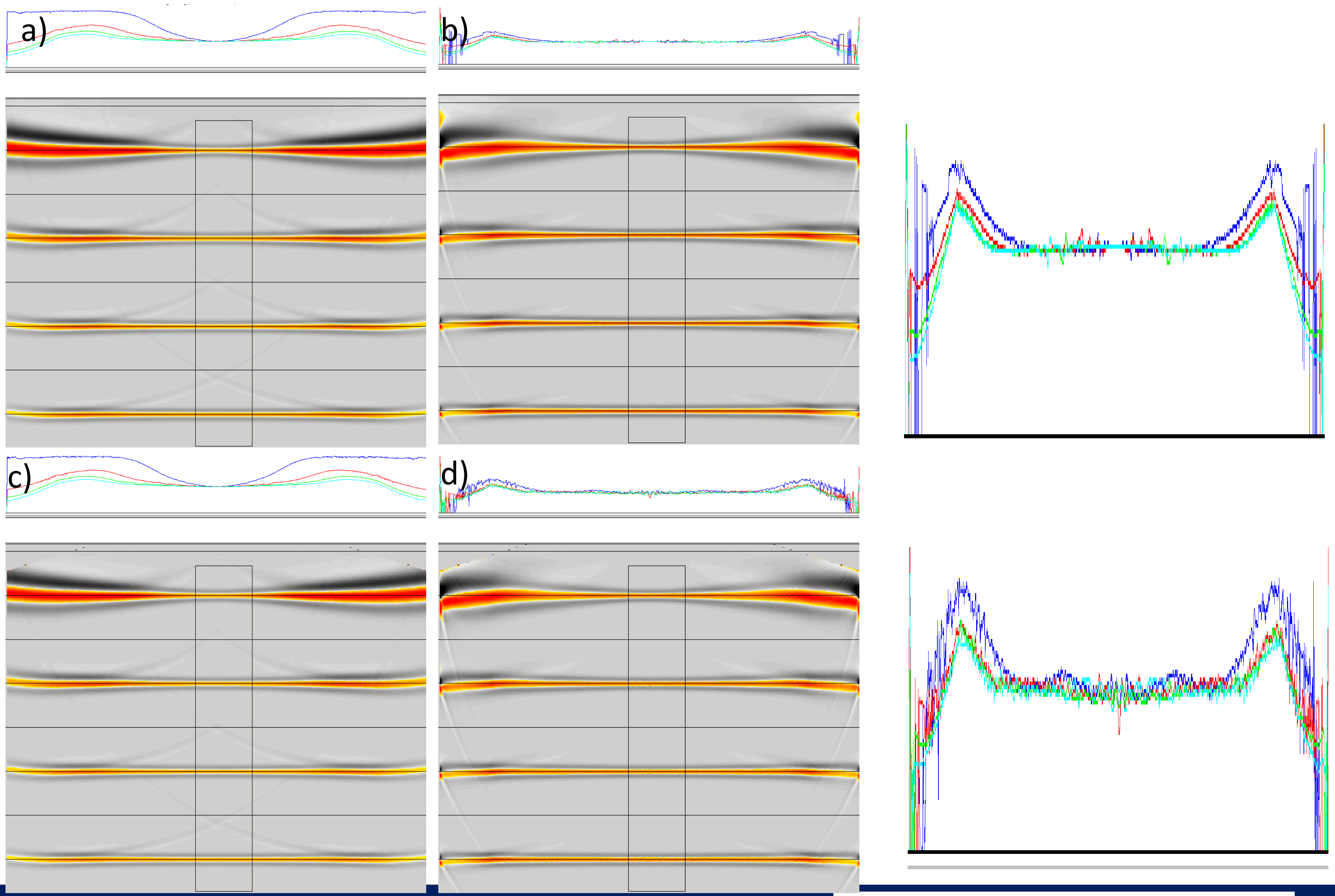
$$d(\xi, t) = LW_m m(x, z)$$
$$W_m = tri(x) \times tri(y) \times tri(offset)$$
$$tri(x) = rect(x) * rect(x)$$



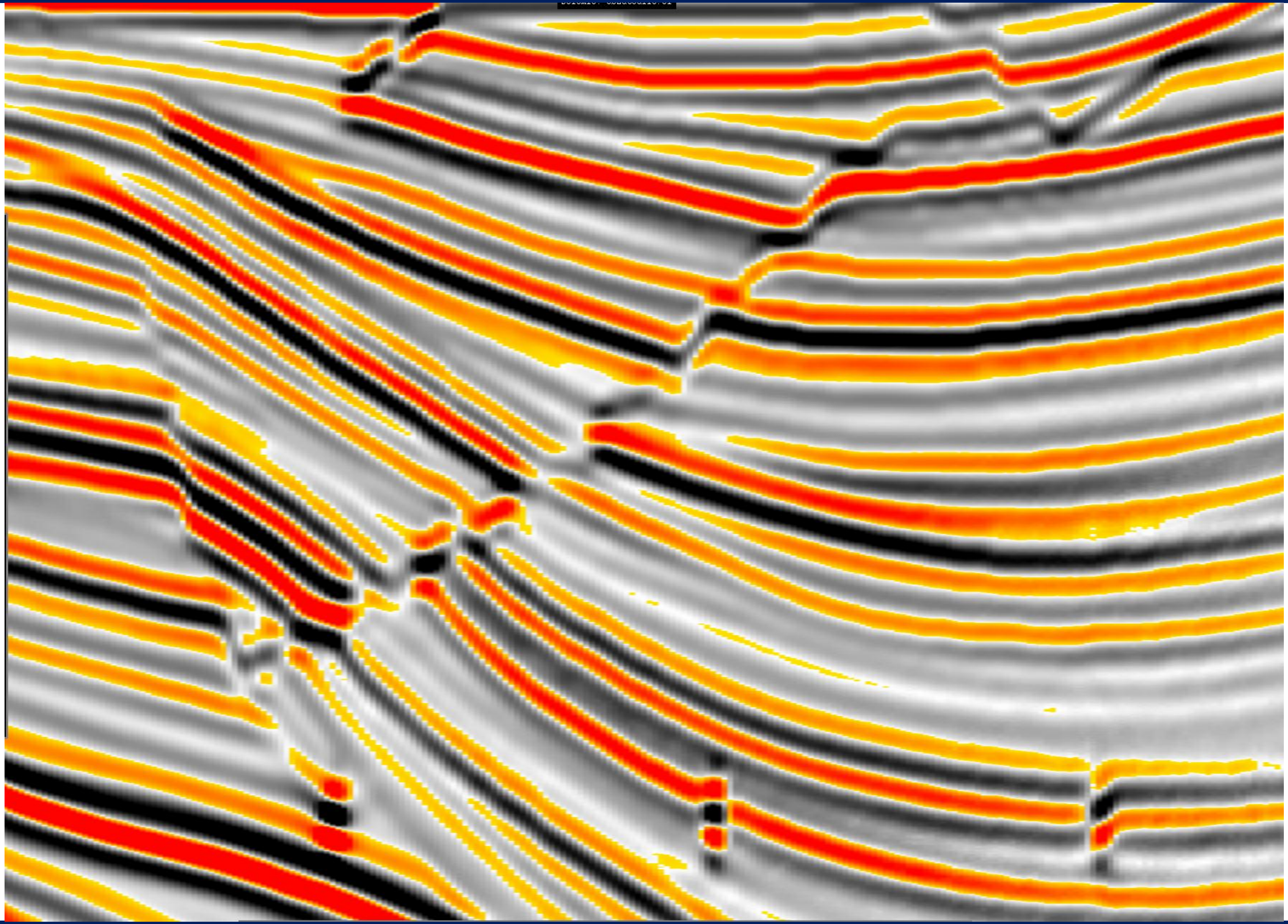
Noise from numerical traveltime table discontinuities

Analytical traveltime
(constant velocity)

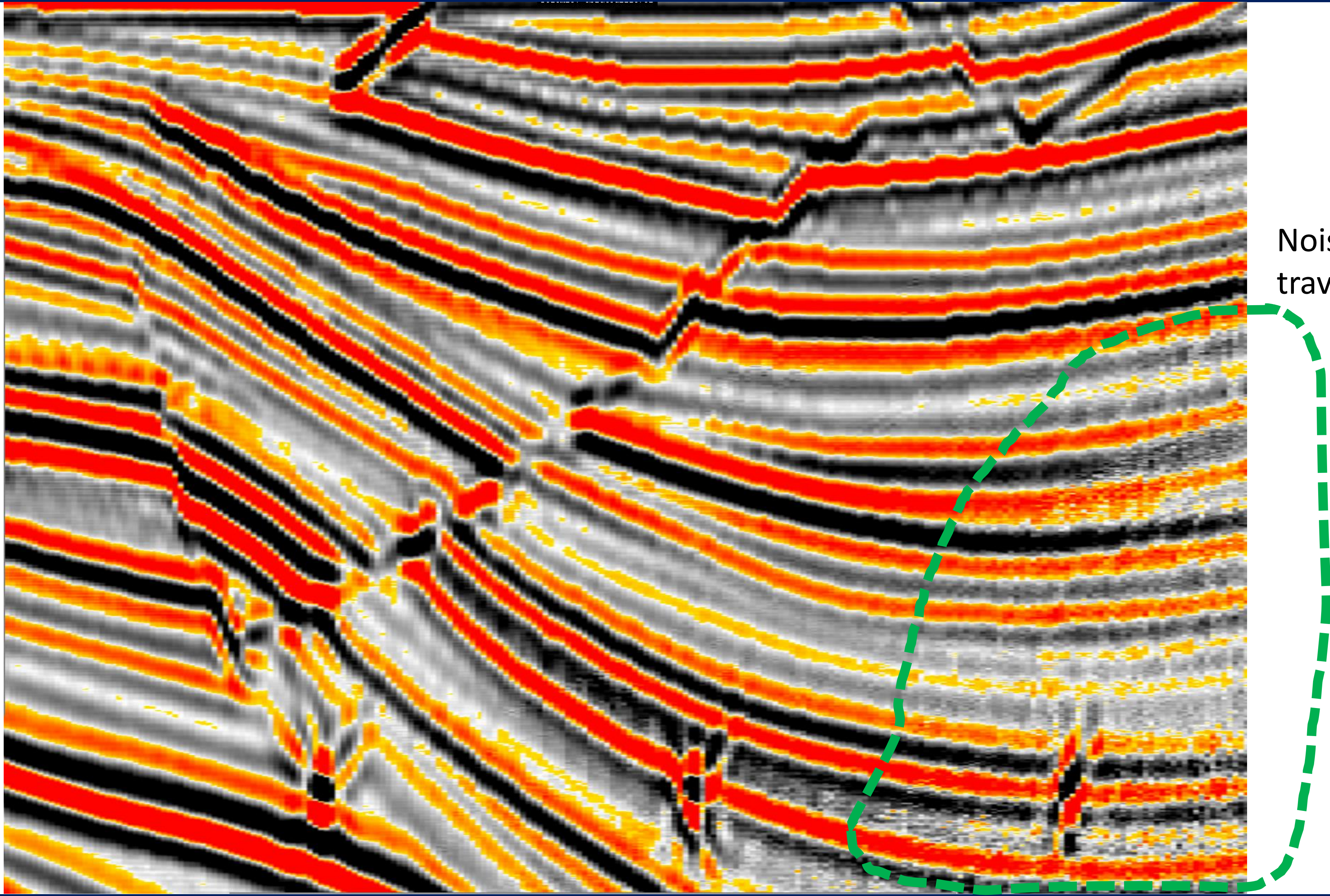
Numerical traveltime
(ray tracing)



Migration (Sigsbee2a)



LSMIG (Sigsbee2a)



Noise from traveltime tables

Conclusions

- Least squares migration has the potential to remove acquisition signature
- Amplitude weights used in LSMIG are not exactly the same as in migration
- Noise accumulates because inconsistencies between operator and physics
- Often this is hidden if the data fit the operator, instead of the reverse
- In Kirchhoff algorithm noise is more obvious than in RTM
- Noise control is essential to stabilize inversion, but we need to distinguish effects.

Acknowledgements

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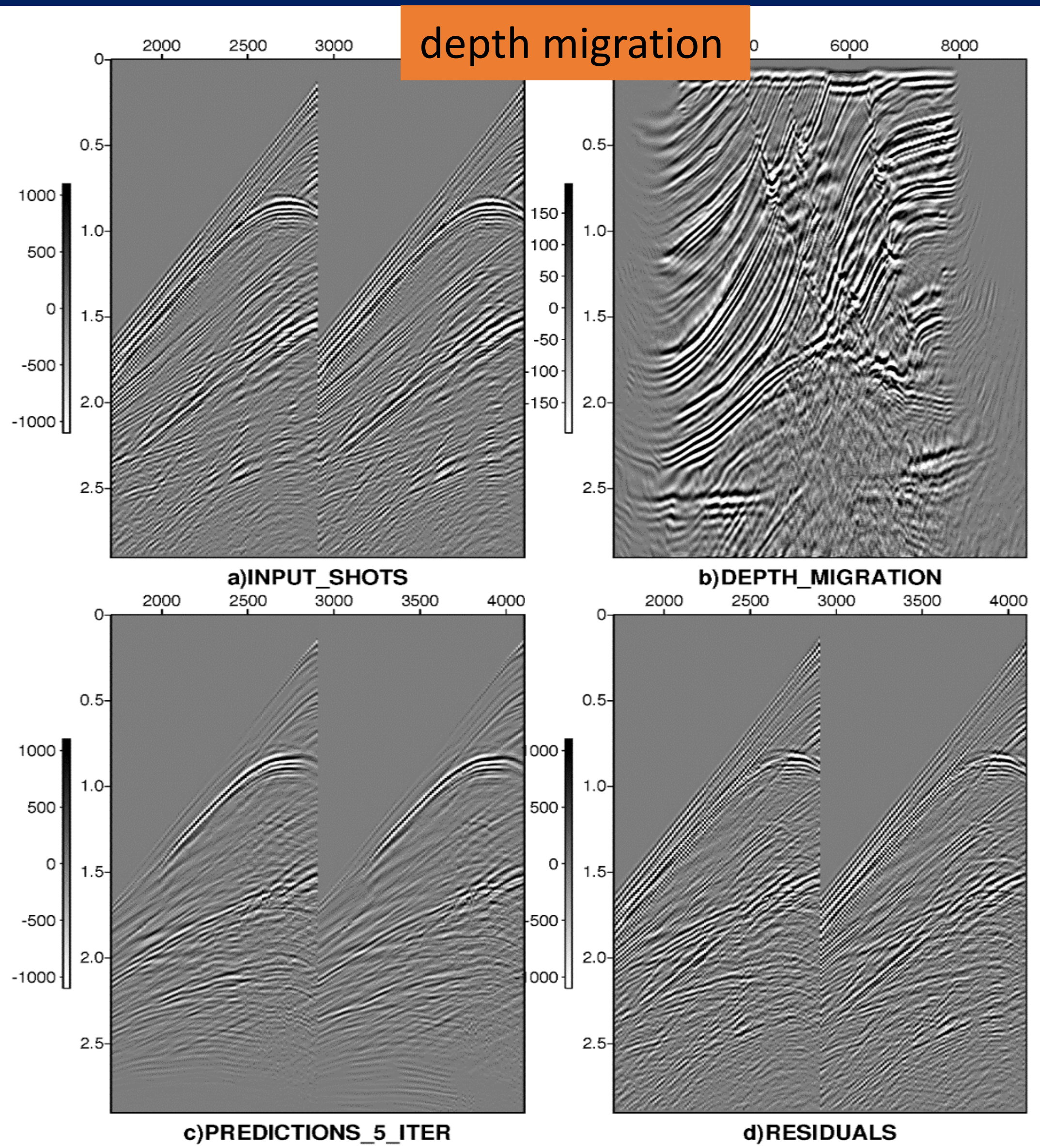
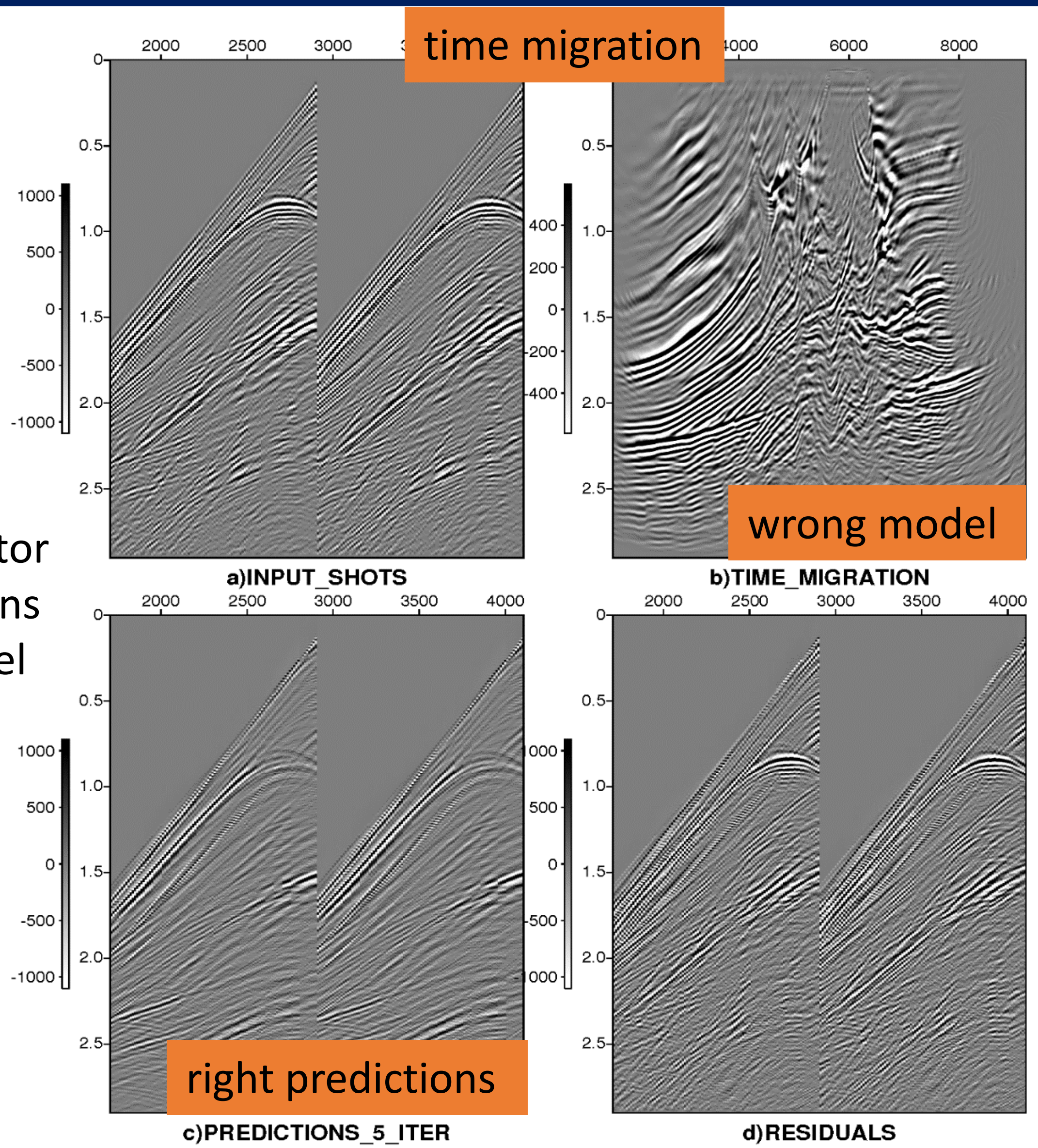
Sam Gray and Lorenzo Casasanta

Interpolation with Green's functions



d=Lm

same data,
different operator
same predictions
different model



Object-Oriented Implementation