Multicomponent internal multiple prediction analysis with elastic stolt-migration, time-stretching, best-fitting by high resolution radon transform

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Outline

- Multicomponent internal multiple prediction (MIMP) algorithm
- Monotonicity of actual depth and pseudo-depth/vertical-traveltime
- Several approximate solutions for input preparation
- Synthetic implementation and discussion
- Conclusion
- Acknowledgements

Multicomponent IMP

1.5D formulation of MIMP (Matson, 1997; Sun and Innanen, 2016):

$$b_3^{ij}(k_g,\omega) = -\int_{-\infty}^{+\infty} \mathrm{d}z_1 e^{\mathrm{i}(\nu^m+\nu^i)z_1} b_1^{im}(k_g,z_1) \int_{-\infty}^{z_1-\epsilon} \mathrm{d}z_2 e^{-\mathrm{i}(\nu^n+\nu^m)z_2} b_1^{mn}(k_g,z_2) \times \int_{z_2+\epsilon}^{+\infty} \mathrm{d}z_3 e^{\mathrm{i}(\nu^j+\nu^n)z_3} b_1^{nj}(k_g,z_3)$$

Inputs: $b_1^{ij}(k_g,z) = \mathrm{i}2\nu^j D^{ij}(k_g,z),\ \{i,j\} \in \{P, SV\}.$

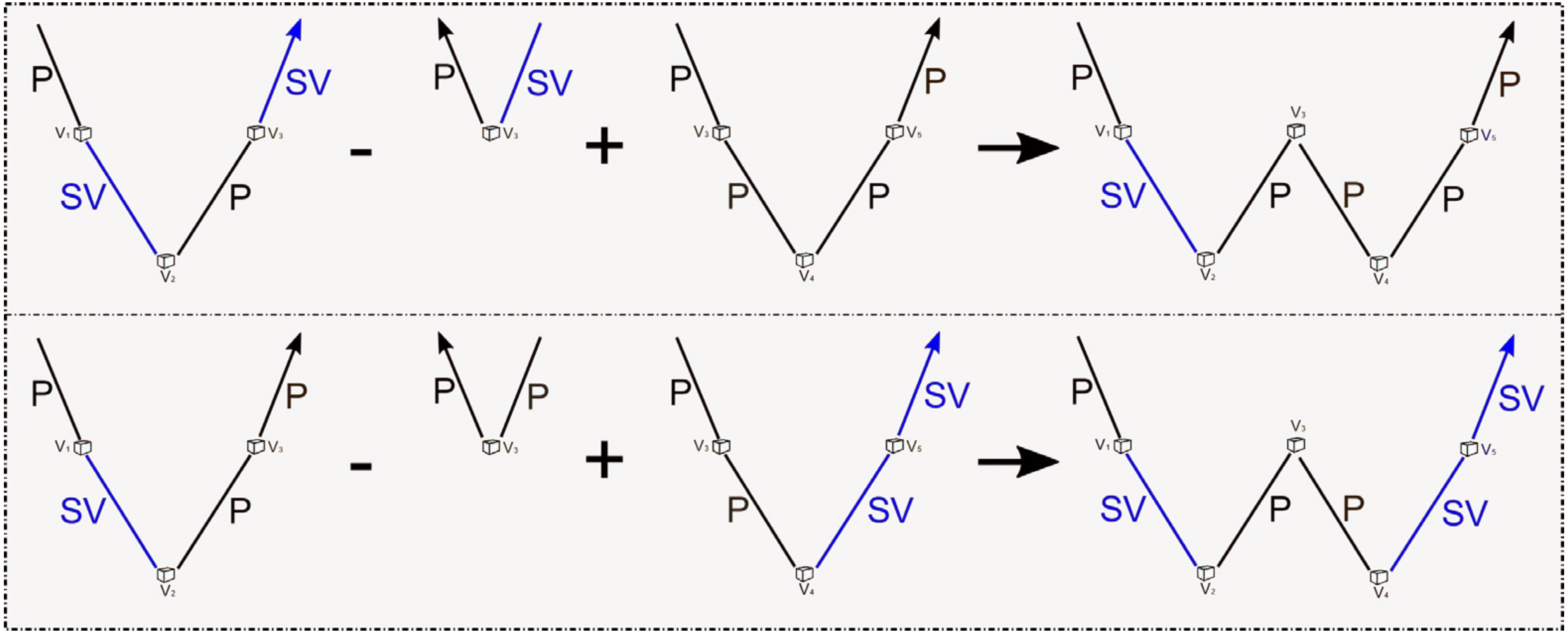
For P-wave source only:

$$b_3^{\acute{P}\grave{P}} = \Theta_1(b_1^{\acute{P}\grave{P}})\Theta_2(b_1^{\acute{P}\grave{P}})\Theta_3(b_1^{\acute{P}\grave{P}}) + \Theta_1(b_1^{\acute{P}\grave{S}})\Theta_2(b_1^{\acute{S}\grave{P}})\Theta_3(b_1^{\acute{P}\grave{P}}) + \Theta_1(b_1^{\acute{P}\grave{P}})\Theta_2(b_1^{\acute{P}\grave{S}})\Theta_3(b_1^{\acute{P}\grave{P}})$$

$$b_3^{\acute{S}\grave{P}} = \Theta_1(b_1^{\acute{S}\grave{P}})\Theta_2(b_1^{\acute{P}\grave{P}})\Theta_3(b_1^{\acute{P}\grave{P}}) + \Theta_1(b_1^{\acute{S}\grave{P}})\Theta_2(b_1^{\acute{P}\grave{S}})\Theta_3(b_1^{\acute{S}\grave{P}})$$

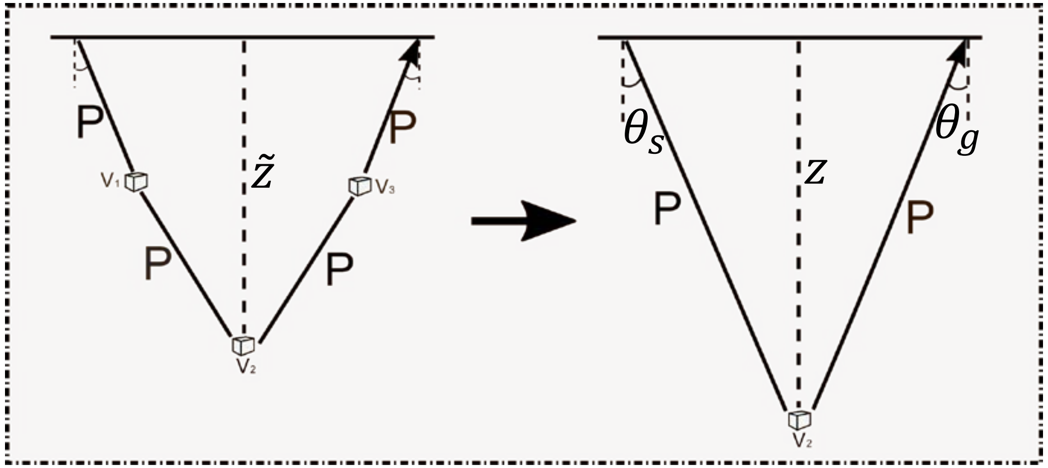
Multicomponent IMP

Lower-Higher-Lower combinations in actual depth:



Monotonicity of actual & pseudo-depth

For acoustic cases:



$$z = \frac{v\tau}{cos\theta_s + cos\theta_g}$$

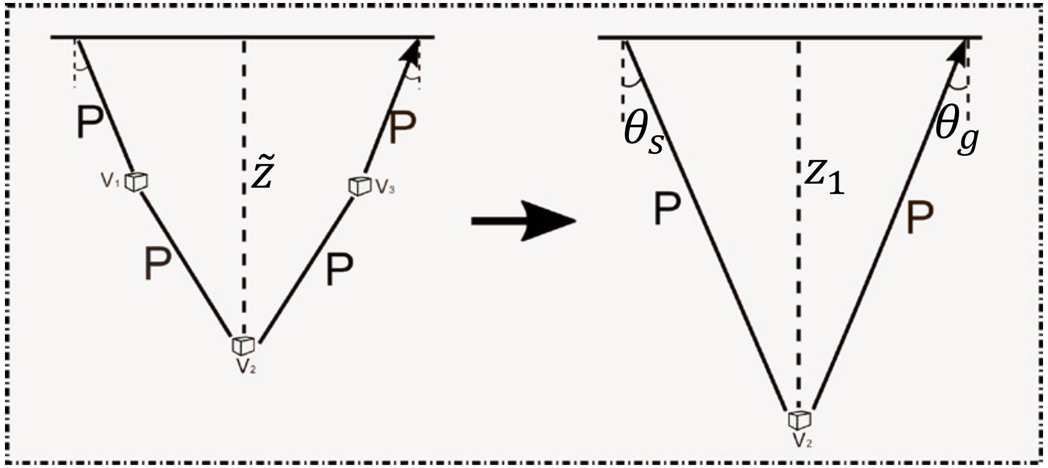
$$\tilde{z}_1 < \tilde{z}_2 \leftrightarrow z_1 < z_2$$

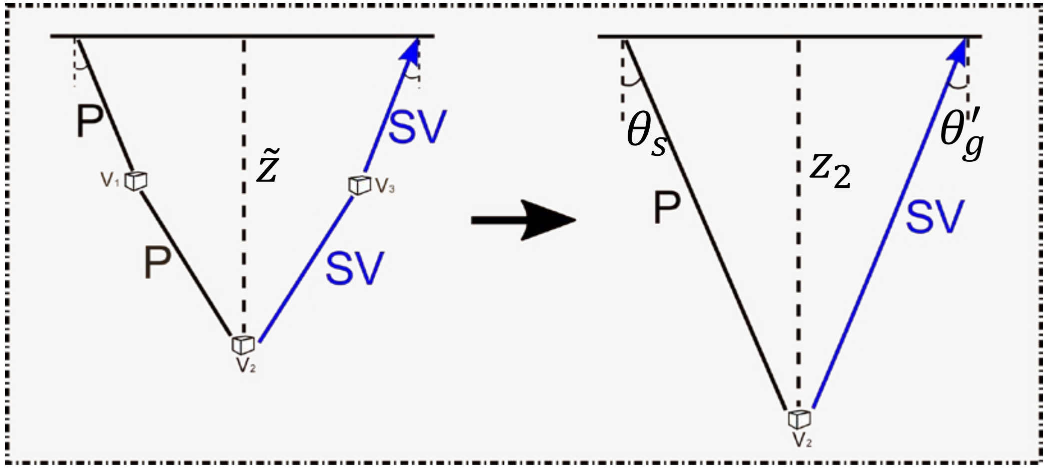
$D(x,t) \rightarrow D(k/p, \omega(k_z)) \rightarrow D(k/p, k_z) \rightarrow D(k/p, z)$ ⇐ Stolt migration

$D(x,t) \rightarrow D(p,\tau)$ ⇐ $\tau - p$ transform

Monotonicity of actual & pseudo-depth

For elastic cases:





$$z_1 = \frac{\alpha \tau_{pp}}{cos\theta_s + cos\theta_g}$$

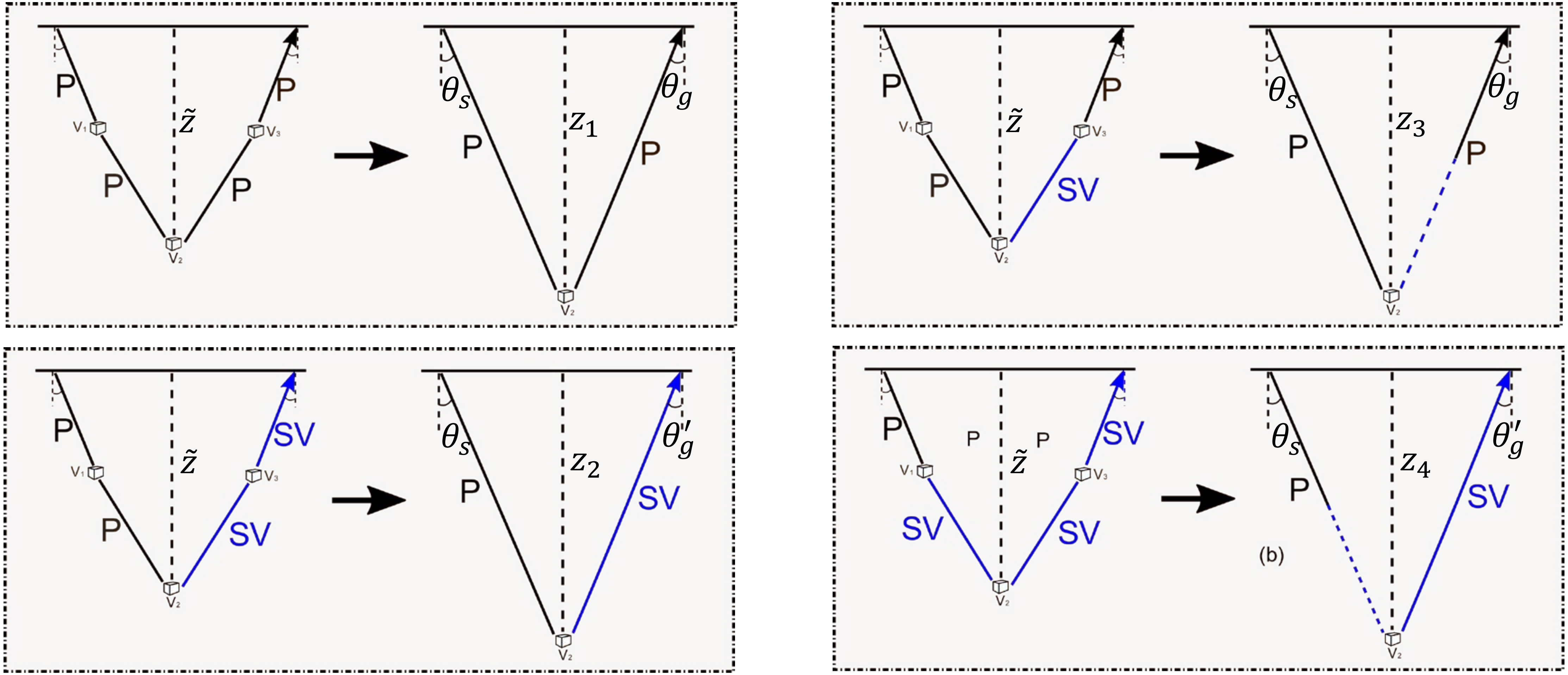
$$z_1 \approx z_2$$

$$z_2 = \frac{v \tau_{ps}}{cos\theta_s + cos\theta_g'}$$

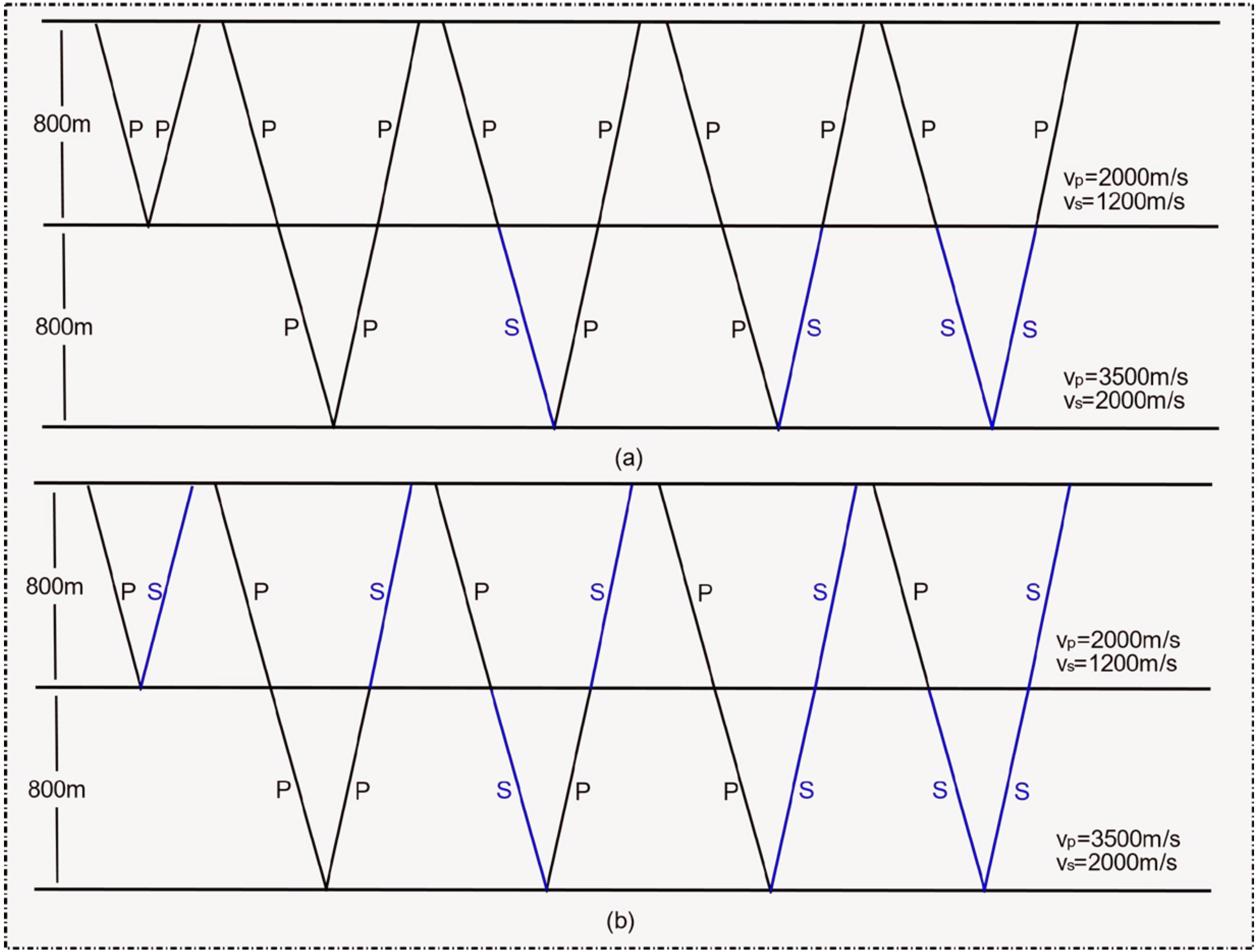
$$v = \frac{\alpha\beta}{\alpha + \beta}$$

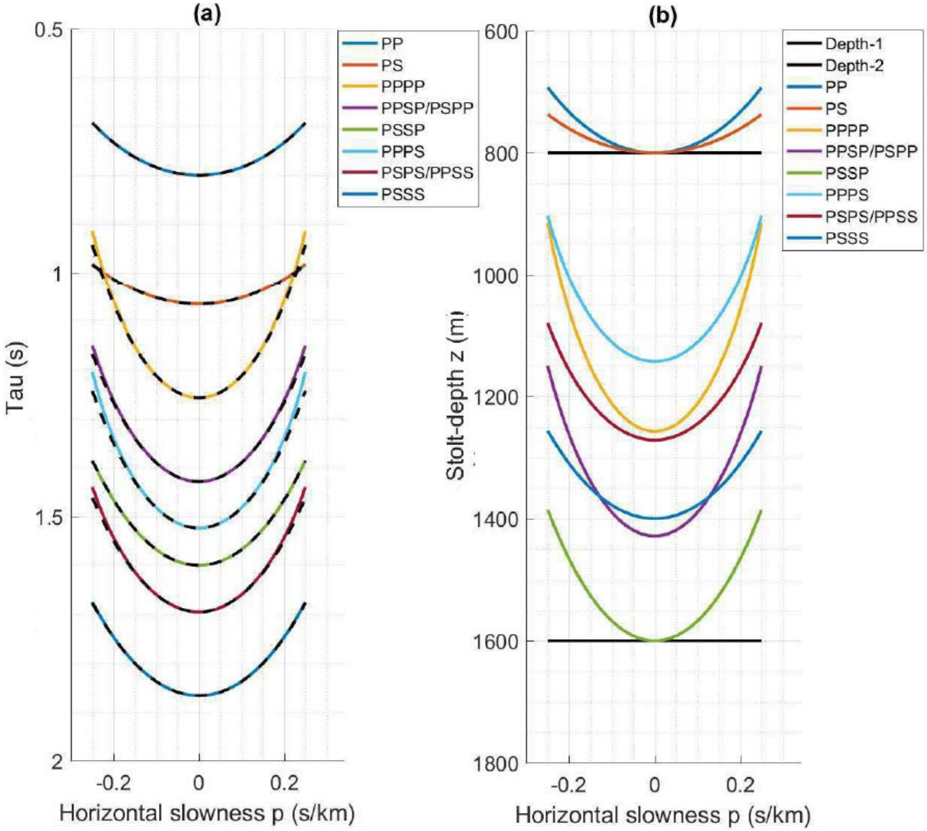
Monotonicity of actual & pseudo-depth

For elastic cases:



Approximate solutions for input preparation





Approximate solutions for input preparation

$$b_3^{ij}(p_g,\omega) = -\int_{-\infty}^{+\infty} \mathrm{d}\tau_2^{mn} e^{-\mathrm{i}\omega\tau_2^{mn}} b_1^{mn}(p_g,\tau_2^{mn}) \int_{\Upsilon(\tau_2^{mn}|\tau_3^{nj})+\epsilon}^{+\infty} \mathrm{d}\tau_3^{nj} e^{\mathrm{i}\omega\tau_3^{nj}} b_1^{nj}(p_g,\tau_3^{nj})$$
$$\times \int_{\Upsilon(\tau_2^{mn}|\tau_1^{im})+\epsilon}^{+\infty} \mathrm{d}\tau_1^{im} e^{\mathrm{i}\omega\tau_1^{im}} b_1^{im}(p_g,\tau_1^{im})$$

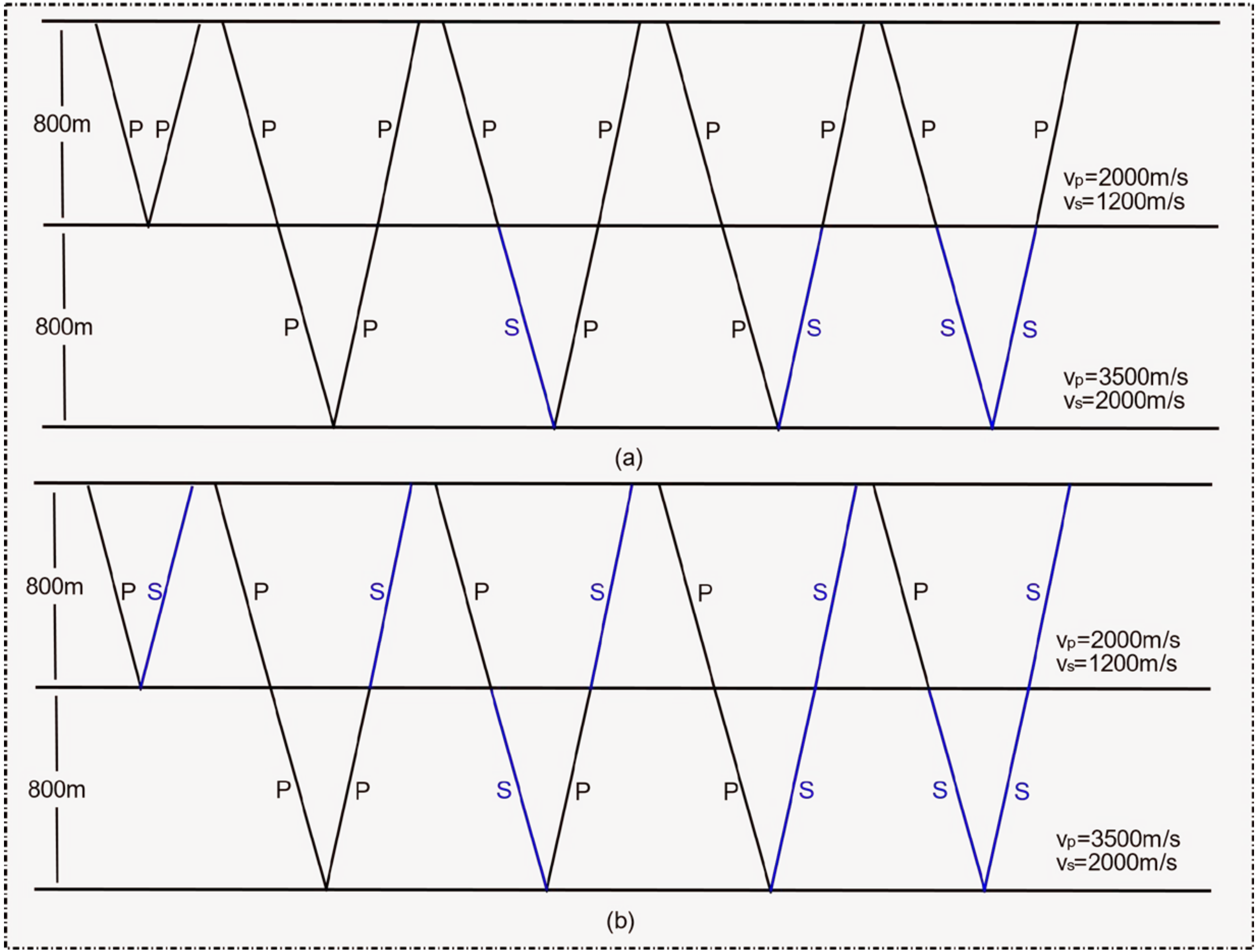
$$\Upsilon(\tau_2^{mn}|\tau_1^{nj}) = \begin{cases} \tau_2^{mn}, & j = m; \\ \frac{\alpha+\beta}{2\beta}\tau_2^{mn}, & j = S \;\&\; m = P; \\ \frac{2\beta}{\alpha+\beta}\tau_2^{mn}, & j = P \;\&\; m = S; \end{cases}$$

Approximate solutions for input preparation

$$b_3^{ij}(p_g,\omega) = -\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} \mathrm{d}\tau_2^{mn} e^{-\mathrm{i}\omega\tau_2^{mn}} b_1^{mn}(p_g,\tau_2^{mn}) \int_{\Gamma(mn,nj)+\epsilon}^{+\infty} \mathrm{d}\tau_3^{nj} e^{\mathrm{i}\omega\tau_3^{nj}} b_1^{nj}(p_g,\tau_3^{nj}) \times \int_{\Gamma(mn,im)+\epsilon}^{+\infty} \mathrm{d}\tau_1^{im} e^{\mathrm{i}\omega\tau_1^{im}} b_1^{im}(p_g,\tau_1^{im})$$

$$\Gamma(X,Y) = \begin{cases} \frac{\alpha}{\beta}, & X = SS; \\ \frac{\alpha+\beta}{2\beta}, & X \neq SS \& Y = PS \text{ or } SP; \\ 1, & others. \end{cases}$$

Approximate solutions for input preparation



$$t^2 = t_0^2 + \frac{x^2}{v_s^2}$$

$$m(\tau, v) = \int_{h_{min}}^{h_{max}} d(t = \sqrt{\tau^2 + h^2/v^2}, h)\mathrm{d}h$$

$$\mathbf{d} = \mathbf{L}\mathbf{m}$$

$$J = \| \mathbf{W}_d(\mathbf{L}\mathbf{m} - \mathbf{d}) \|^2 + \| \mathbf{W}_m\mathbf{m} \|^2$$

$$\hat{d}(\tau, p) = \int_{v_{min}}^{v_{max}} m(\tau_0 = \frac{\tau}{\sqrt{1 - p^2v^2}}, v)\mathrm{d}v$$

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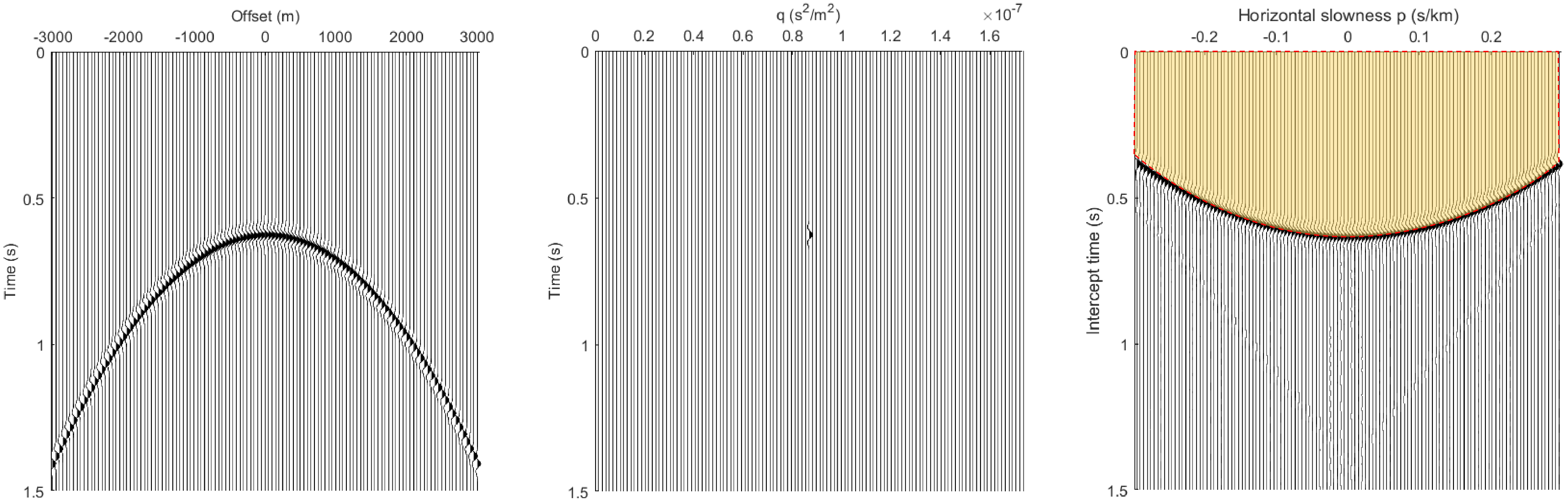
Approximate solutions for input preparation



$D(x, t)$ $m(v, \tau)$ $\widetilde{D}\,(p, \tau)$

Approximate solutions for input preparation

- Approximate solutions for input preparation:
  - Elastic stolt migration (First mentioned by Matson, 1997)
  - Time-stretching method
  - Best-fitting approach by high-resolution hyperbolic radon

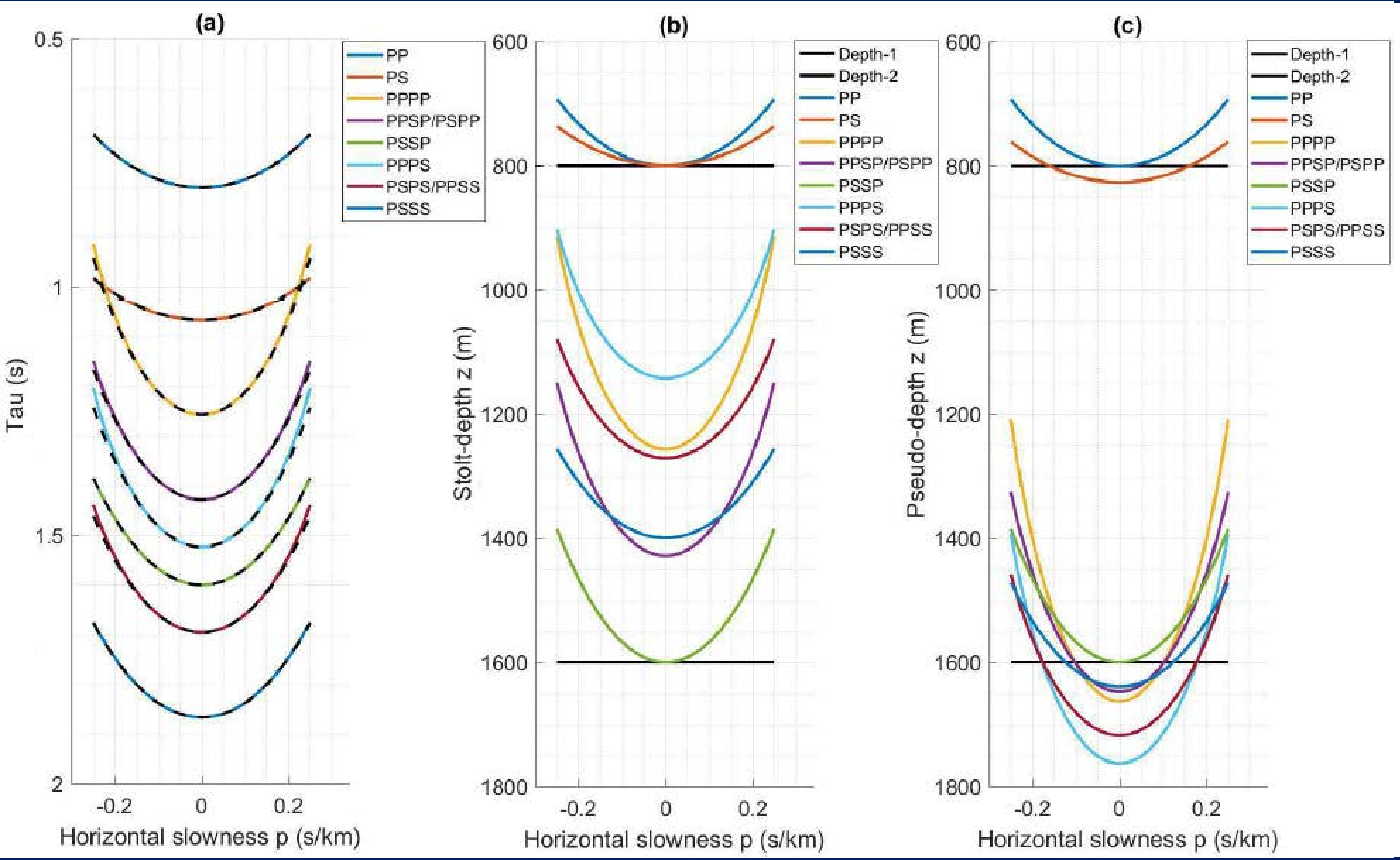
$$b_3^{ij}(k_g,\omega) = \int_{-\infty}^{+\infty} \mathrm{d}z_1 e^{\mathrm{i}(\nu^m+\nu^i)z_1} b_1^{im}(k_g,z_1) \int_{-\infty}^{z_1-\epsilon} \mathrm{d}z_2 e^{-\mathrm{i}(\nu^n+\nu^m)z_2} b_1^{mn}(k_g,z_2) \times \int_{z_2+\epsilon}^{+\infty} \mathrm{d}z_3 e^{\mathrm{i}(\nu^n+\nu^j)z_3} b_1^{nj}(k_g,z_3)$$

$$b_3^{ij}(p_g,\omega) = -\int_{-\infty}^{+\infty} \mathrm{d}\tau_1^{im} e^{\mathrm{i}\omega\tau_1^{im}} b_1^{im}(p_g,\tau_1^{im}) \int_{-\infty}^{\Gamma(\tau_1^{im}|\tau_2^{mn})-\epsilon} \mathrm{d}\tau_2^{mn} e^{-\mathrm{i}\omega\tau_2^{mn}} b_1^{mn}(p_g,\tau_2^{mn}) \times \int_{\Gamma(\tau_2^{mn}|\tau_3^{nj})+\epsilon}^{+\infty} \mathrm{d}\tau_3^{nj} e^{\mathrm{i}\omega\tau_3^{nj}} b_1^{nj}(p_g,\tau_3^{nj})$$

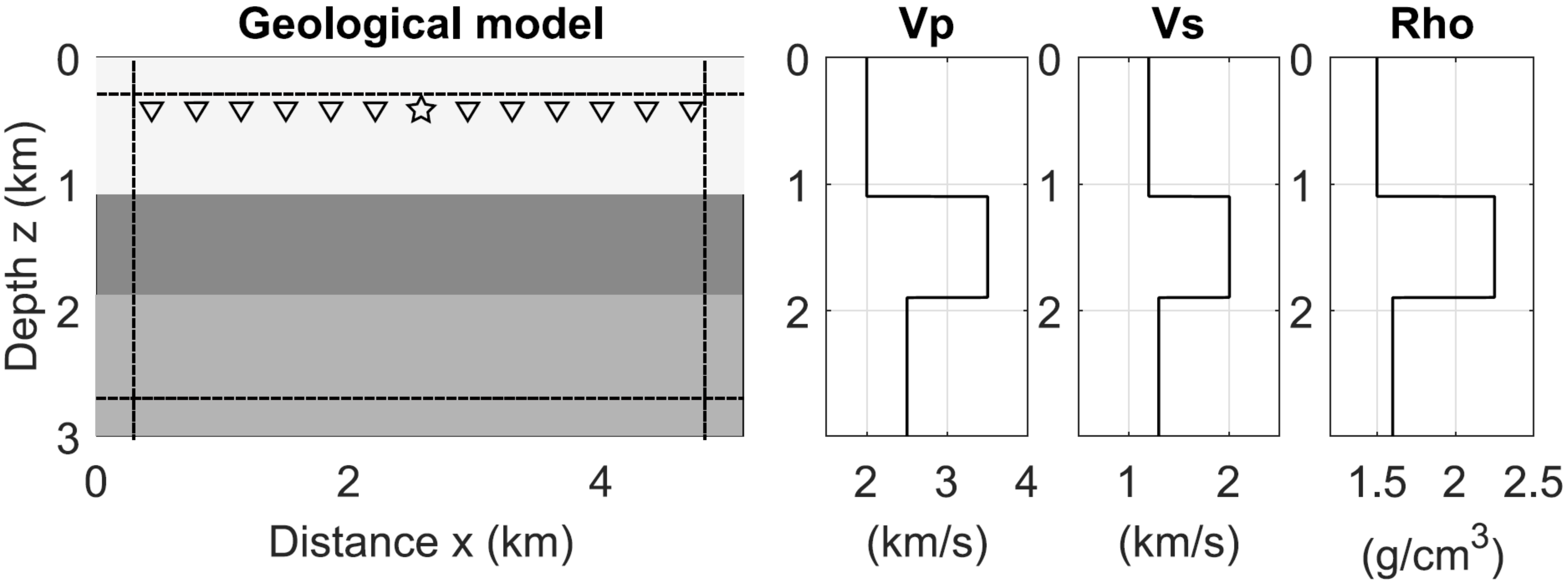
$$\Gamma(\tau_2^{mn}|\tau_1^{nj}) = \frac{\upsilon^{mn}(p_g,\tau_2^{mn})}{\upsilon^{nj}(p_g,\tau_1^{nj})}\tau_2^{mn}$$

$$P \,\&\, m = S;$$

Approximate solutions for input preparation



Synthetic -- model



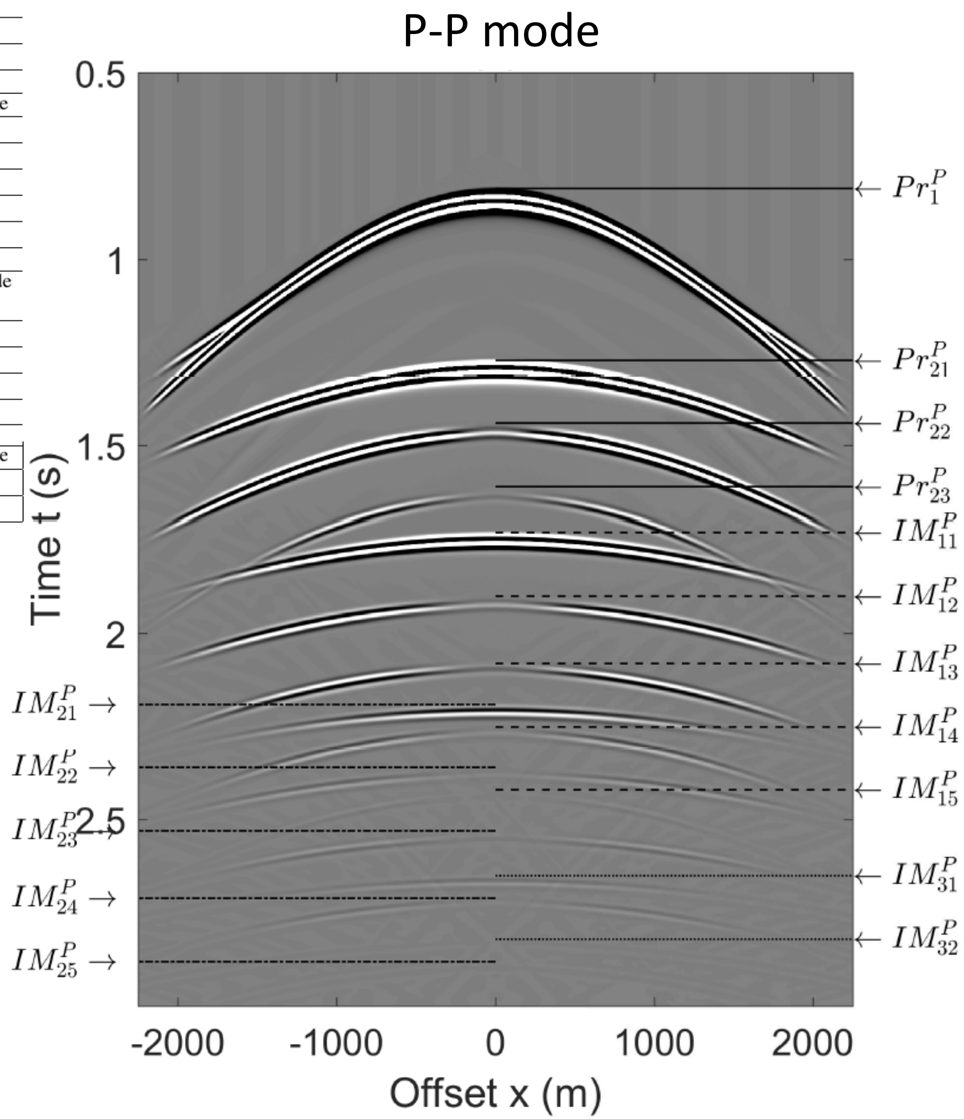
Synthetic -- shot profile

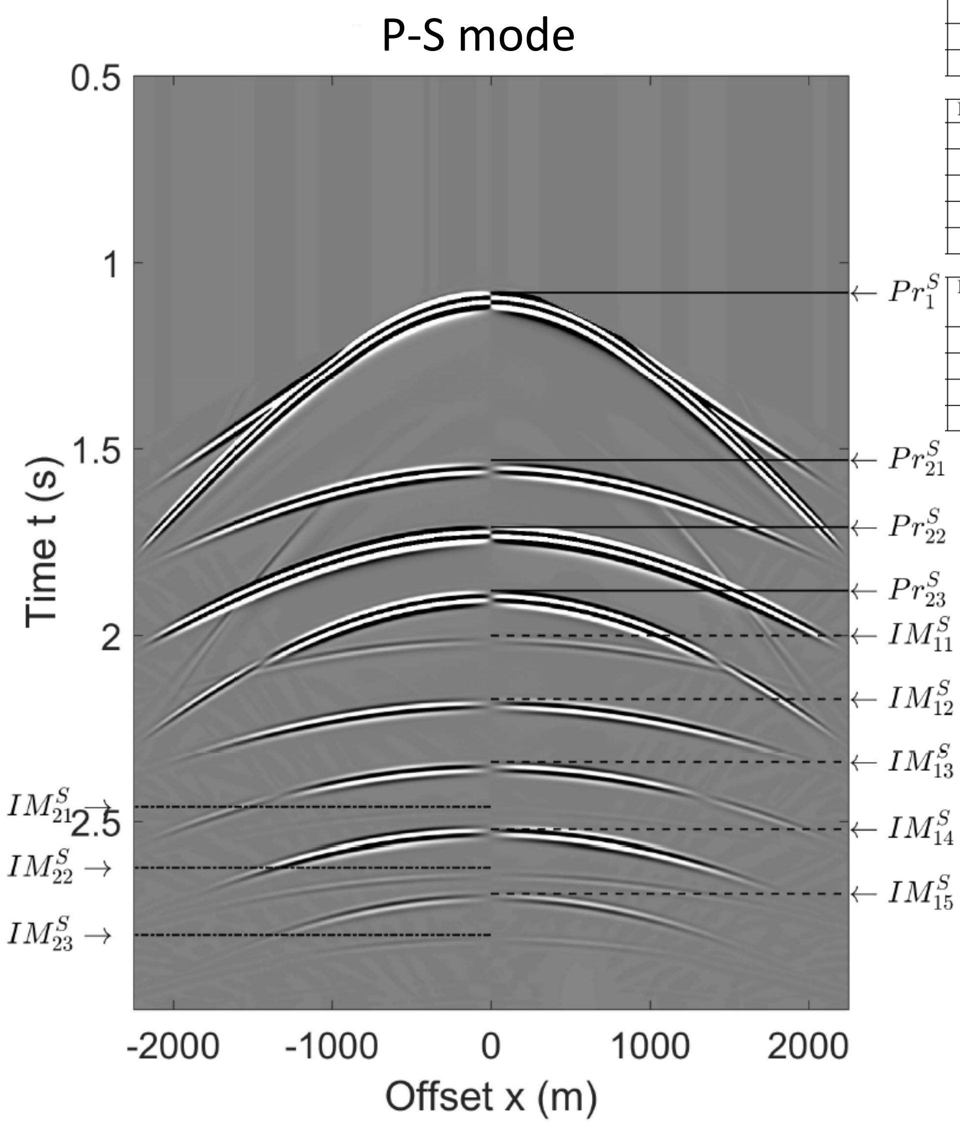
Label-P	Primaries in P-mode
Pr_1^P	$\grave{P}\acute{P}$
Pr_{21}^P	$\grave{P}\grave{P}\acute{P}\acute{P}$
Pr_{22}^P	$\grave{P}\grave{P}\acute{S}\acute{P}$ & $\grave{P}\grave{S}\acute{P}\acute{P}$
Pr_{23}^P	$\grave{P}\grave{S}\acute{S}\acute{P}$

Label-P	1st-order IMs in P-mode
IM_{11}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{P}$
IM_{12}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{P}$
IM_{13}^P	$\grave{P}\grave{P}\acute{P}\grave{S}\acute{S}\acute{P}$
IM_{14}^P	$\grave{P}\grave{P}\acute{S}\grave{S}\acute{S}\acute{P}$
IM_{15}^P	$\grave{P}\grave{S}\acute{S}\grave{S}\acute{S}\acute{P}$

Label-P	2nd-order IMs in P-mode
IM_{21}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{P}$
IM_{22}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{P}$
IM_{23}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{S}\acute{S}\acute{P}$
IM_{24}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{S}\grave{S}\acute{S}\acute{P}$
IM_{25}^P	$\grave{P}\grave{P}\acute{P}\grave{S}\acute{S}\grave{S}\acute{S}\acute{P}$

Label-P	3rd-order IMs in P-mode
IM_{31}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{P}$
IM_{32}^P	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{P}$



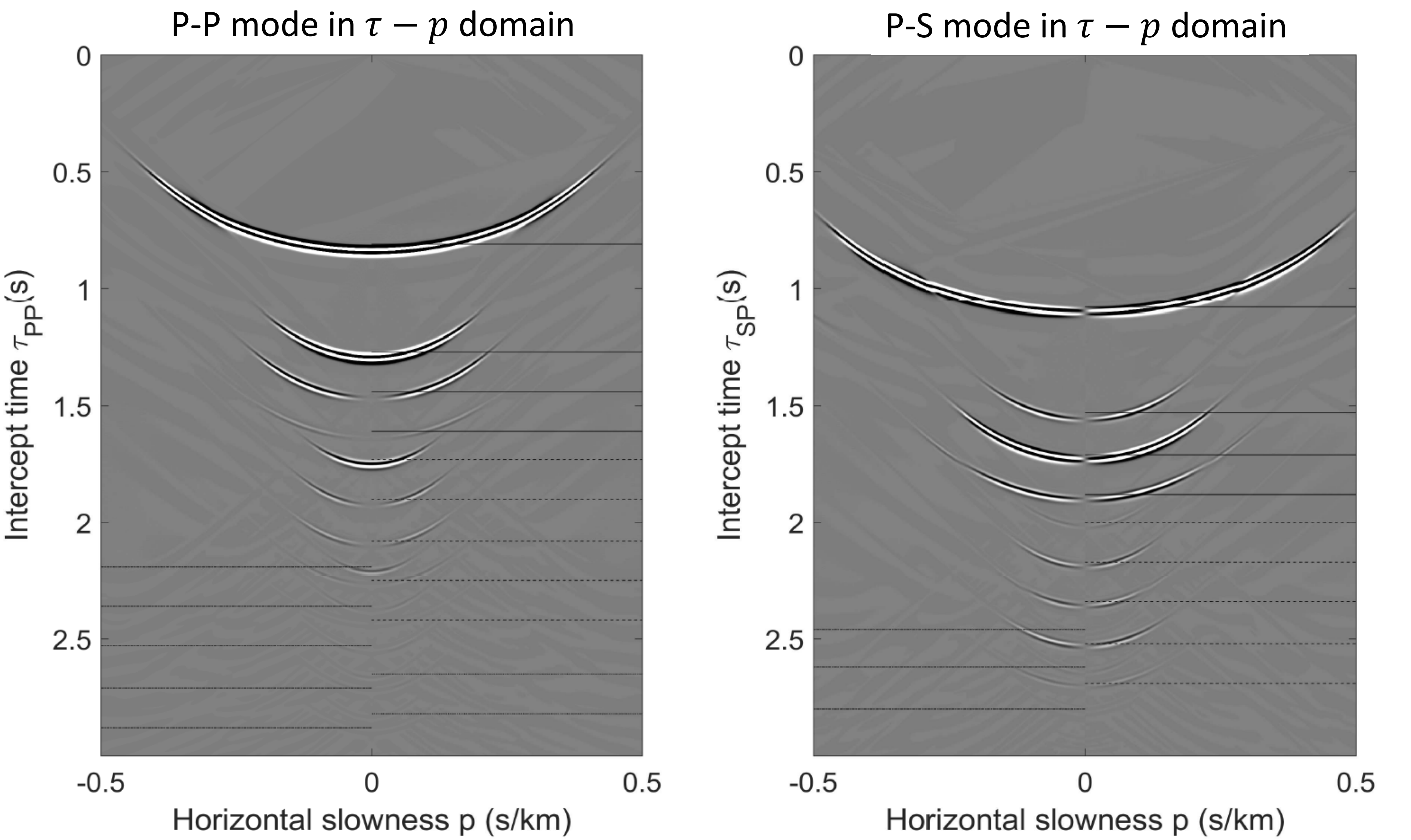


Label-S	Primaries S-mode
Pr_1^S	$\grave{P}\acute{S}$
Pr_{21}^S	$\grave{P}\grave{P}\acute{P}\acute{S}$
Pr_{22}^S	$\grave{P}\grave{P}\acute{S}\acute{S}$ & $\grave{P}\grave{S}\acute{P}\acute{S}$
Pr_{23}^S	$\grave{P}\grave{S}\acute{S}\acute{S}$

Label-S	1st-order IMs in S-mode
IM_{11}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{S}$
IM_{12}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{S}$
IM_{13}^S	$\grave{P}\grave{P}\acute{P}\grave{S}\acute{S}\acute{S}$
IM_{14}^S	$\grave{P}\grave{P}\acute{S}\grave{S}\acute{S}\acute{S}$
IM_{15}^S	$\grave{P}\grave{S}\acute{S}\grave{S}\acute{S}\acute{S}$

Label-S	2nd-order IMs in S-mode
IM_{21}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{P}\acute{S}$
IM_{22}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{P}\acute{S}\acute{S}$
IM_{23}^S	$\grave{P}\grave{P}\acute{P}\grave{P}\acute{P}\grave{S}\acute{S}\acute{S}$

Synthetic -- linear radon

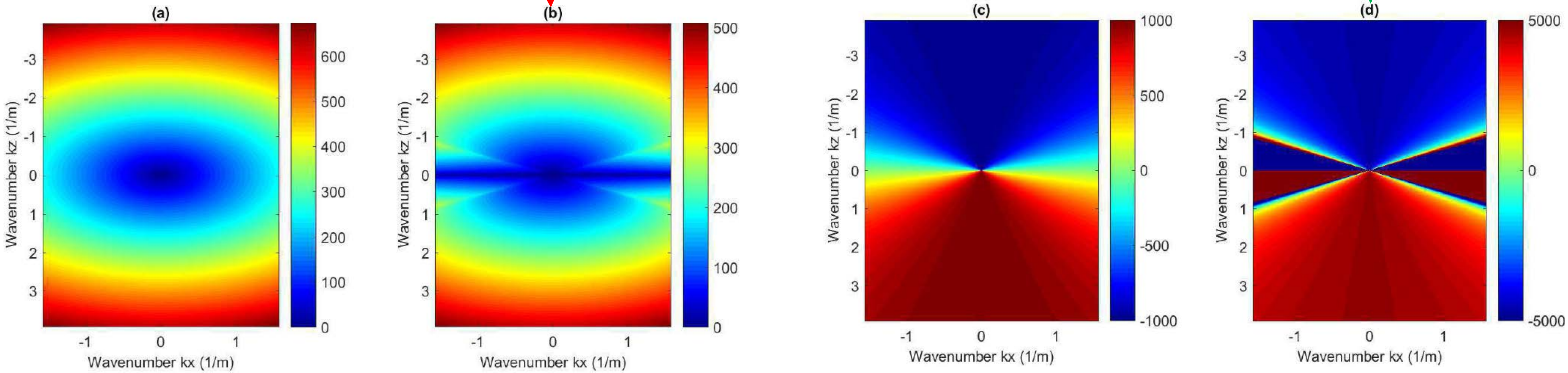


Synthetic -- elastic stolt-migration

Elastic stolt-migration (Etgen, 1988):

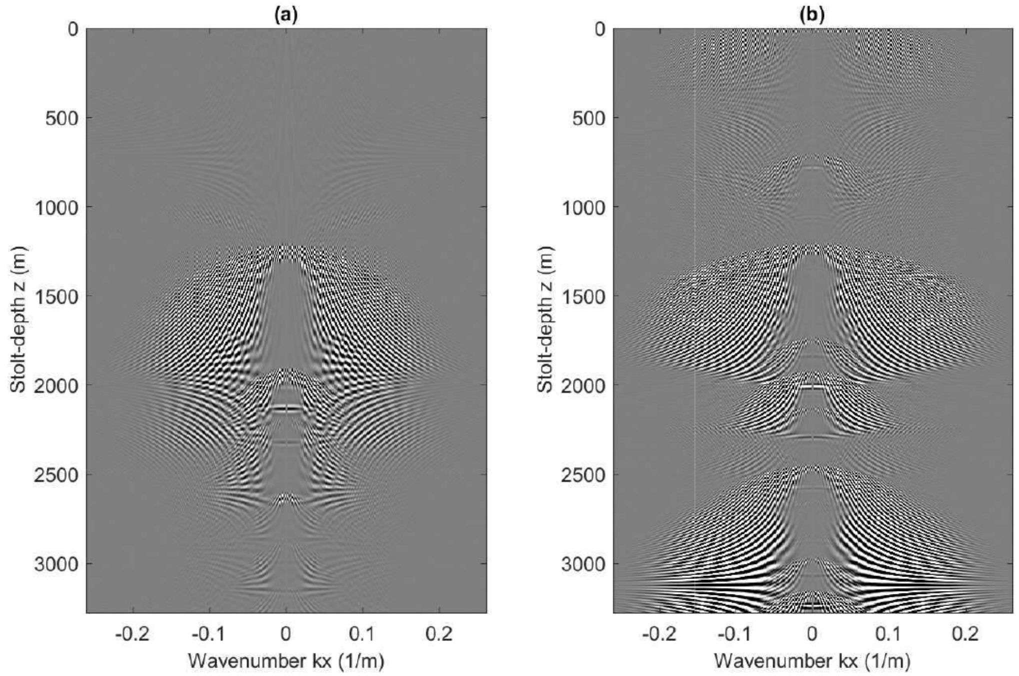
$$R_{PP}(k_m, z) = \int \mathrm{d}k_h \int \mathrm{d}k_{zP} \left| \frac{\mathrm{d}\omega}{\mathrm{d}k_{zP}} \right| P(k_m, k_h, w(k_{zP})) e^{ik_{zP}z}$$

$$R_{PSV}(k_m, z) = \int \mathrm{d}k_h \int \mathrm{d}k_{zSV} \left| \frac{\mathrm{d}\omega}{\mathrm{d}k_{zSV}} \right| P(k_m, k_h, w(k_{zSV})) e^{ik_{zSV}z}$$



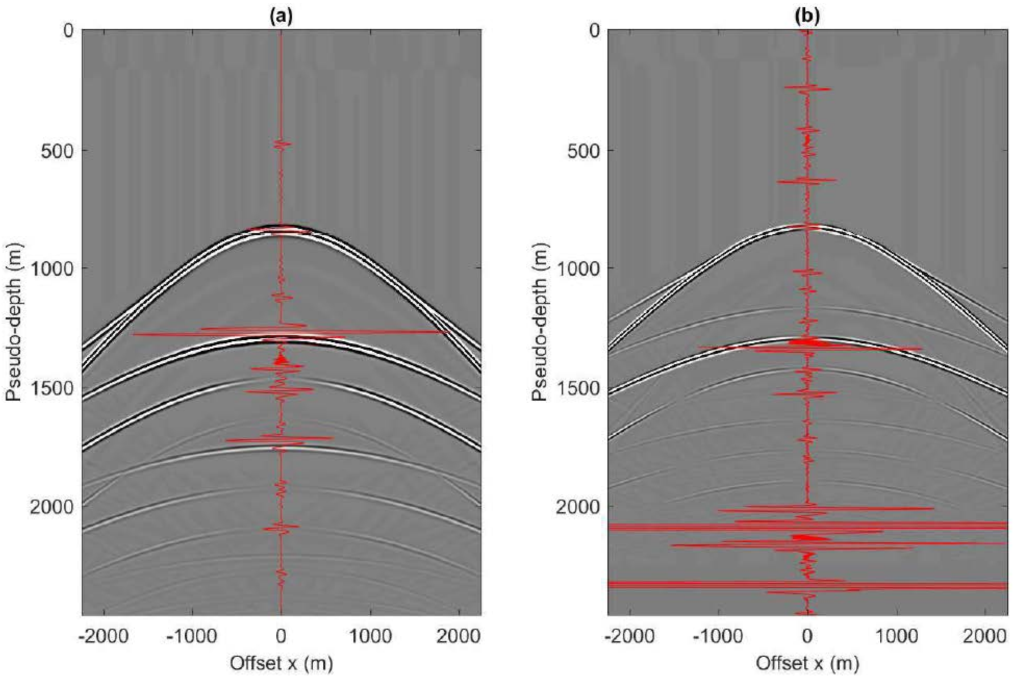
Synthetic -- elastic stolt-migration

Elastic stolt-migration with wavenumber:



$R_{PP}(k_x, z)$

$R_{PS}(k_x, z)$



$R_{PP}(z)$

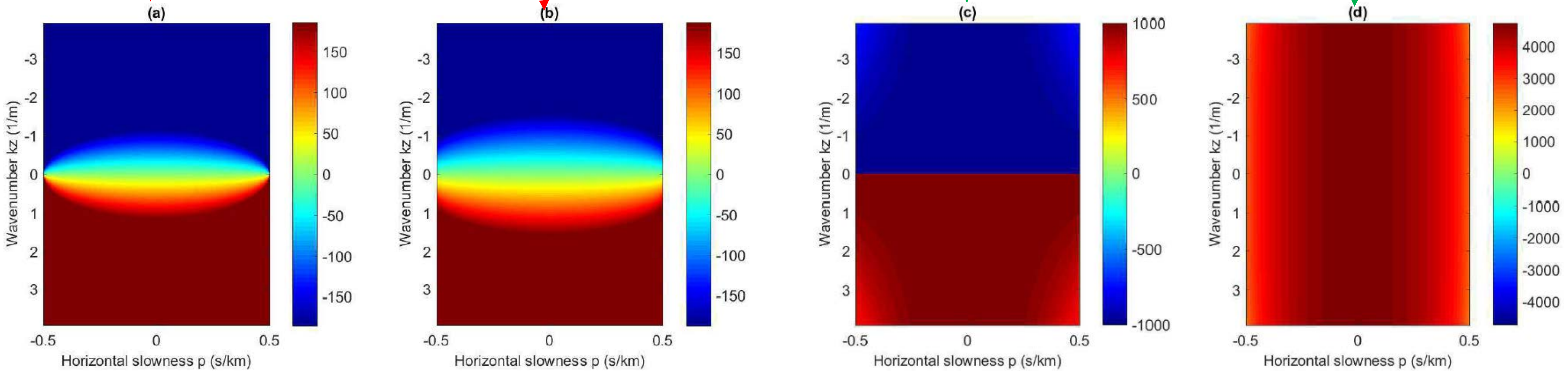
$R_{PS}(z)$

Synthetic -- elastic stolt-migration

Elastic stolt-migration with horizontal slowness:

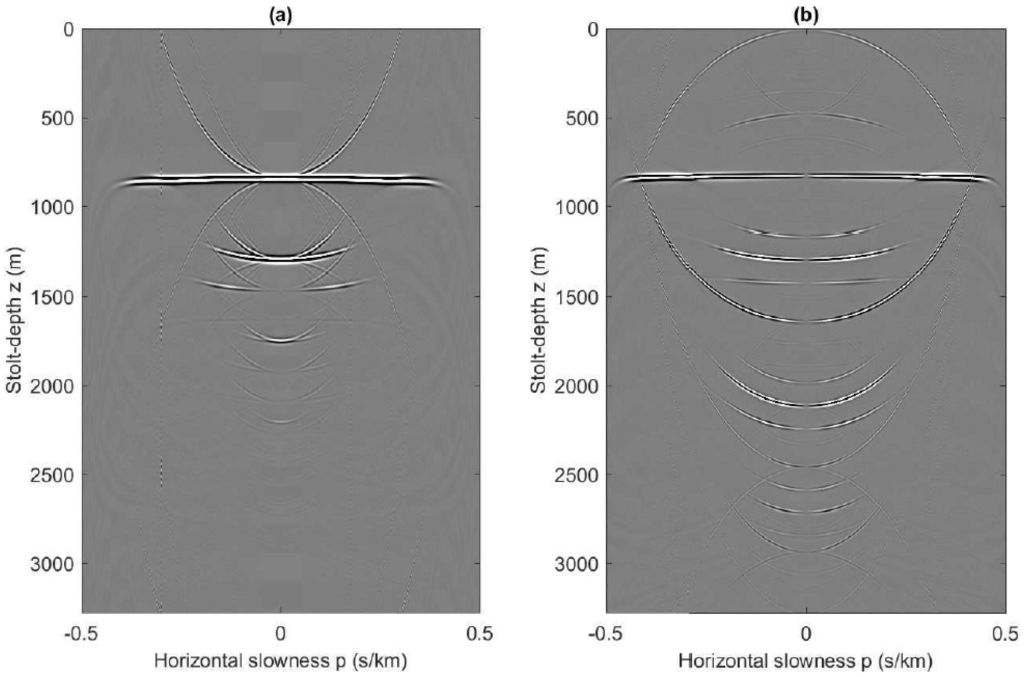
$$R_{PP}(p_m, z) = \int \mathrm{d}p_h \int \mathrm{d}k_{zP} \left|\frac{\mathrm{d}\omega}{\mathrm{d}k_{zP}}\right| P(p_m, p_h, w(k_{zP})) e^{ik_{zP} z}$$

$$R_{PSV}(p_m, z) = \int \mathrm{d}p_h \int \mathrm{d}k_{zSV} \left|\frac{\mathrm{d}\omega}{\mathrm{d}k_{zSV}}\right| P(p_m, p_h, w(k_{zSV})) e^{ik_{zSV} z}$$

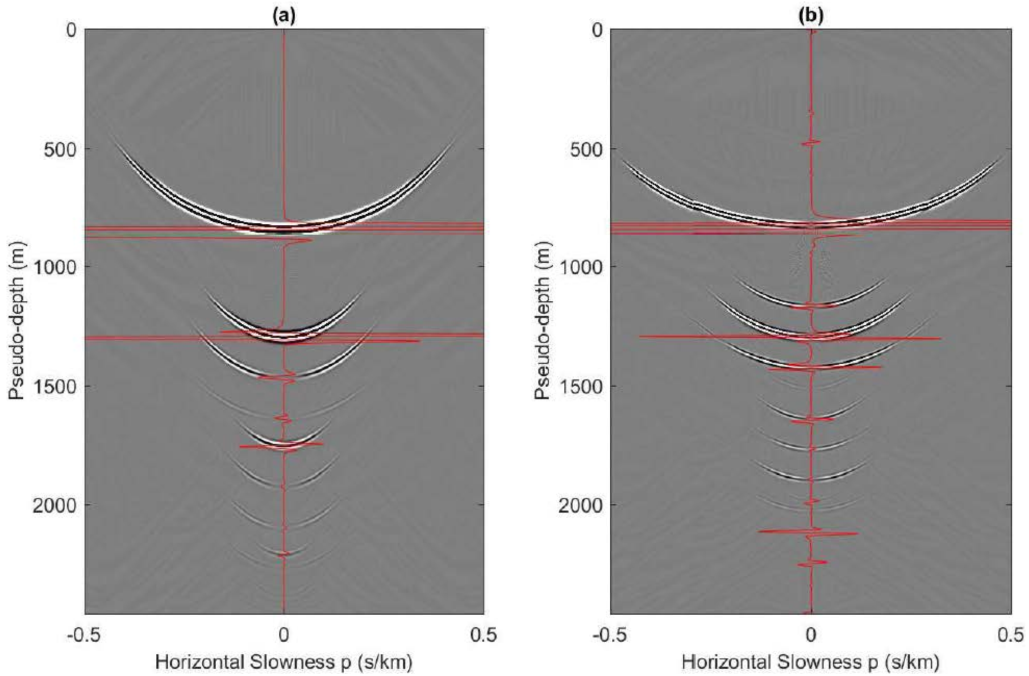


Synthetic -- elastic stolt-migration

Elastic stolt-migration with horizontal slowness:



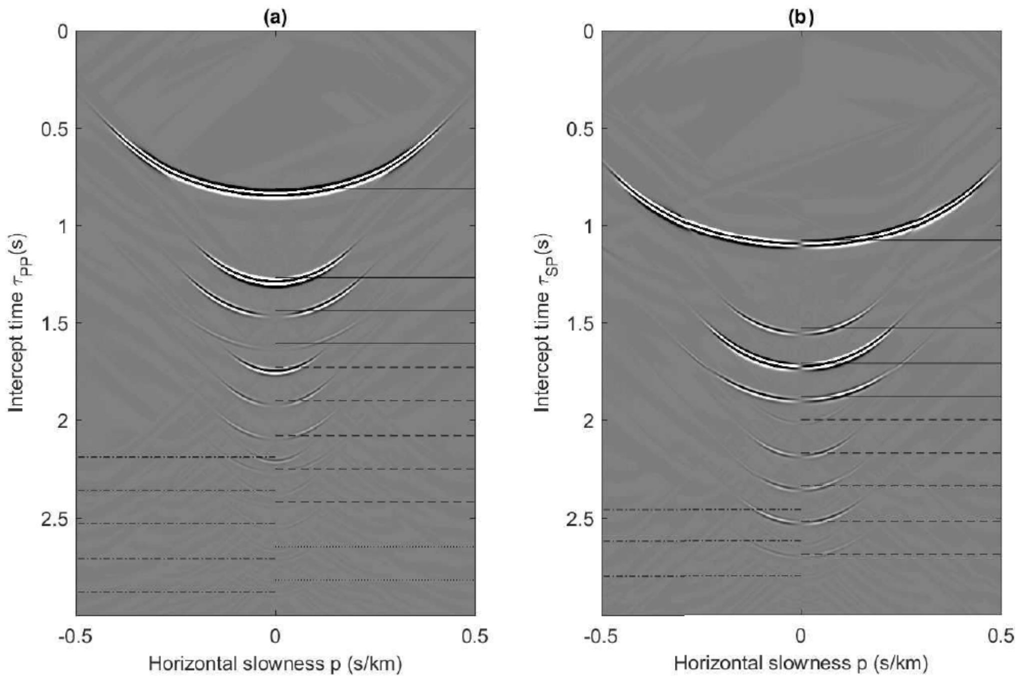
$R_{PP}(p,z)$ $R_{PS}(p,z)$

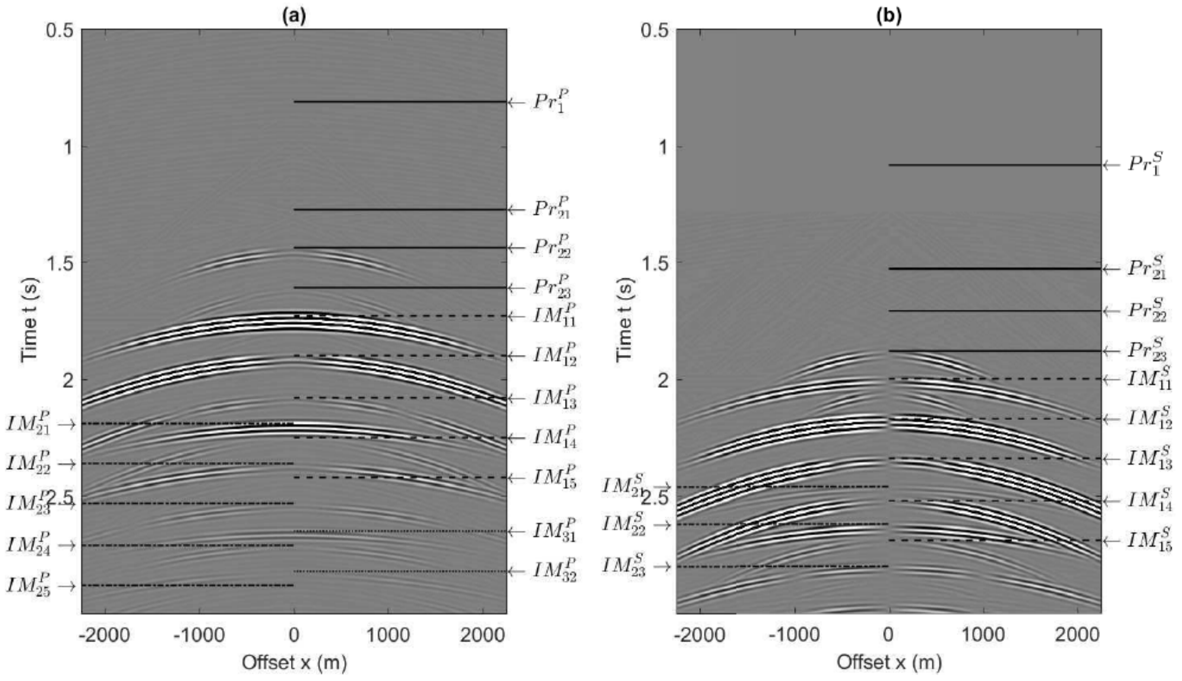


$R_{PP}(z)$ $R_{PS}(z)$

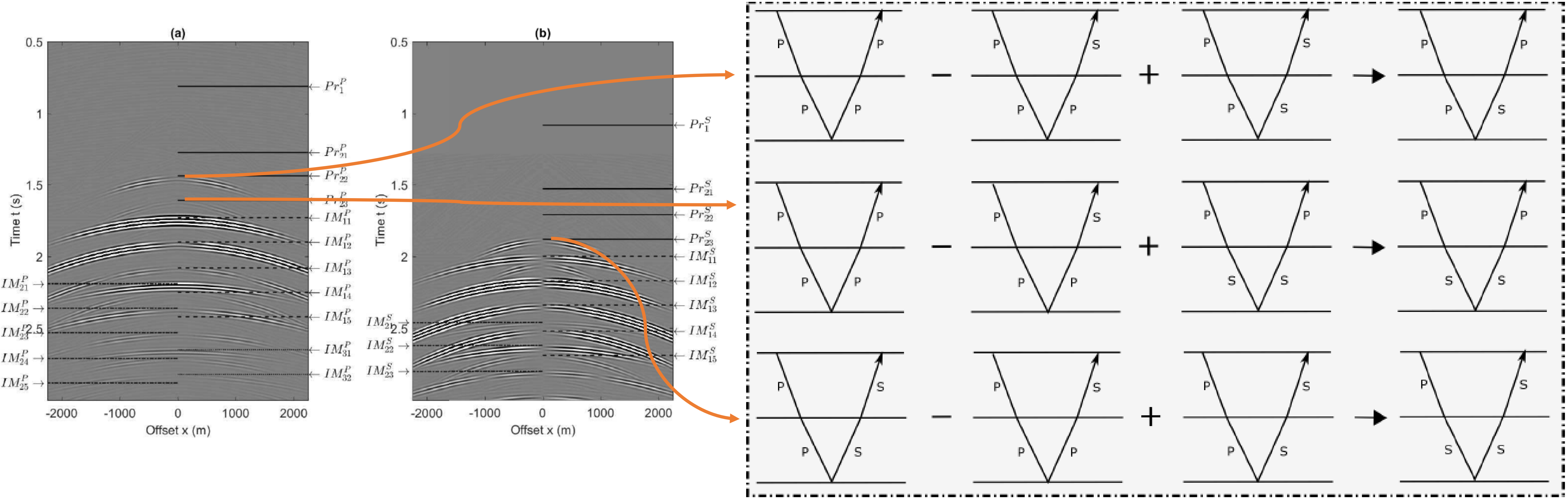
Synthetic -- elastic stolt-migration/time-stretching

Prediction with time-stretching condition ($\epsilon = 96ms$):

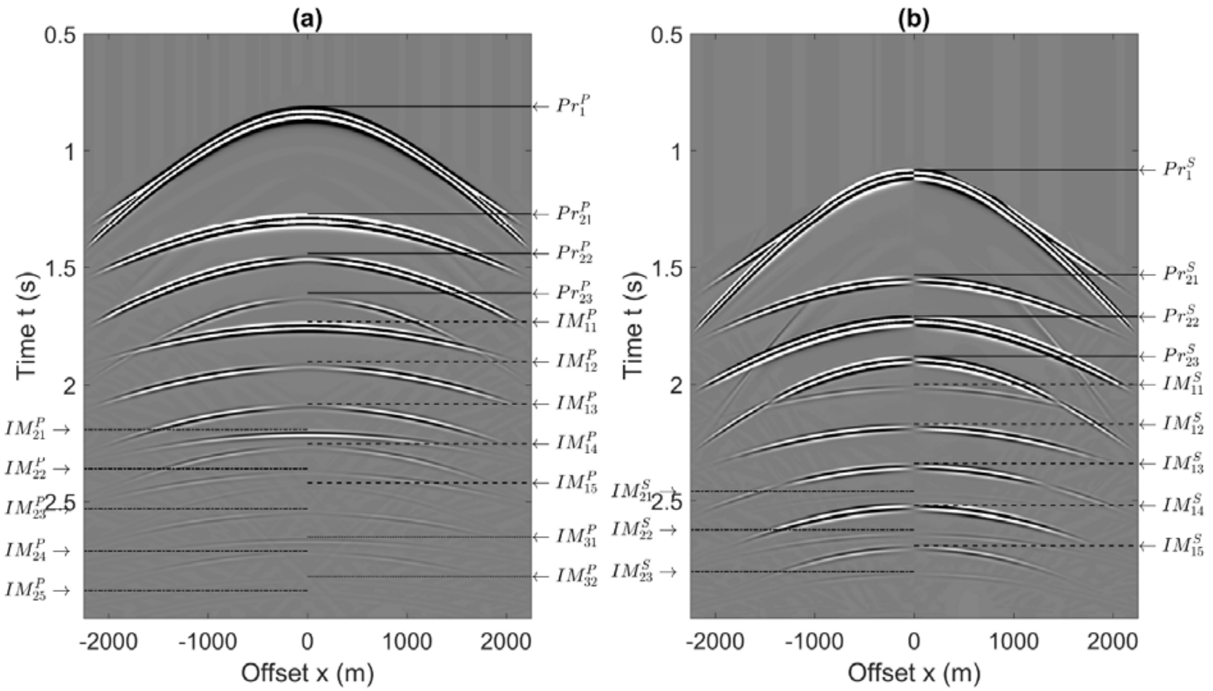


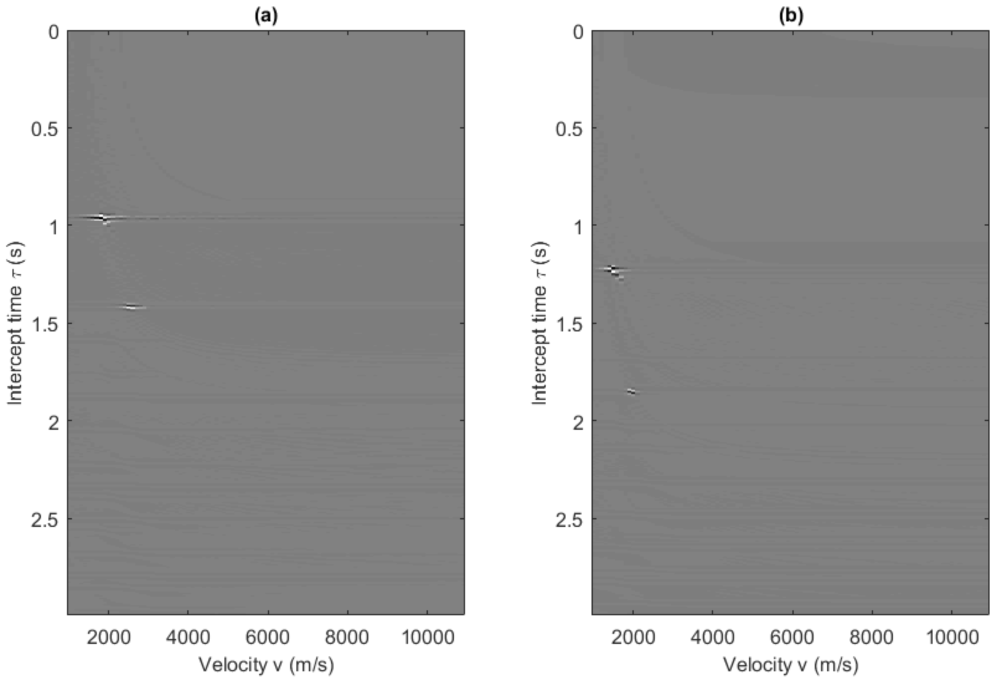


Synthetic -- elastic stolt-migration/time-stretching



Synthetic -- best-fitting approach

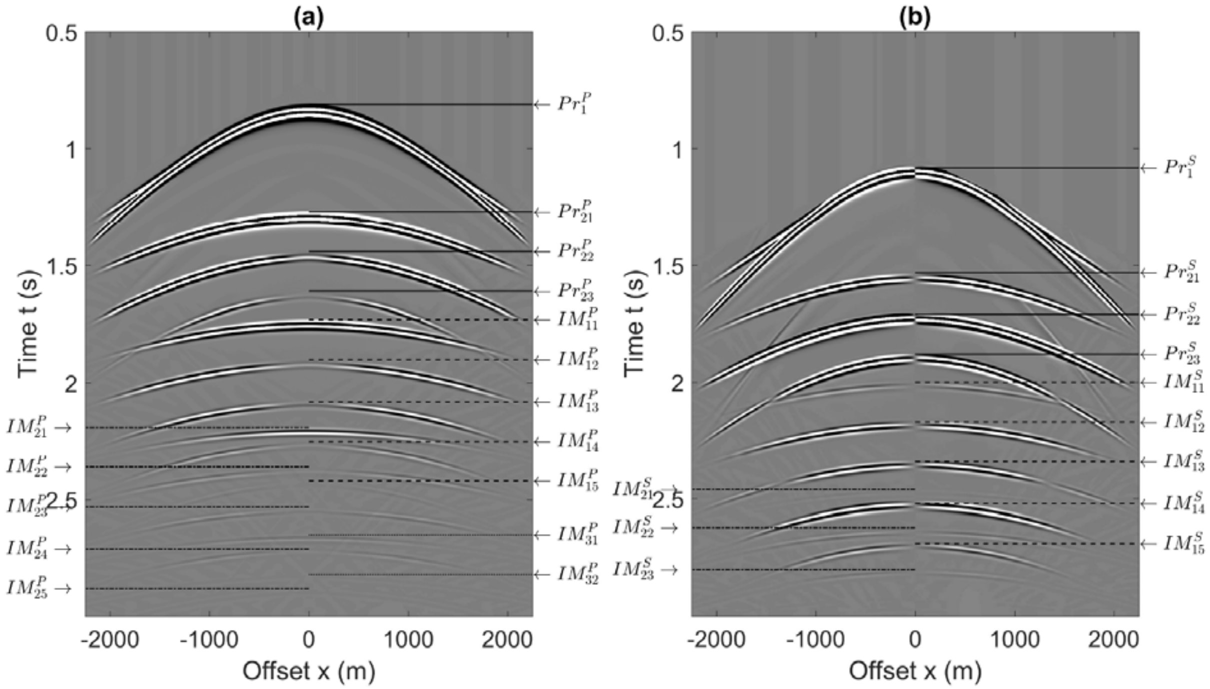


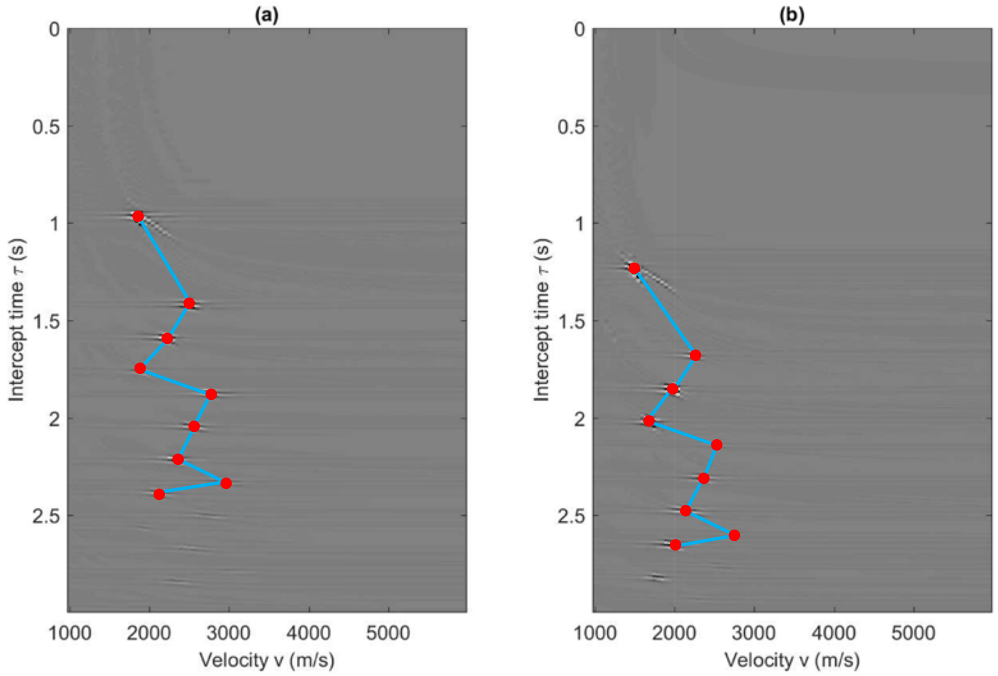


$$\Delta v_{iq+1} = \Delta v_{iq}(1+\epsilon) + \Delta v_{min} \;\; if\; {}^{n}/_{2} < iq < n$$

$$\Delta v_{iq-1} = \Delta v_{iq}(1+\epsilon) - \Delta v_{min} \;\; if\; 1 < iq < {}^{n}/_{2}$$

Synthetic -- best-fitting approach



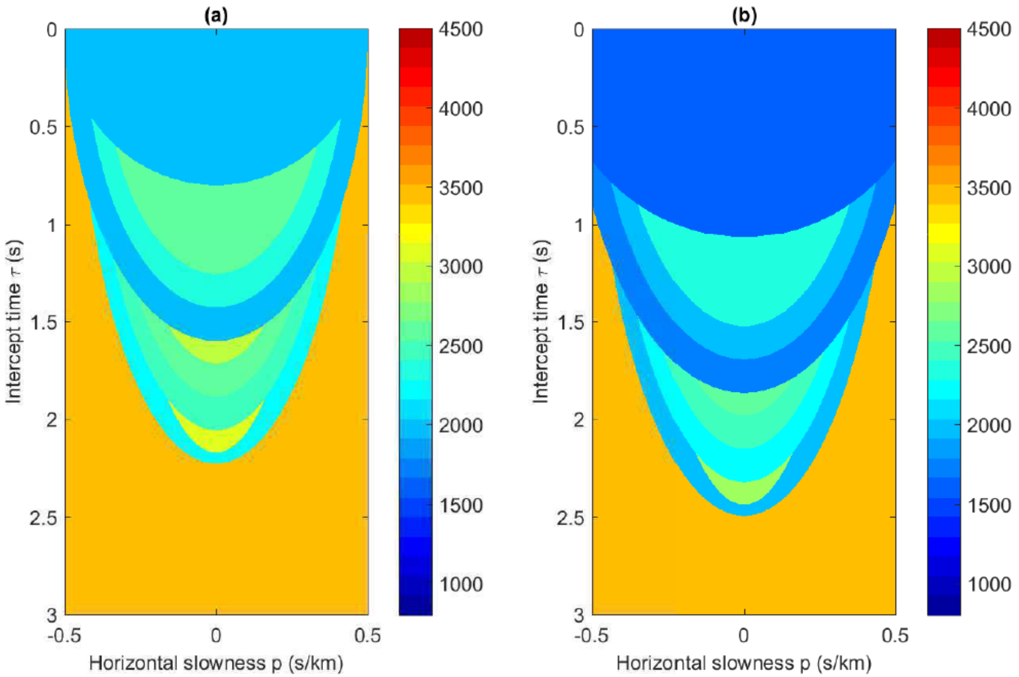


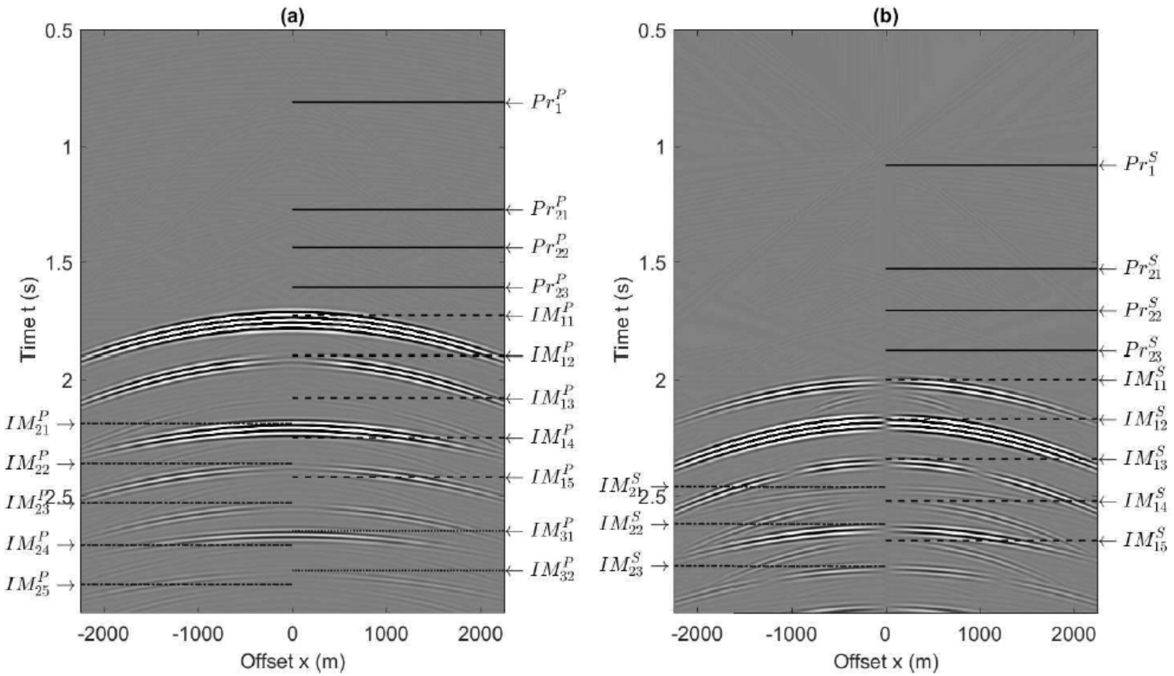
Data = Data * t^{tpow}, Here $tpow$ =4.

$$\Delta v_{iq+1} = \Delta v_{iq} + \Delta v_{min} \;\; if \; {}^{n}/_{2} < iq < n$$
$$\Delta v_{iq-1} = \Delta v_{iq} - \Delta v_{min} \;\; if \; 1 < iq < {}^{n}/_{2}$$

Synthetic – best-fitting approach

Prediction with best-fitting condition ($\epsilon = 200ms$):





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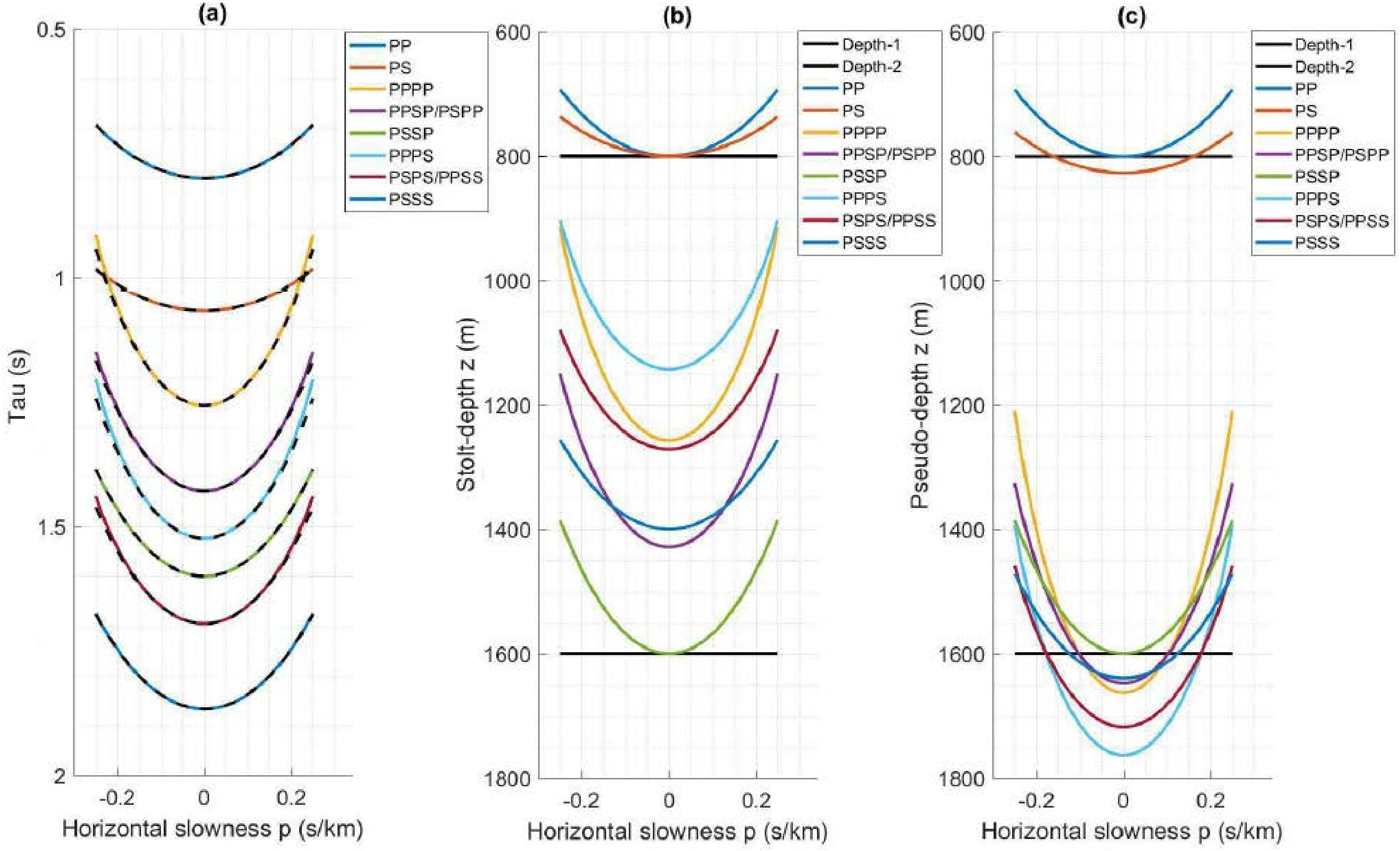
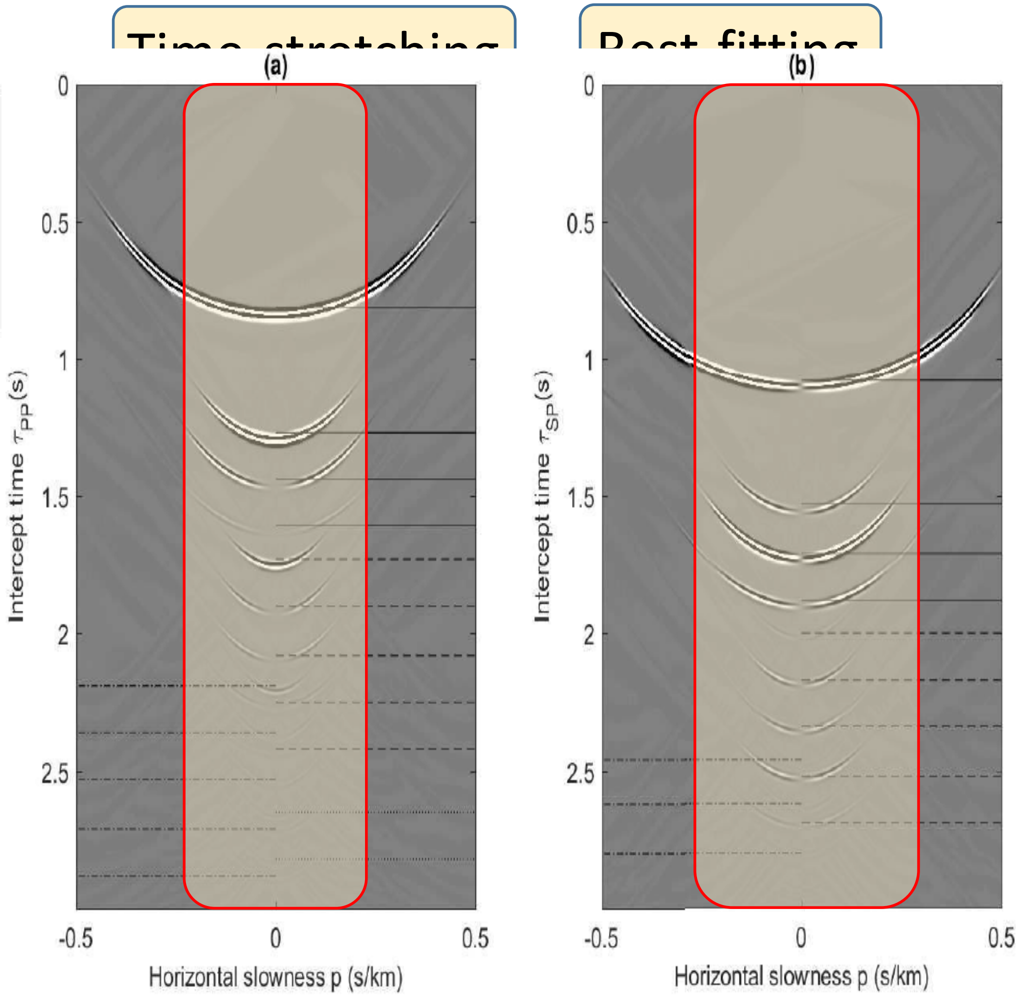
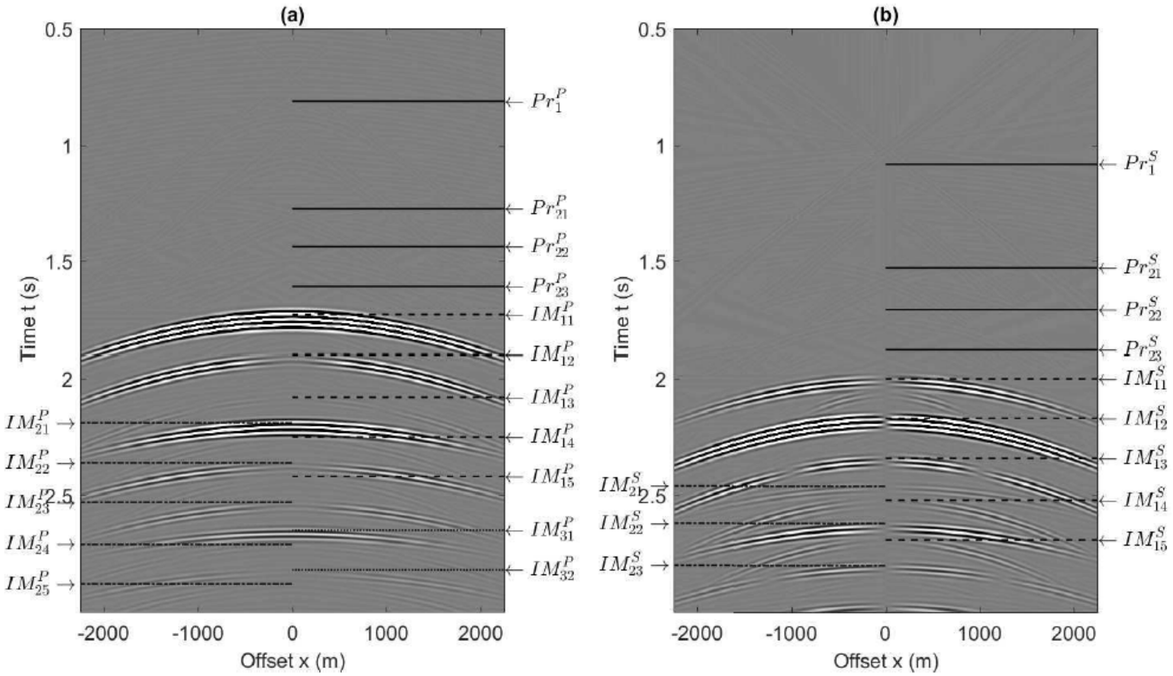
Approximate solutions for input preparation



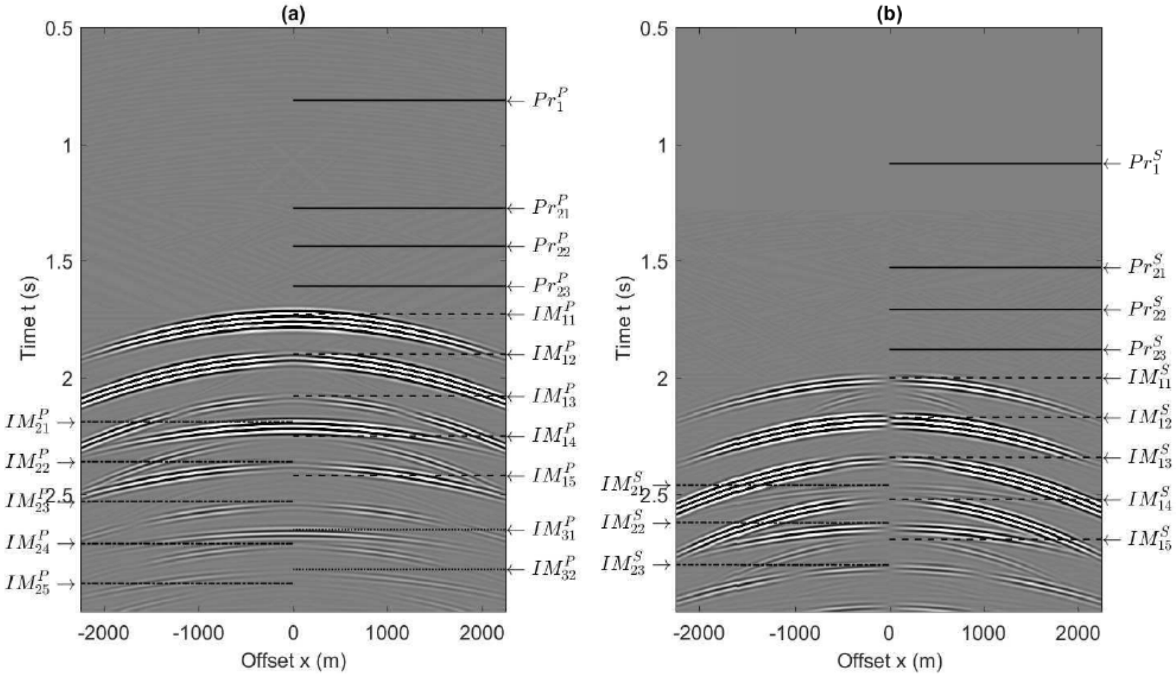


Synthetic – best-fitting & time-stretching

Best-fitting ($\epsilon = 200ms$)



Cross-validate ($\epsilon = 96ms$)



Conclusions

- We disclosed and discussed the issues of the multicomponent internal multiple prediction using inverse scattering series.
- Elastic stolt-migration with two background velocities is not suitable for input preparation.
- Time-stretching method retain the monotonic relationship of vertical traveltime and pseudo-depth.
- Best-fitting velocity by high-resolution hyperbolic radon provided an approximate solution which requires a large search parameter.
- Based on the opposite sorting order, the cross-validate condition obtained by time-stretching and best-fitting velocity can produce an exact prediction while allowing a regular constant epsilon.

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- NSERC (CRDPJ 46117913)
- All CREWES staff and students

Thank You!

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