

# Seismic responses in fractured reservoir rocks with induced attenuation

---Estimation of fracture weaknesses and integrated attenuation factors

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# Outline

- Introduction
- Stiffness parameters of fractured and attenuative rocks
- Linearized reflection coefficient and seismic amplitude difference inversion
- Examples
- Discussions and conclusions

# Introduction

- Expression of the normal fracture weakness for fluid saturated fractured rocks (Bakulin et al., 2000)

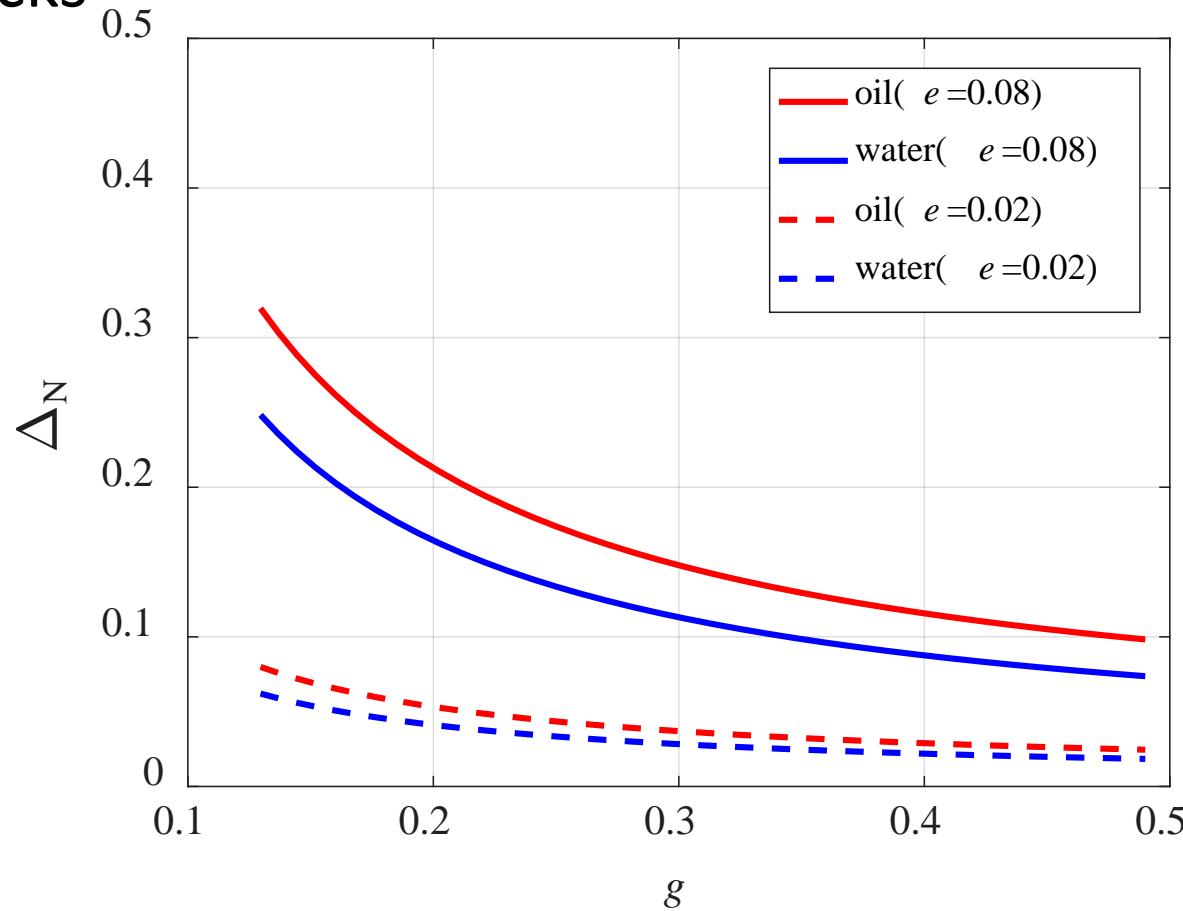
$$\Delta_N = \frac{4e}{3g(1-g) \left[ 1 + \frac{1}{\pi(1-g)} \left( \frac{k_f + 4/3\mu_f}{\mu\alpha} \right) \right]}$$

- where  $e$  is fracture density,  $k_f$  and  $\mu_f$  are effective moduli of mixture of water and oil,  $\alpha$  is fracture aspect ratio,  $g$  and  $\mu$  are elastic parameters of isotropic and elastic background.

	$K_f$ (GPa)	$\mu_f$ (GPa)
Water	2.15	0
Oil	1.5	0

# Introduction

- Variations of the normal fracture weakness for oil- and water saturated rocks



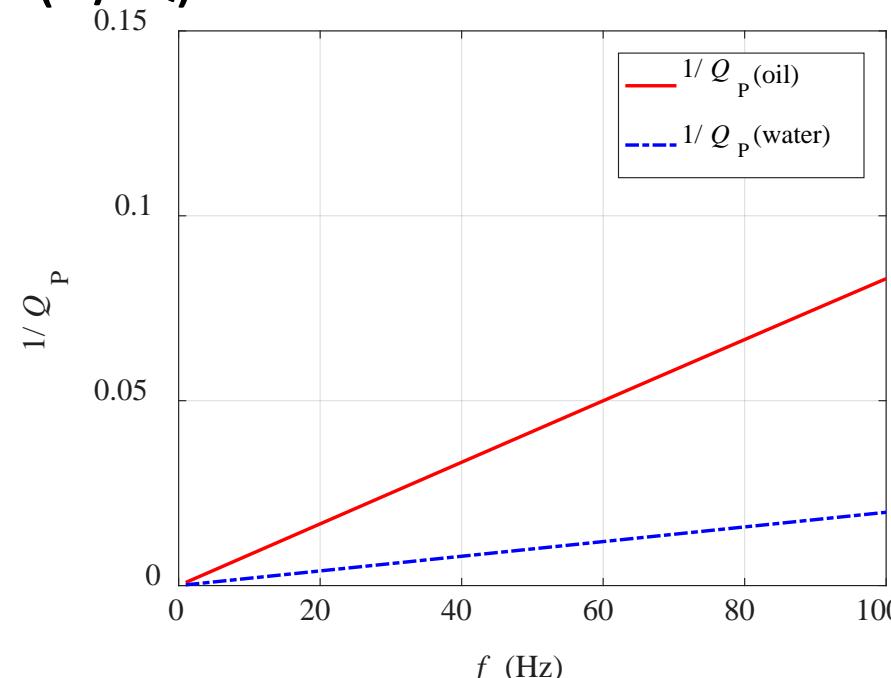
The normal  
fracture weakness  
is not sensitive to  
distinguish oil and  
water reservoirs.

# Introduction

- Incorporating fluid viscosity influences

	$\eta_f$ (cp)
Water	1
Oil	8

- Attenuation factor ( $1/Q$ ) variations



Results obtained using the extended squirt model (Dvorkin et al, 1996).

# Stiffness parameters of fractured and attenuative rocks

- Elastic and anisotropic linear-slip theory for HTI media (Schoenberg and Sayers, 1995)

$$\mathbf{C} = \begin{bmatrix} M(1-\Delta_N) & \lambda(1-\Delta_N) & \lambda(1-\Delta_N) & 0 & 0 & 0 \\ \lambda(1-\Delta_N) & M(1-\chi^2\Delta_N) & \lambda(1-\chi\Delta_N) & 0 & 0 & 0 \\ \lambda(1-\Delta_N) & \lambda(1-\chi\Delta_N) & M(1-\chi^2\Delta_N) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu(1-\Delta_T) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu(1-\Delta_T) \end{bmatrix}$$

- Introducing the complex normal and tangential fracture weaknesses (Chichinina et al, 2006)

$$\Delta_N \rightarrow \overline{\Delta}_N = \Delta_N - i \frac{1}{Q_N} \Delta_N,$$
$$\Delta_T \rightarrow \overline{\Delta}_T = \Delta_T - i \frac{1}{Q_T} \Delta_T,$$

Induced  
attenuation  
factors

# Stiffness parameters of fractured and attenuative rocks

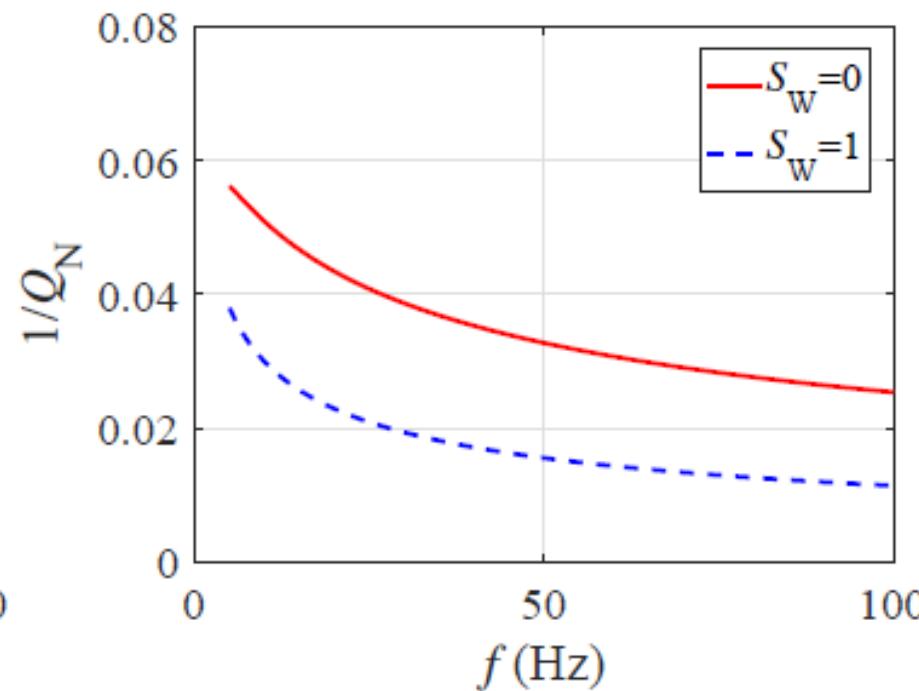
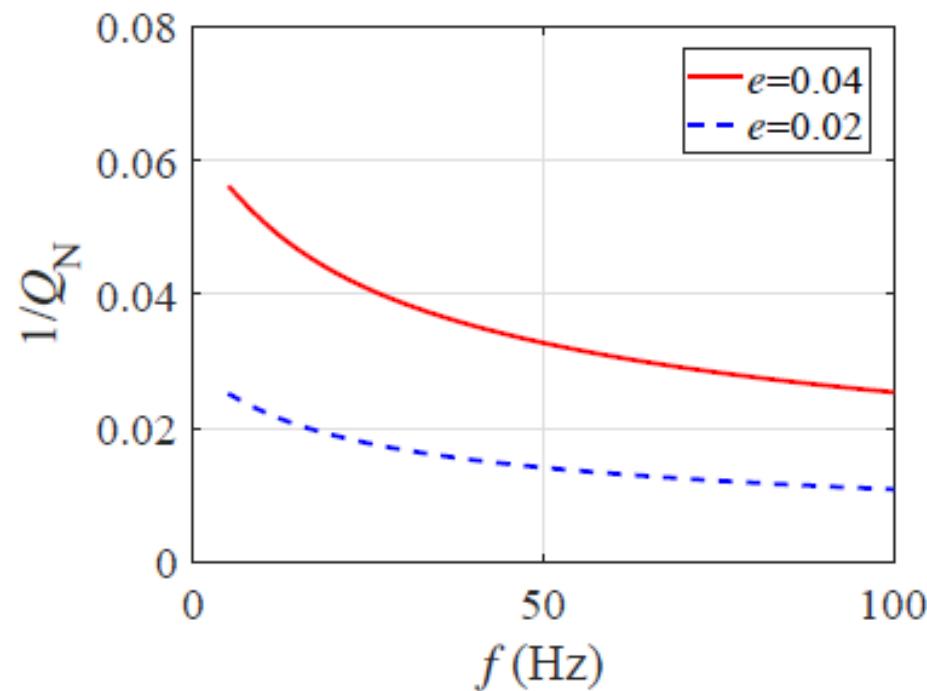
- Relating complex fracture weaknesses to fracture properties (fracture density) and fluid parameters (fluid moduli and viscosity)

$$\overline{\Delta_N} = \frac{M}{\mu} \overline{U}_{33} e, \quad \overline{\Delta_T} = \overline{U}_{11} e$$

- where  $\overline{U}_{11} = \frac{16}{3} \frac{M}{3\lambda + 4\mu} \frac{1}{1 + \overline{\Gamma}(\omega)}$  and  $\overline{\Gamma}(\omega) = \frac{4a}{\pi c} \left( \frac{i\omega\eta_f}{\mu} \right) \left( \frac{M}{3\lambda + 4\mu} \right)$
- $\overline{U}_{33} = \frac{4}{3} \frac{M}{\lambda + \mu} \frac{1}{1 + \overline{\Psi}(\omega)}$   $\overline{\Psi}(\omega) = \frac{a}{\pi c} \frac{K_f}{\mu} \frac{M}{\lambda + \mu} \left[ 1 - \frac{3(1-i)}{2c} \sqrt{\frac{\phi K_f P_m}{2\omega\eta_f}} \right]^{-1}$

- In the seismic frequency range (1-100Hz),  $\overline{\Gamma}(\omega)$  is approximately equal to zero (Pointer et al., 2000), which indicates the tangential fracture weakness is real and  $1/Q_T$  is zero.

# Stiffness parameters of fractured and attenuative rocks



Induced attenuation factor increases with fracture density.

Oil saturated rocks exhibit higher induced attenuation than water saturated rocks.

# Stiffness parameters of fractured and attenuative rocks

- Combining the intrinsic and induced attenuations

$$\bar{\mathbf{C}} = \begin{bmatrix} \bar{M}(1 - \bar{\Delta}_N) & \bar{\lambda}(1 - \bar{\Delta}_N) & \bar{\lambda}(1 - \bar{\Delta}_N) & 0 & 0 & 0 \\ \bar{\lambda}(1 - \bar{\Delta}_N) & \bar{M}(1 - \bar{\chi}^2 \bar{\Delta}_N) & \bar{\lambda}(1 - \bar{\chi} \bar{\Delta}_N) & 0 & 0 & 0 \\ \bar{\lambda}(1 - \bar{\Delta}_N) & \bar{\lambda}(1 - \bar{\chi} \bar{\Delta}_N) & \bar{M}(1 - \bar{\chi}^2 \bar{\Delta}_N) & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{\mu} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{\mu}(1 - \bar{\Delta}_T) & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{\mu}(1 - \bar{\Delta}_T) \end{bmatrix}$$

- where

$$\bar{M} = \rho \left[ \alpha_E \left( 1 + \frac{i}{2Q_P} \right) \right]^2 \approx M \left( 1 + \frac{i}{Q_P} \right), \quad \bar{\mu} = \rho \left[ \beta_E \left( 1 + \frac{i}{2Q_S} \right) \right]^2 \approx \mu \left( 1 + \frac{i}{Q_S} \right)$$

the intrinsic  
attenuation

# Stiffness parameters of fractured and attenuative rocks

- Real parts of simplified stiffness parameters involving influences of integrated attenuation factors

$$C_{11} \approx M - \frac{M}{Q_{PN}} - M\Delta_N$$

$$C_{12} \approx \lambda - \lambda\Delta_N - \frac{M}{Q_{PN}} + \frac{2\mu}{Q_{SN}}$$

$$C_{23} \approx \lambda(1 - \Delta_N) + 2\lambda g \Delta_N - M \frac{1}{Q_{PN}} + 2Mg \frac{1}{Q_{PN}} + 2\mu \frac{1}{Q_{SN}} - 4\mu g \frac{1}{Q_{SN}}$$

$$C_{33} = M(1 - \Delta_N) + 4Mg \Delta_N - 4Mg^2 \Delta_N + 4Mg \frac{1}{Q_{PN}} - 4Mg^2 \frac{1}{Q_{PN}}$$

$$C_{44} = \mu$$

$$C_{55} = \mu - \mu\Delta_T$$

where

$$\frac{1}{Q_{PN}} = \frac{1}{Q_P} \frac{1}{Q_N}$$

$$\frac{1}{Q_{SN}} = \frac{1}{Q_S} \frac{1}{Q_N}$$

integrated  
attenuation factors

# Linearized reflection coefficient

- Linearized Rpp for weakly anisotropic media (Shaw and Sen, 2004)

$$R_{PP} = \frac{1}{4\rho \cos^2 \theta} S$$

- $S$  is the scattering function,  $\rho$  is density, and  $\theta$  is the angle of incidence.

$$S = \Delta\rho\xi + \Delta C_{IJ}\eta$$

- Linearized reflection coefficient

$$R_{PP}(\theta, \phi) = R_{PP}^{\text{iso-elastic}}(\theta) + R_{PP}^{\text{ani-visco}}(\theta, \phi)$$

$$R_{PP}^{\text{iso-elastic}}(\theta) = \frac{1}{4} \sec^2 \theta \frac{\Delta M}{M} - 2g \sin^2 \theta \frac{\Delta \mu}{\mu} + \frac{\cos 2\theta}{4 \cos^2 \theta} \frac{\Delta \rho}{\rho},$$

$$R_{PP}^{\text{ani-visco}}(\theta, \phi) = P_{Q_{PN}}(\theta, \phi) \Delta \left( \frac{1}{Q_{PN}} \right) + P_{Q_{SN}}(\theta, \phi) \Delta \left( \frac{1}{Q_{SN}} \right) + P_{\Delta_N}(\theta, \phi) \delta_{\Delta_N} + P_{\Delta_T}(\theta, \phi) \delta_{\Delta_T}.$$

# Seismic amplitude difference inversion

- Differences between reflection coefficients along different azimuthal angles

$$\Delta R_{PP1} = R_{PP}^{\text{ani-visco}}(\theta, \phi_2) - R_{PP}^{\text{ani-visco}}(\theta, \phi_1)$$

$$\Delta R_{PP2} = R_{PP}^{\text{ani-visco}}(\theta, \phi_3) - R_{PP}^{\text{ani-visco}}(\theta, \phi_1)$$

$$\Delta R_{PP3} = R_{PP}^{\text{ani-visco}}(\theta, \phi_4) - R_{PP}^{\text{ani-visco}}(\theta, \phi_1)$$

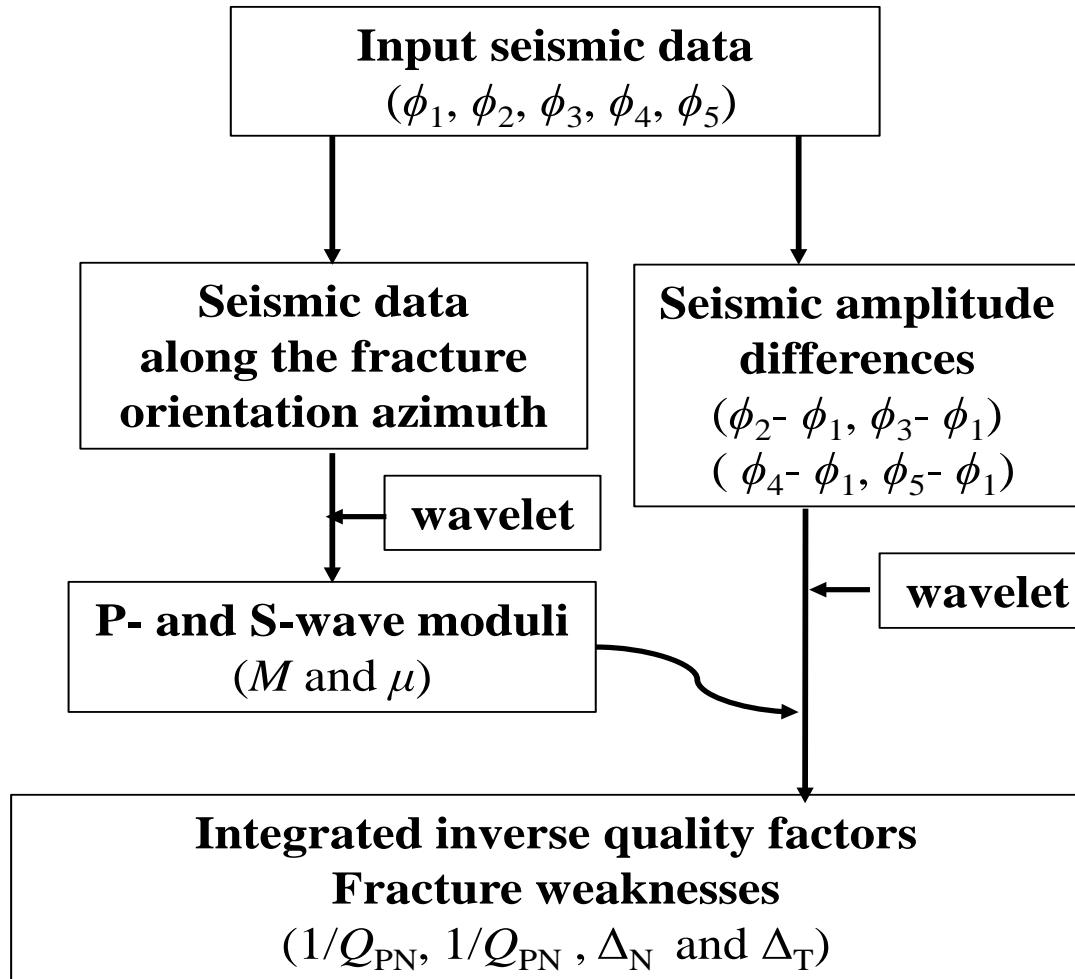
$$\Delta R_{PP4} = R_{PP}^{\text{ani-visco}}(\theta, \phi_5) - R_{PP}^{\text{ani-visco}}(\theta, \phi_1)$$

- Seismic differences

$$\Delta S_{PP} = W \Delta R_{PP}^{\text{ani-visco}}$$

- where  $W$  is seismic wavelet.

# Seismic amplitude difference inversion



Workflow of seismic inversion

# Seismic amplitude difference inversion

- Step1 : Utilizing seismic data along the fracture orientation azimuth to predict elastic parameters

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

- where

$$\mathbf{d} = \begin{bmatrix} \mathbf{s}(\theta_1) \\ \vdots \\ \mathbf{s}(\theta_m) \end{bmatrix}_{mn \times 1}, \mathbf{G} = \begin{bmatrix} \mathbf{WP}_M(\theta_1) & \mathbf{WP}_\mu(\theta_1) & \mathbf{WP}_\rho(\theta_1) \\ \vdots & \vdots & \vdots \\ \mathbf{WP}_M(\theta_m) & \mathbf{WP}_\mu(\theta_1) & \mathbf{WP}_\rho(\theta_1) \end{bmatrix}_{mn \times 3n}, \mathbf{m} = \begin{bmatrix} \mathbf{R}_M \\ \mathbf{R}_\mu \\ \mathbf{R}_\rho \end{bmatrix}_{3n \times 1}$$

- Solution: Least-squares

$$\mathbf{m} = \mathbf{m}_{\text{mod}} + (\mathbf{G}^T \mathbf{G} + \sigma^2)^{-1} \mathbf{G}^T (\mathbf{d} - \mathbf{G}\mathbf{m}_{\text{mod}})$$

# Seismic amplitude difference inversion

- Step2 : Using seismic differences between azimuthal data to estimate fracture weaknesses and attenuation factors

$$\mathbf{B} = \mathbf{AX}$$

- where

$$\mathbf{B} = \begin{bmatrix} \mathbf{b}(\phi_2) - \mathbf{b}(\phi_1) \\ \mathbf{b}(\phi_3) - \mathbf{b}(\phi_1) \\ \mathbf{b}(\phi_4) - \mathbf{b}(\phi_1) \\ \mathbf{b}(\phi_5) - \mathbf{b}(\phi_1) \end{bmatrix}_{4mn \times 1}, \mathbf{X} = \begin{bmatrix} \mathbf{R}_{\varrho_{PN}} \\ \mathbf{R}_{\varrho_{SN}} \\ \mathbf{R}_{\Delta_N} \\ \mathbf{R}_{\Delta_T} \end{bmatrix}_{4n \times 1}$$

- Solution: An iterative approach to obtain the unknown parameter vector based on Bayesian theorem

# Seismic amplitude difference inversion

- In Bayesian theorem, the posterior probability distribution function (PDF)

$$P(\mathbf{X} | \mathbf{B}) \propto P(\mathbf{B} | \mathbf{X}) P(\mathbf{X})$$

- where

Likelihood  
function

$$P(\mathbf{B} | \mathbf{X}) = \frac{1}{\sqrt{2\pi\sigma_{\text{noise}}^2}} \exp\left[ -\frac{-(\mathbf{B} - \mathbf{AX})^T (\mathbf{B} - \mathbf{AX})}{2\sigma_{\text{noise}}^2} \right]$$

Gaussian  
distribution

The prior  
PDF

$$P(\mathbf{X}) = \frac{1}{\pi\sigma_{\mathbf{X}}} \exp\left[ -\ln\left(1 + \frac{\mathbf{X}^2}{\sigma_{\mathbf{X}}^2}\right) \right]$$

Cauchy distribution

# Seismic amplitude difference inversion

- In Bayesian theorem, the posterior probability distribution function (PDF)

$$P(\mathbf{X} | \mathbf{B}) = \frac{1}{\sqrt{2\pi\sigma_{\text{noise}}^2}} \frac{1}{\pi^2 \sigma_{\mathbf{X}}^2} \exp[-J(\mathbf{X})]$$

$$J(\mathbf{X}) = \frac{(\mathbf{B} - \mathbf{AX})^T (\mathbf{B} - \mathbf{AX})}{2\sigma_{\text{noise}}^2} + \ln \left( 1 + \frac{\mathbf{X}^2}{\sigma_{\mathbf{X}}^2} \right)$$

- The objective function

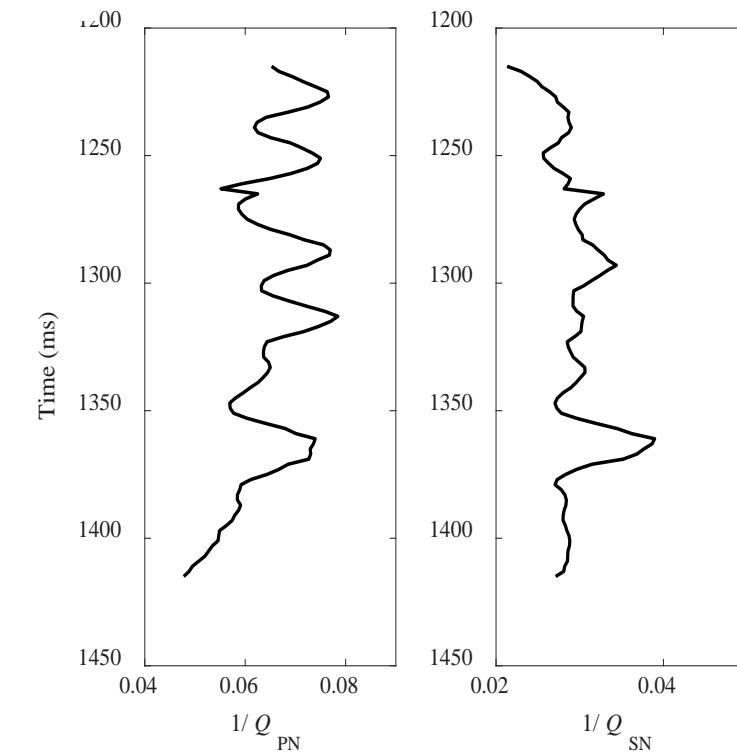
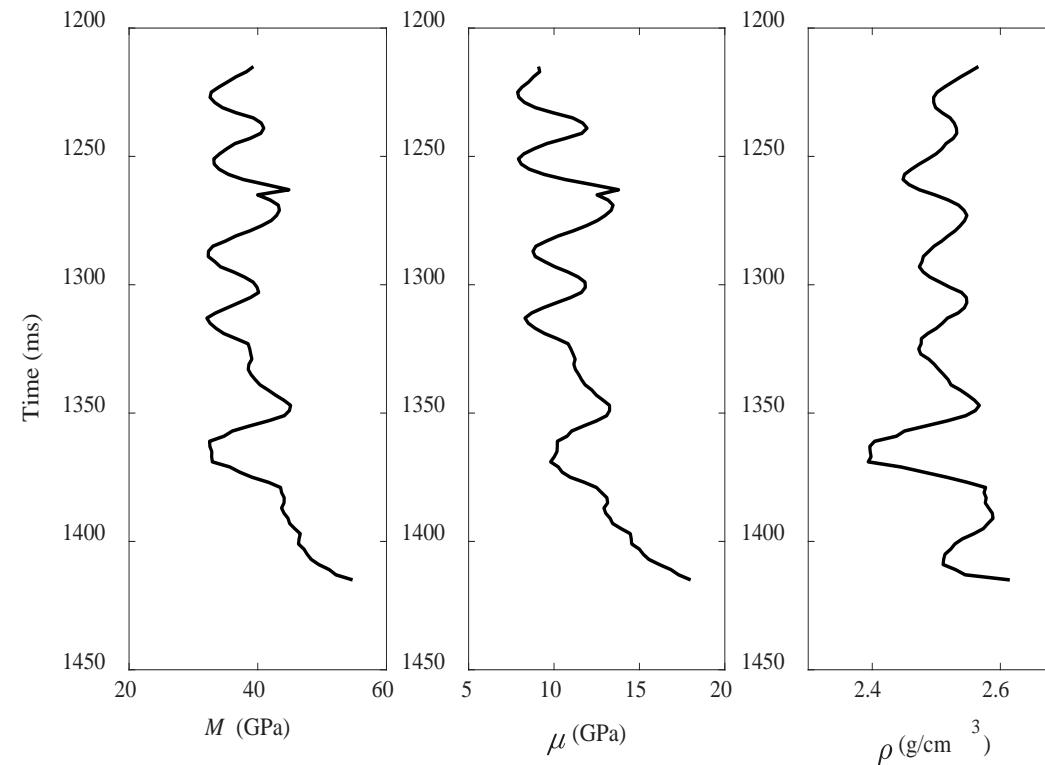
$$\frac{\partial [J(\mathbf{X})]}{\partial \mathbf{X}} = 0, \quad \left( \mathbf{A}^T \mathbf{A} + \frac{2\sigma_{\text{noise}}^2 / \sigma_{\mathbf{X}}^2}{1 + \mathbf{X}^2 / \sigma_{\mathbf{X}}^2} \right) \mathbf{X} = \mathbf{A}^T \mathbf{B}$$

- Solution of unknown parameter vector

$$\mathbf{X}_{i+1} = \mathbf{X}_i + \left( \mathbf{A}^T \mathbf{A} + \frac{2\sigma_{\text{noise}}^2 / \sigma_{\mathbf{X}}^2}{1 + \mathbf{X}_i^2 / \sigma_{\mathbf{X}_i}^2} \right)^{-1} \mathbf{A}^T (\mathbf{B} - \mathbf{AX}_i)$$

# Examples

- Synthetic tests

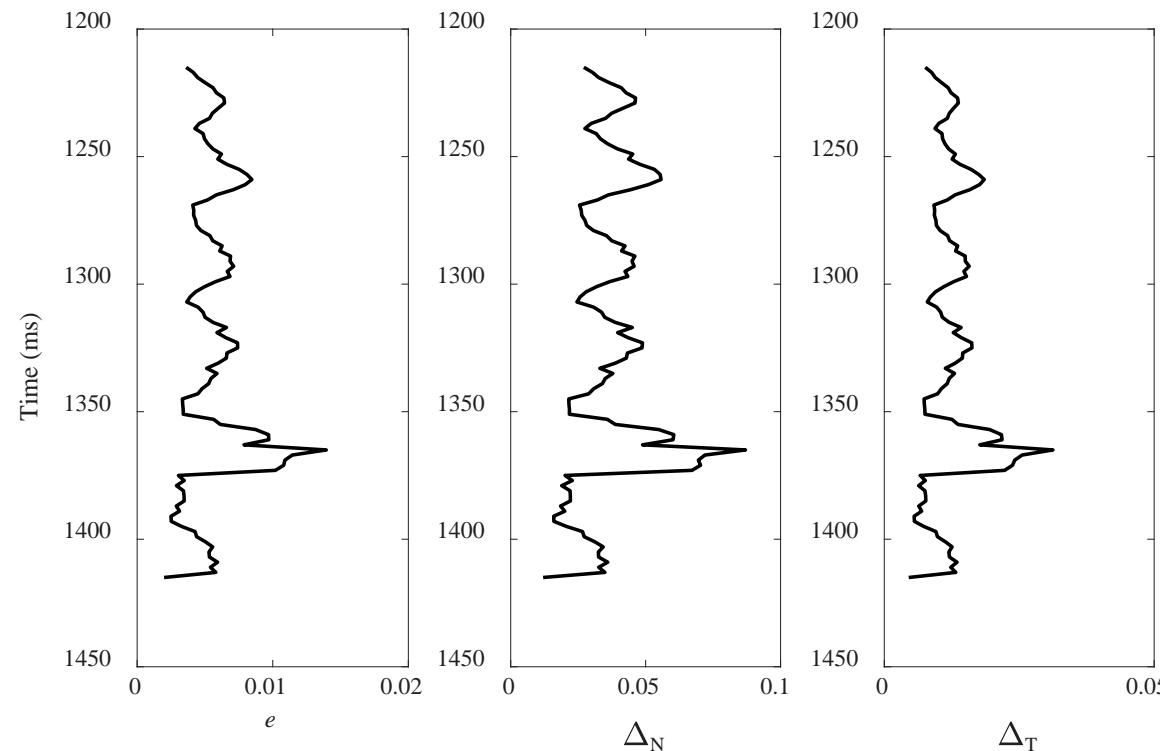


Calculated using empirical relationships  
between elastic parameters and quality factors  
(Mavko et al, 2009)

- Well log model

# Examples

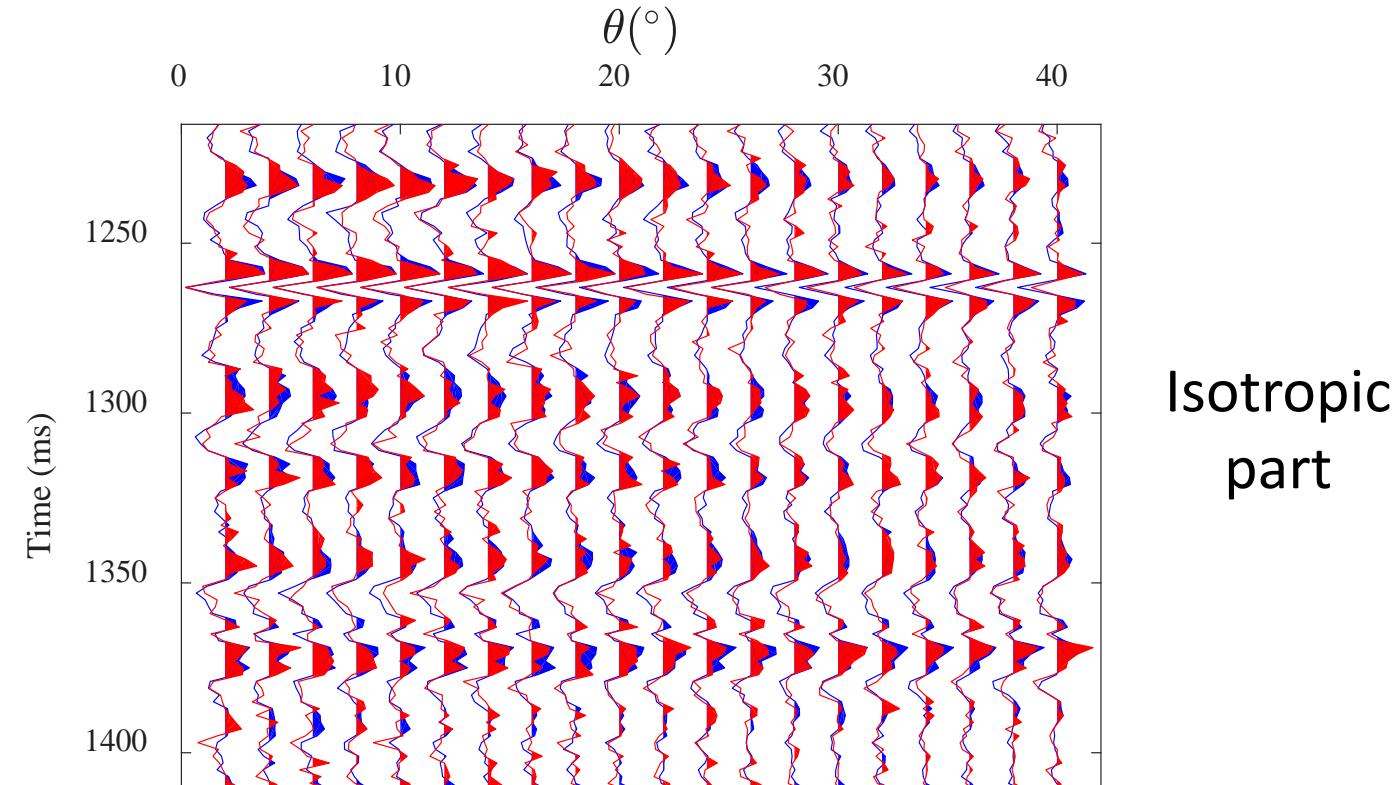
- Synthetic tests



- Well log model

# Examples

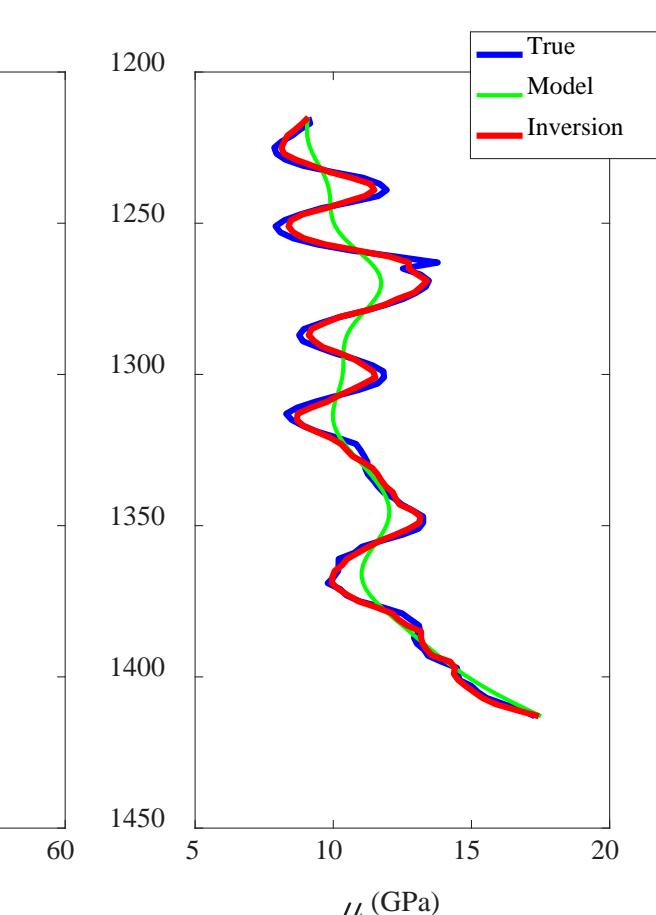
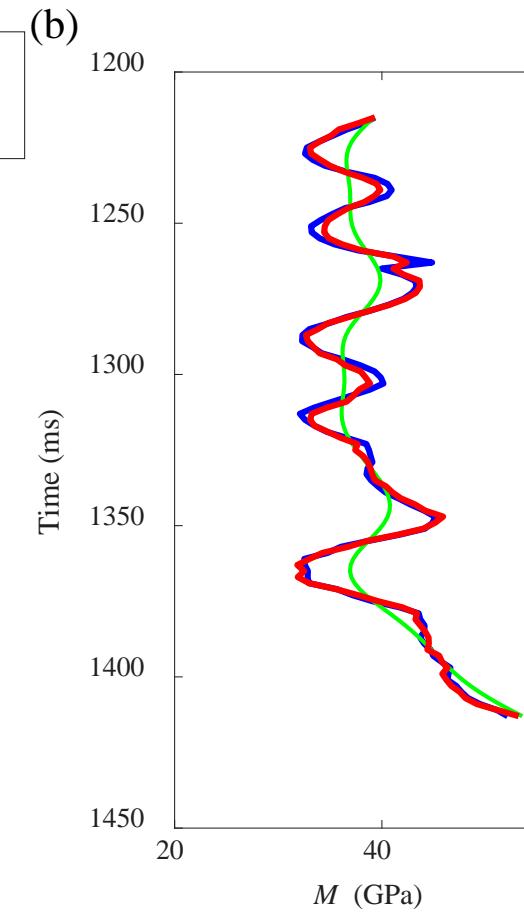
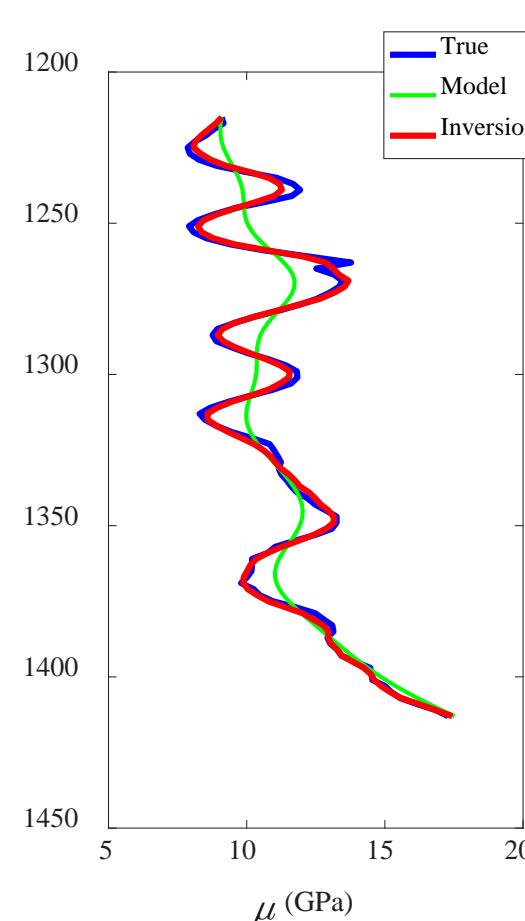
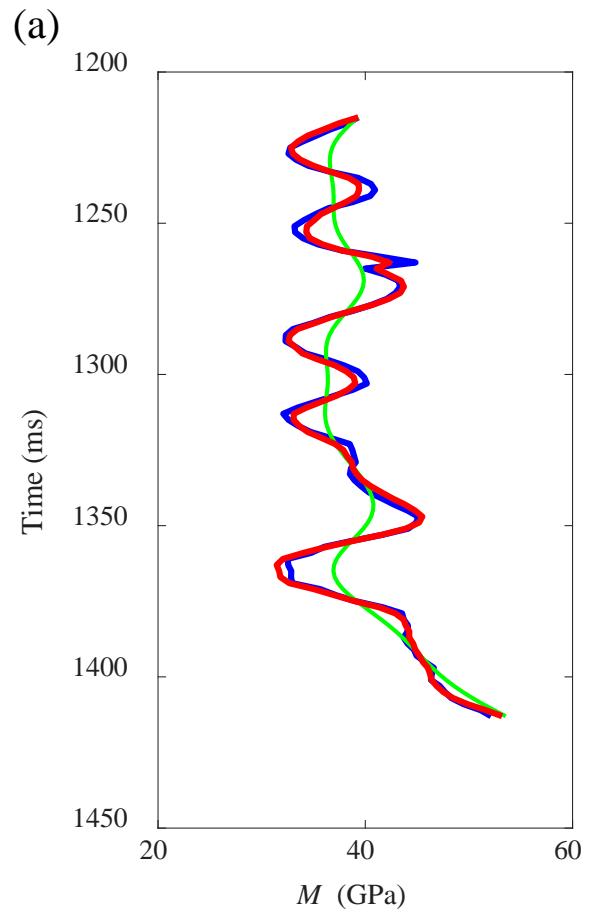
- Synthetic seismic profiles (Convolutional model)



- S/N=5(blue)
- S/N=2(red)

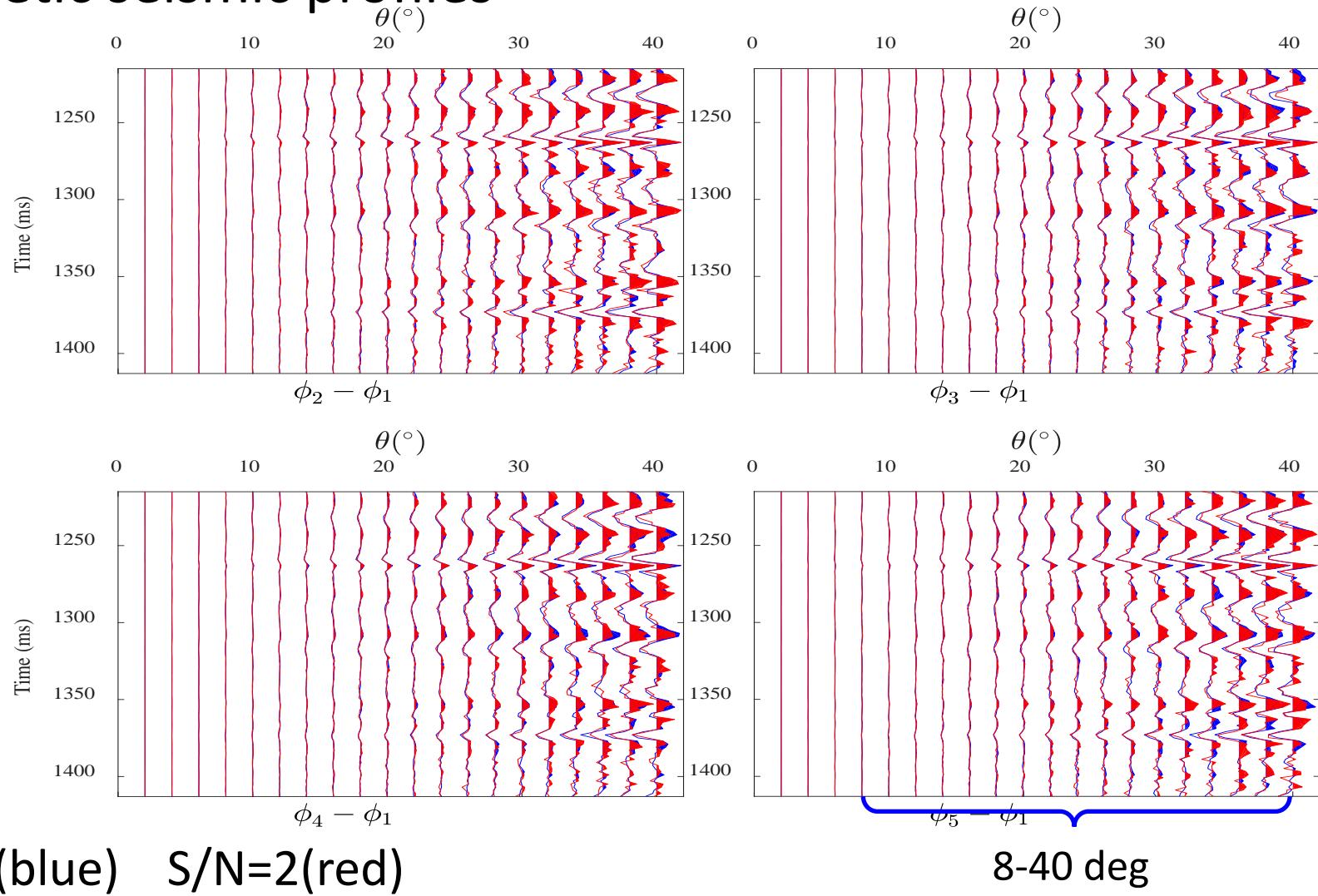
# Examples

- Inversion results of P- and S-wave moduli



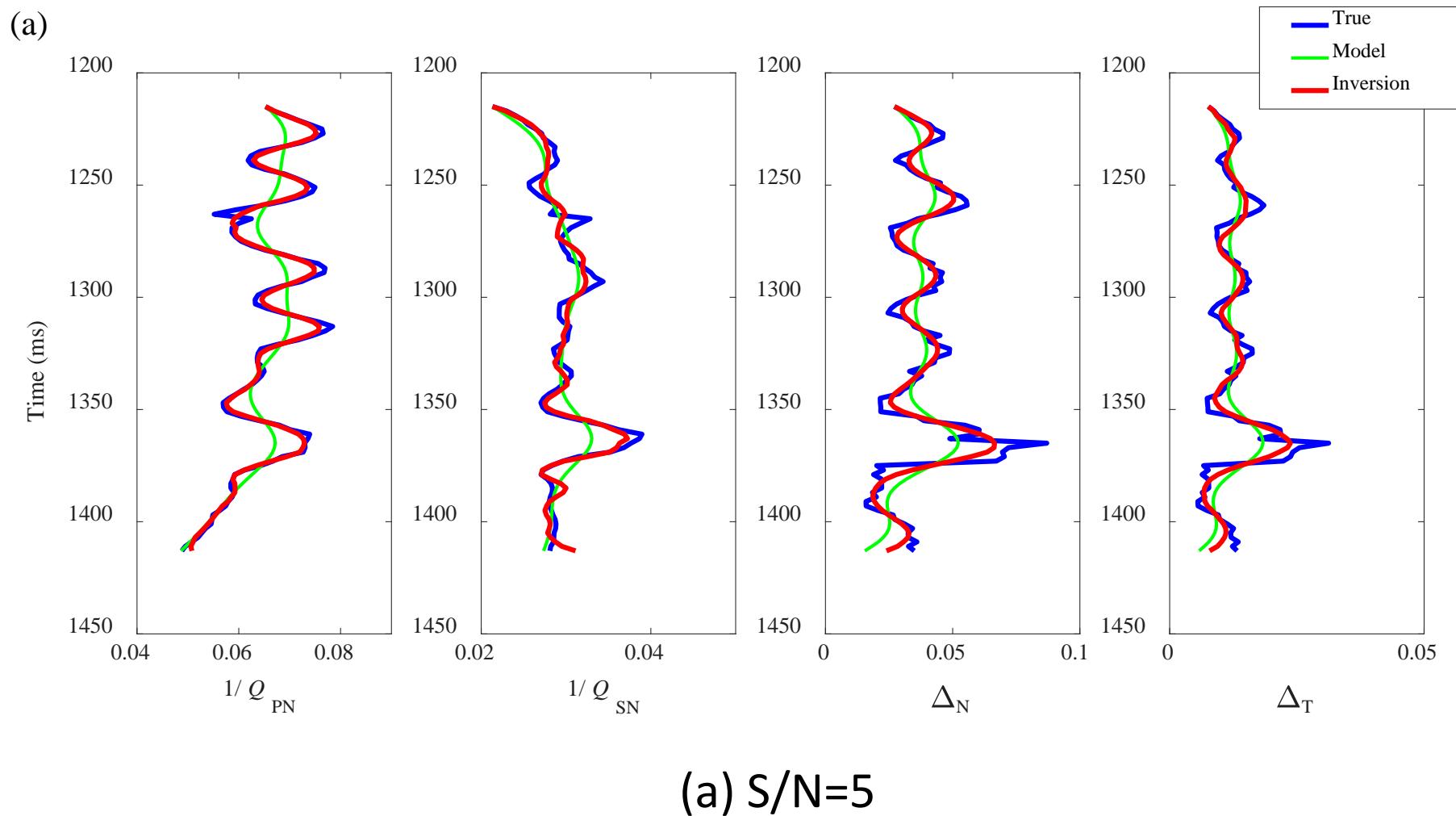
# Examples

- Synthetic seismic profiles



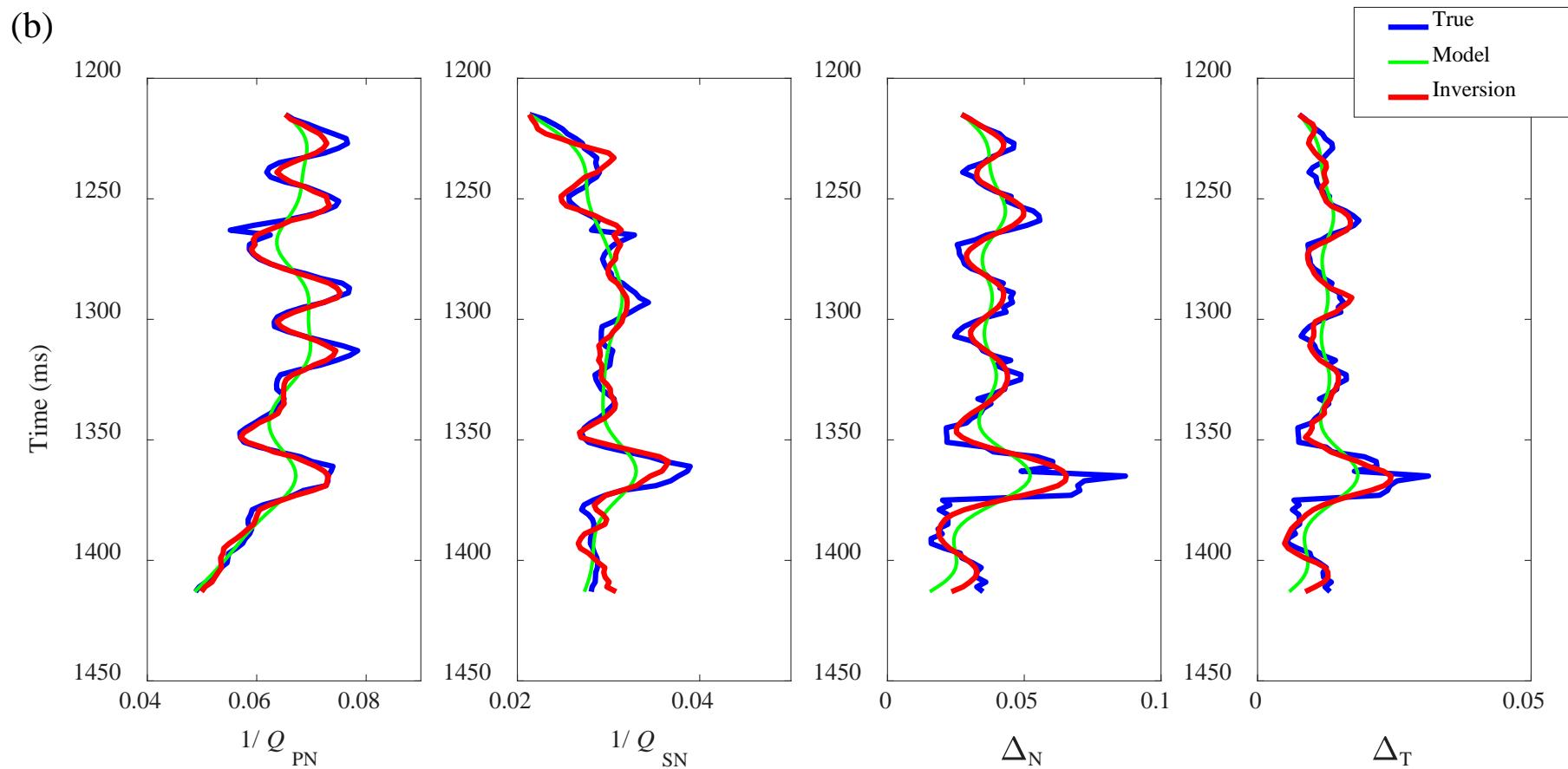
# Examples

- Inversion results of fracture weaknesses and attenuation factors



# Examples

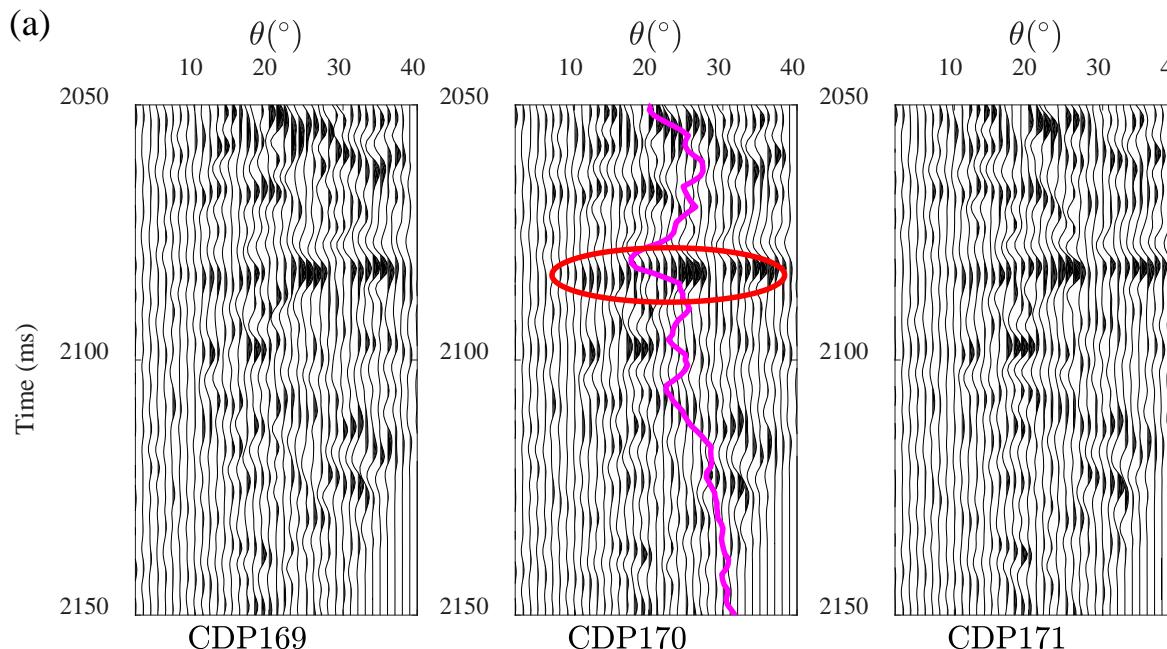
- Inversion results of fracture weaknesses and attenuation factors



(b) S/N=2

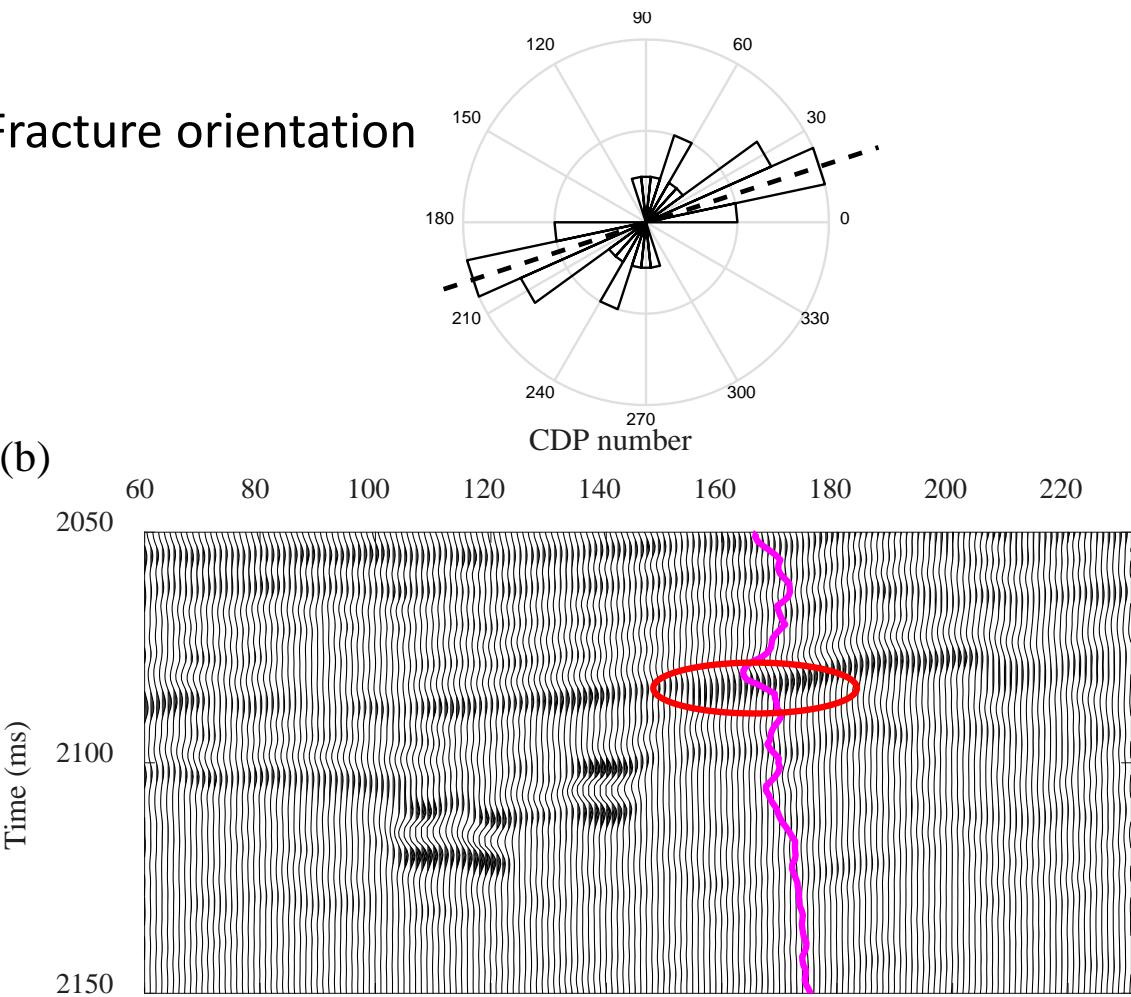
# Examples

- Real data (Fractured carbonate)



(a) Angle gathers

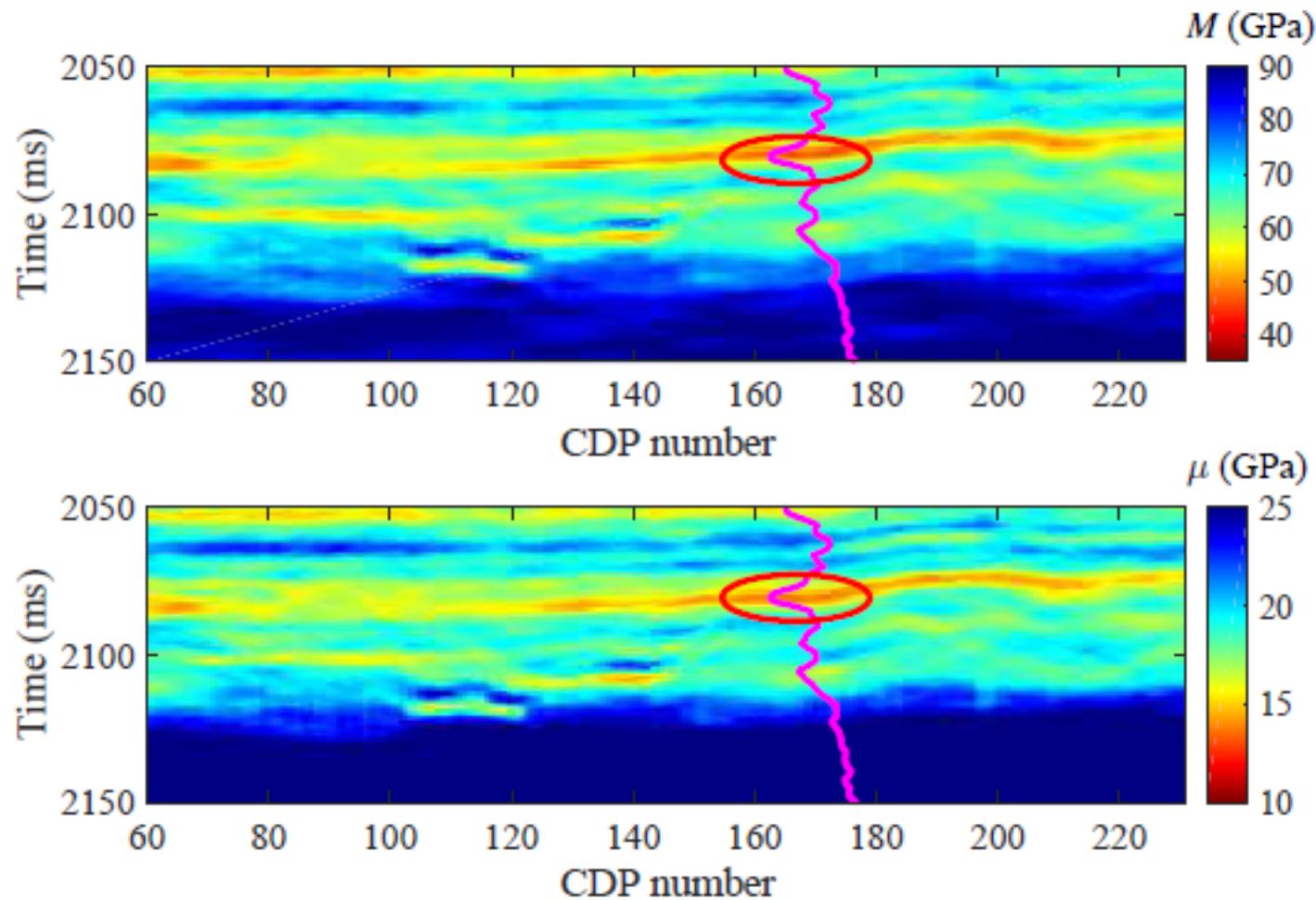
Fracture orientation



(b) Stacked seismic profile

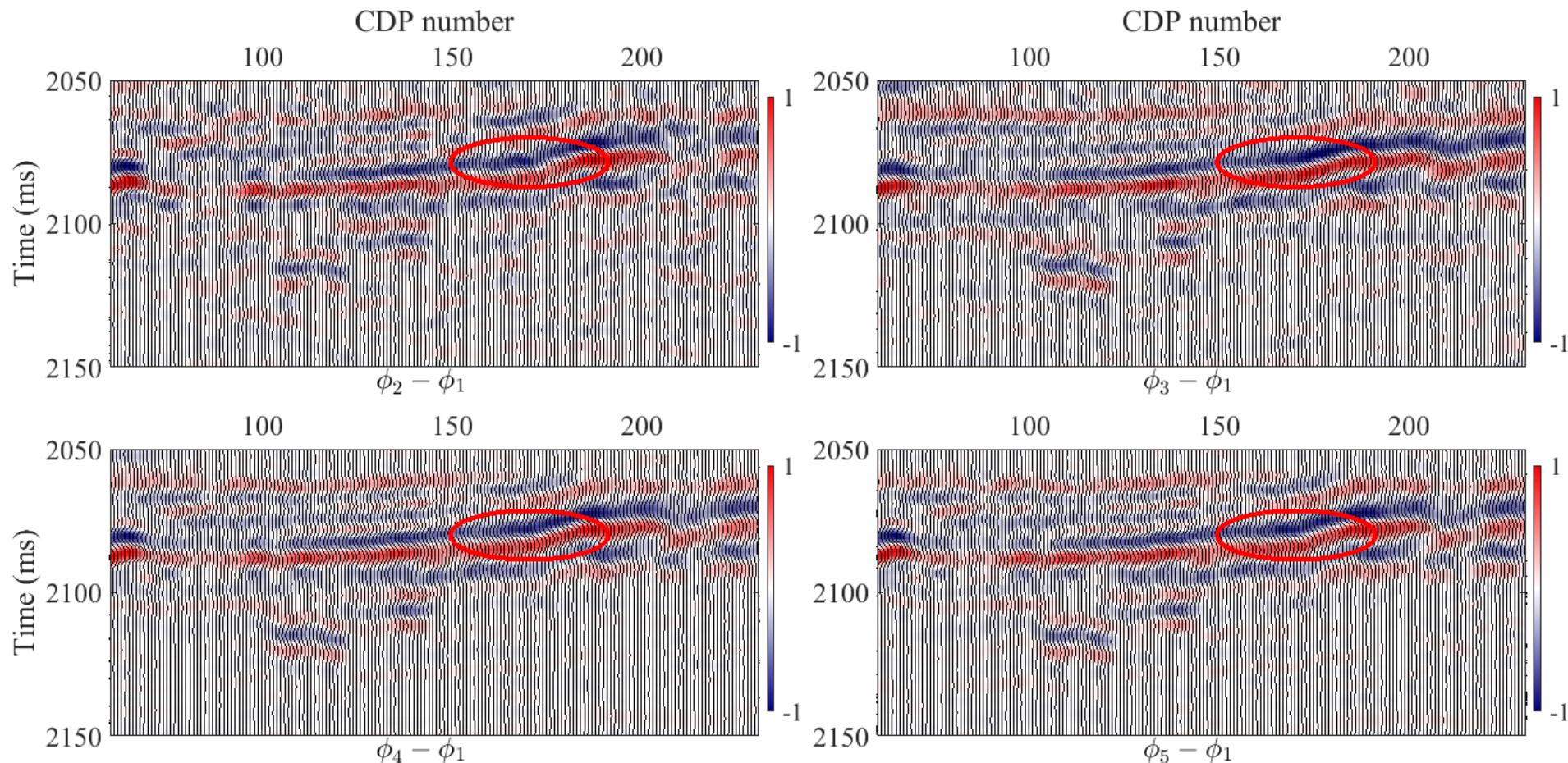
# Examples

- Inversion results of P- and S-wave moduli



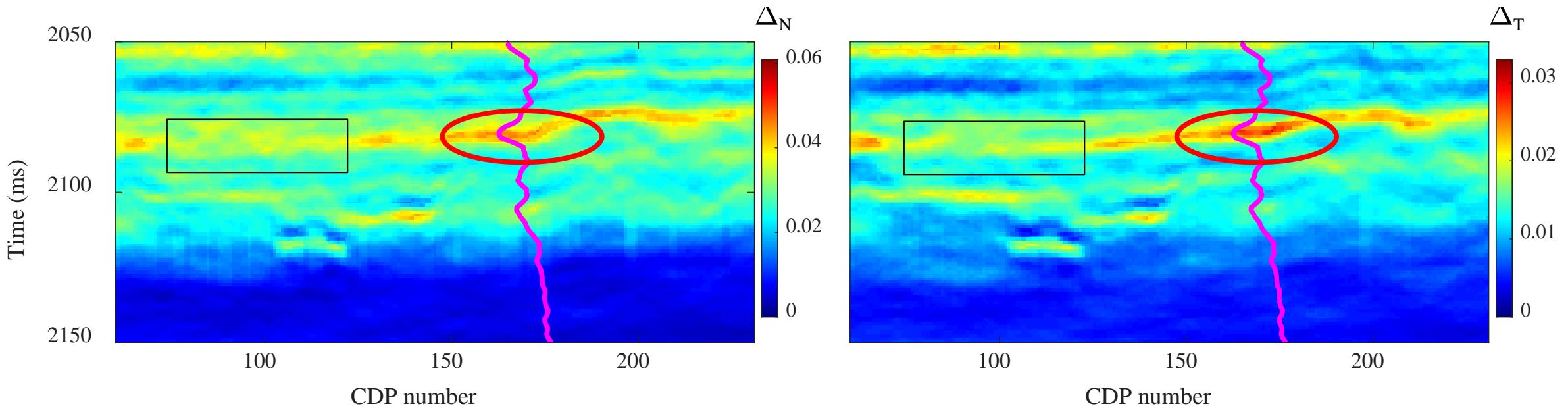
# Examples

- Seismic differences



# Examples

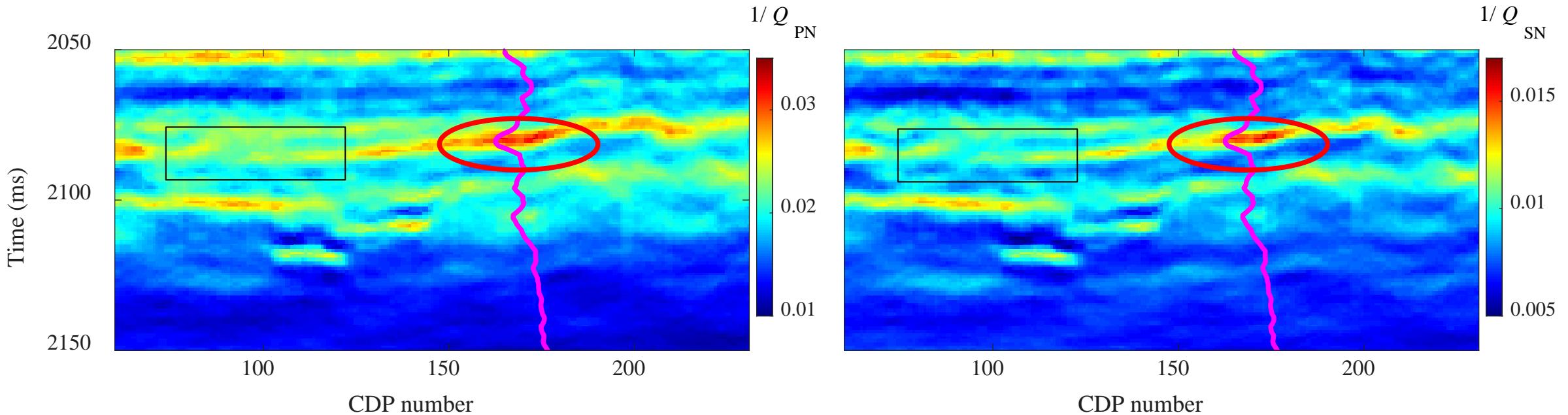
- Inversion results of fracture weaknesses and attenuation factors



There are still some locations that exhibit a high fracture weakness

# Examples

- Inversion results of fracture weaknesses and attenuation factors



Detection of fractured reservoirs is improved using attenuation factors.

# Discussion and Conclusions

- We have derived linearized reflection coefficient related to fracture weaknesses and attenuation factors.
- We proposed a workflow of inversion for fracture weaknesses and attenuation factors from seismic differences among azimuthal data. Tests on synthetic and real data confirm the stability and reliability of the inversion.
- Assumptions: low loss background; small fracture weaknesses; small changes in elastic properties across the interface.
- Future work: relationships among integrated, intrinsic and induced attenuation factors; combining fracture weaknesses and attenuation factors to discriminate oil bearing fractured reservoirs.

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