Quantum computing for seismic problems

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Outline

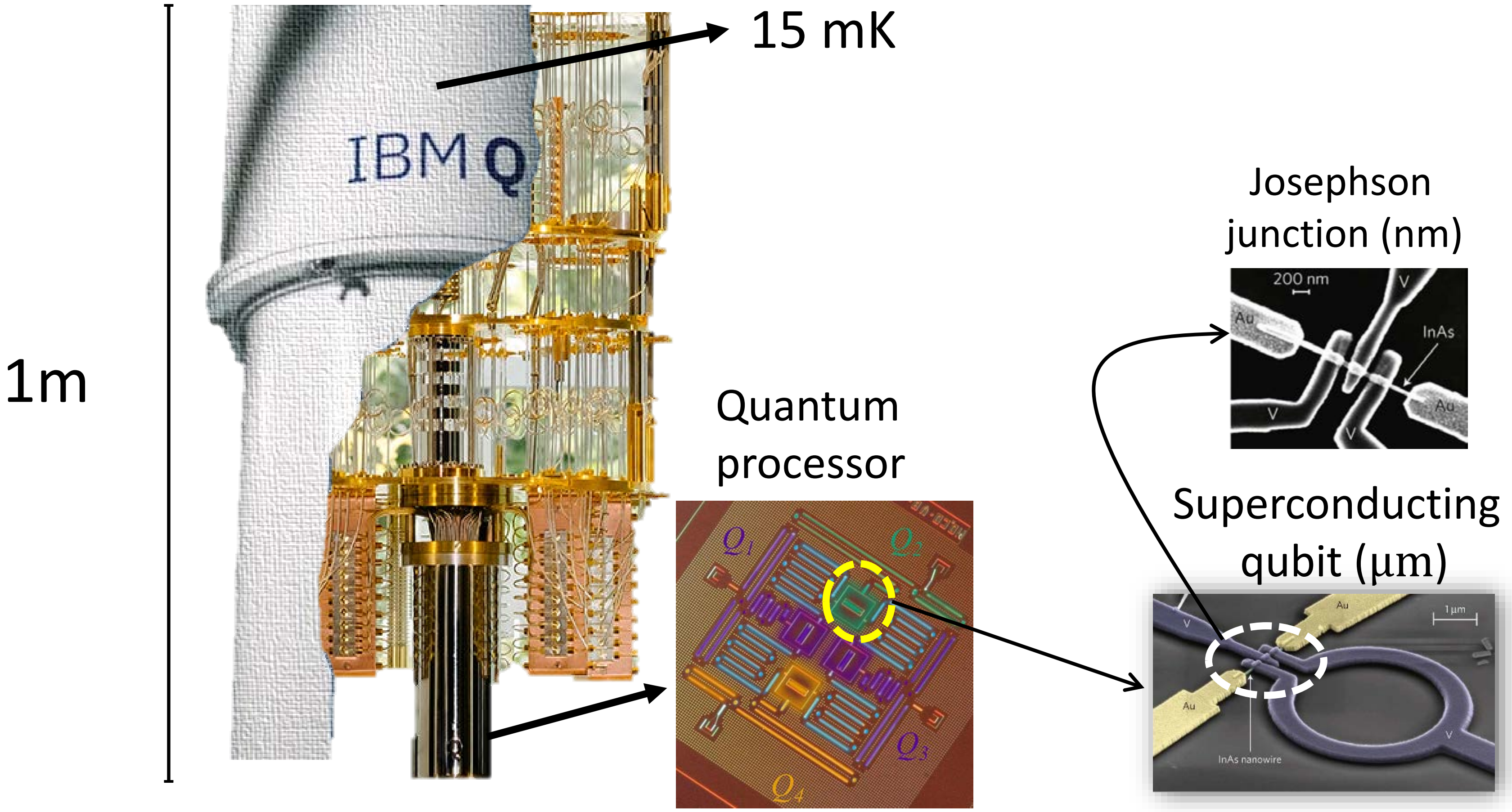
1. Background and motivation
2. Quantum computing
   a) Quantum bit (qubit)
   b) Quantum parallelism
   c) Physical realization of qubit
   d) Mathematics of qubit
3. Applications
   a) Quantum algorithms
   b) Seismic algorithms
4. Summary and future works

Motivation

- Quantum algorithms for seismic wave modeling
- Develop the software package for quantum computing with applications in seismic problems
- Design a quantum simulator for modeling and inversion

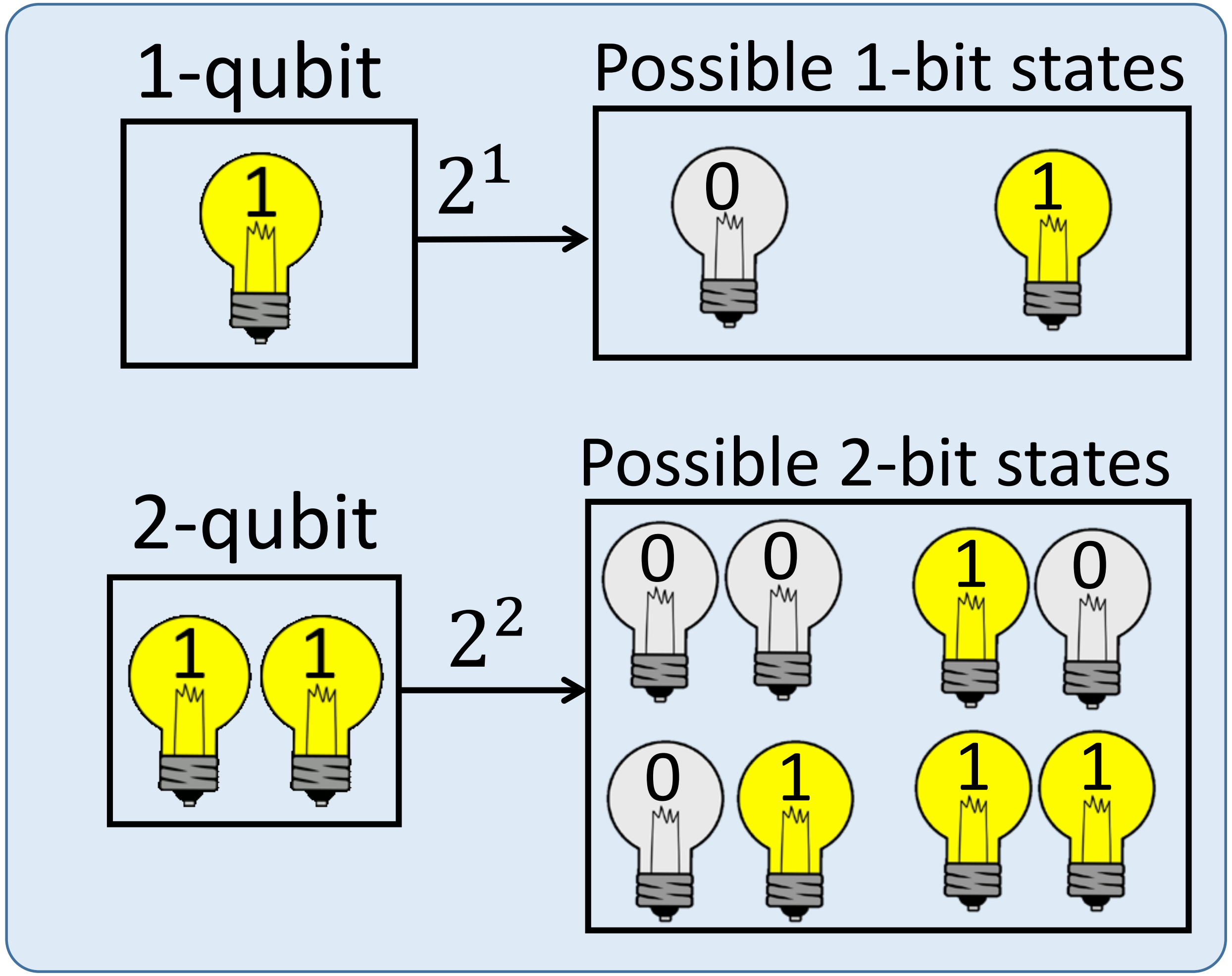
Background

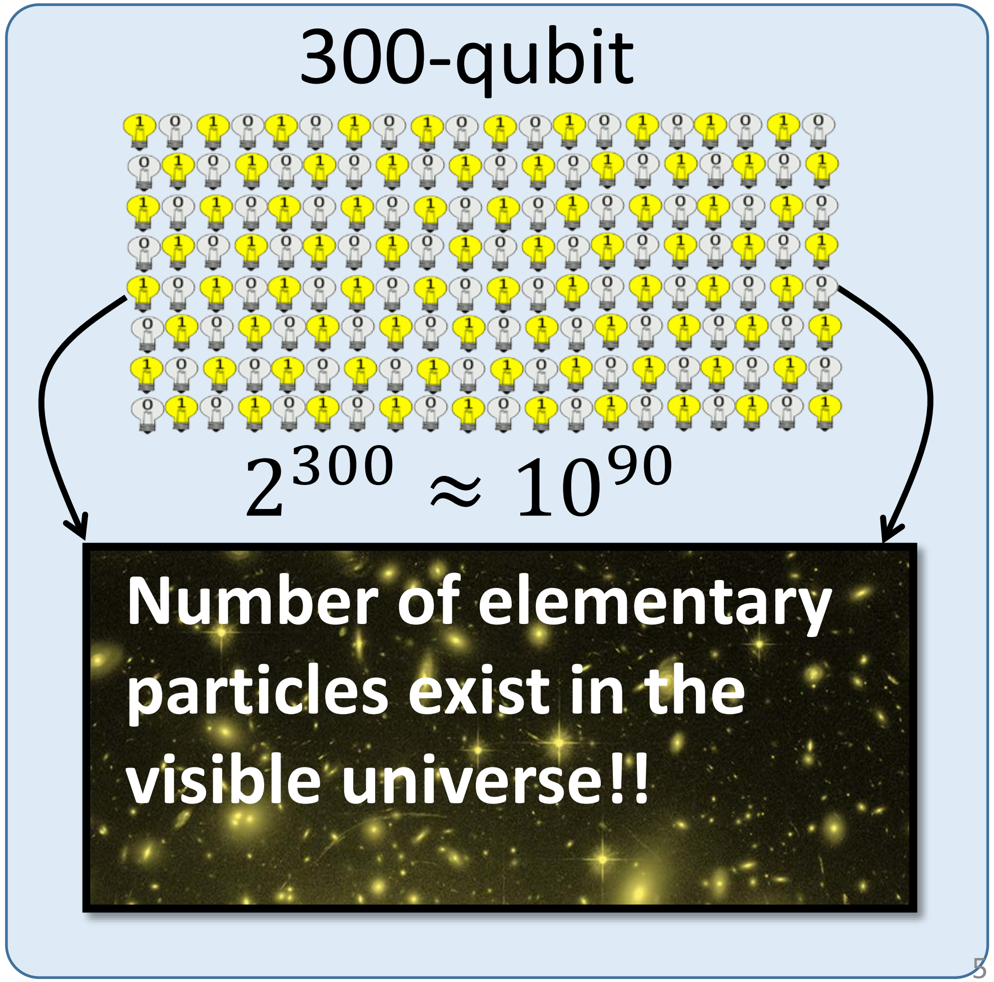
IBMQ (2017) 50 qubit quantum computer



Quantum computing

qubit: the unit of quantum information



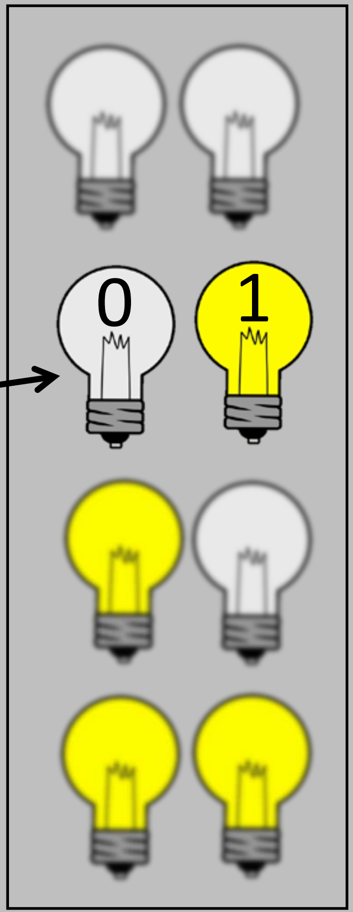


Quantum computing

Quantum parallelism: computational speedup

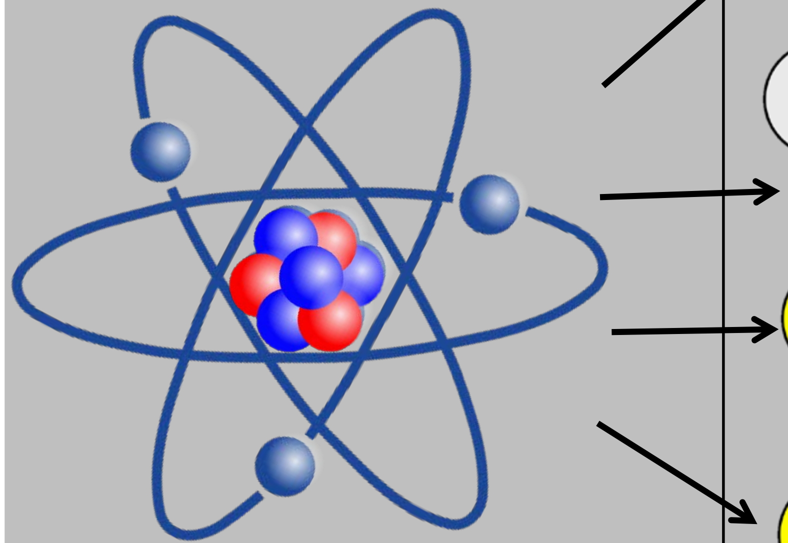
2-bit

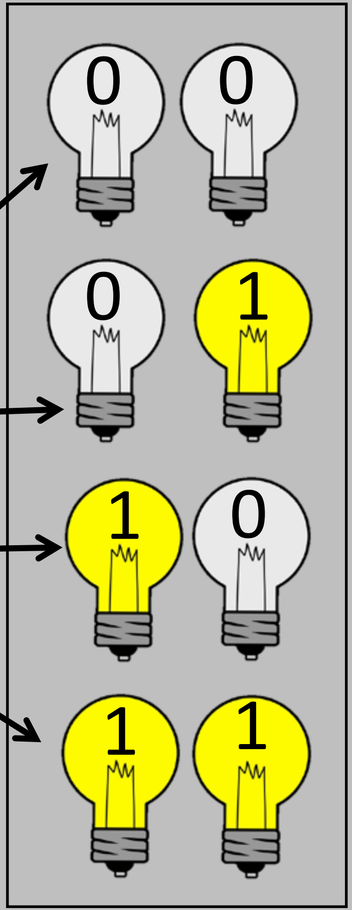
Classical computer



2-qubit

Quantum computer





Quantum computing

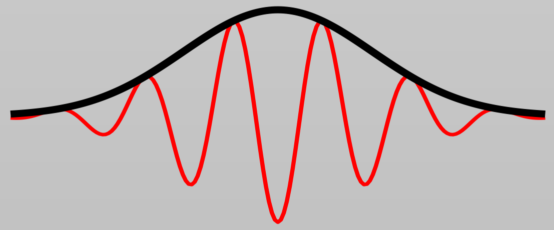
Schrodinger equation: wave equation for atoms

$$(\mathrm{KE} + \mathrm{PE})\Psi = \mathrm{E}\Psi$$

H(Hamiltonian)

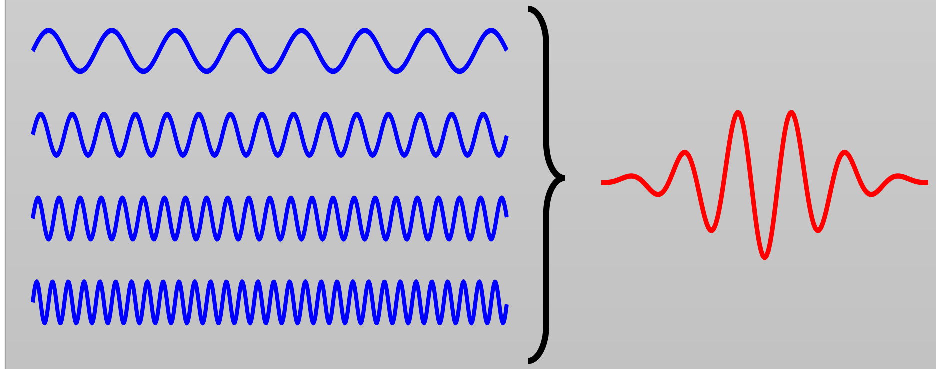
Algorithm output

$|\Psi(x,t)|^2$ **Probability density** for finding the particle at position x and time t



Quantum parallelism

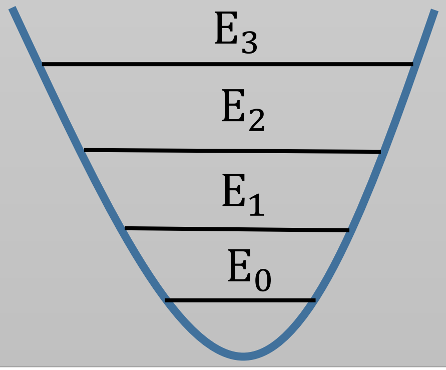
Superposition principle



Hardware (Q proc.)

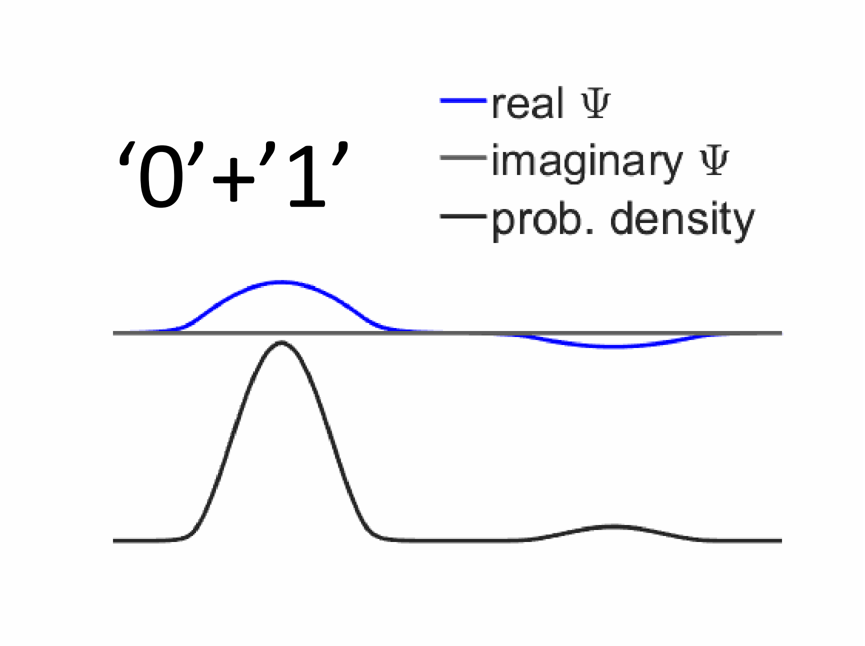
Energy is discretized

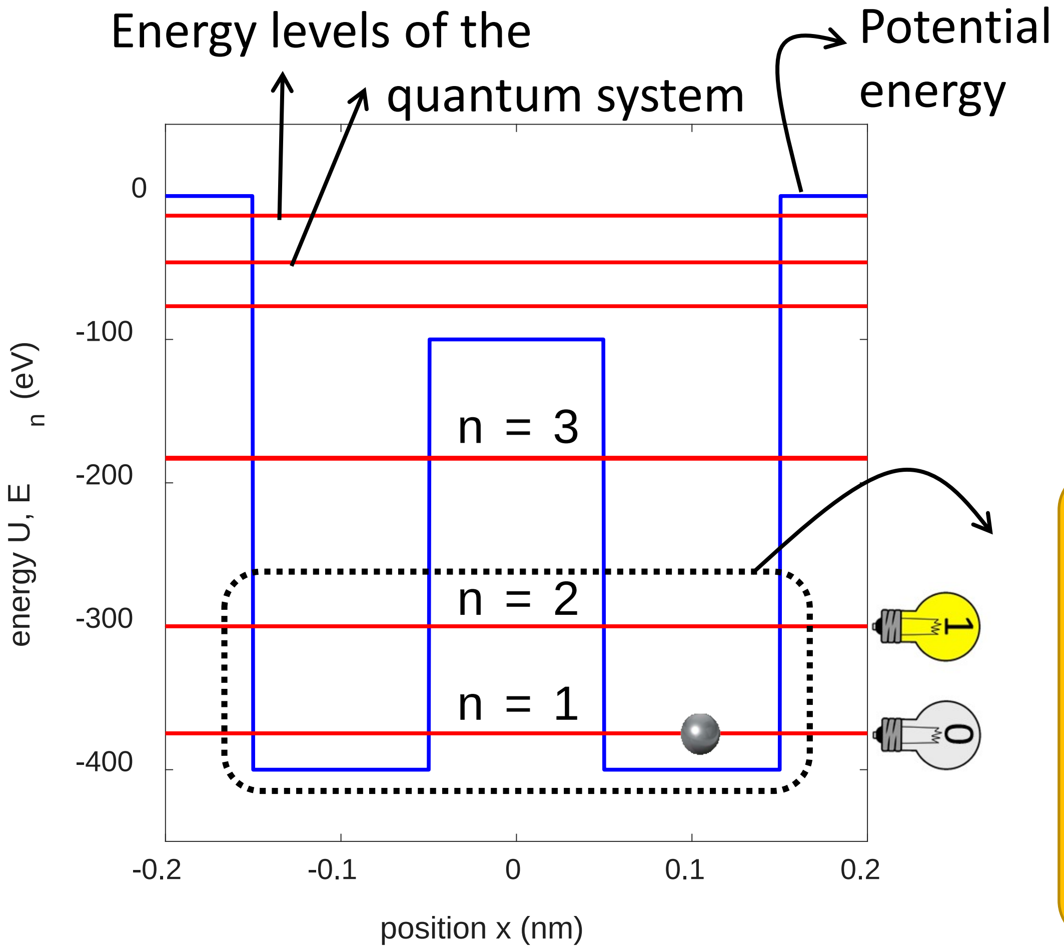
$$\mathrm{H}\Psi = \mathrm{E}\Psi$$



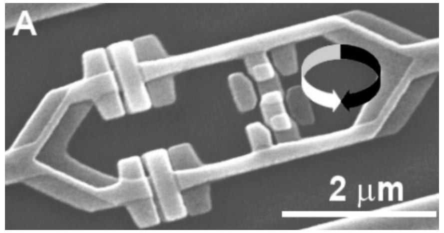
Quantum computing

Physical simulation of qubit





Superconducting phase qubit



Qubit is the superposition of the lowest energy levels of the system near absolute zero temperature

Quantum computing

Mathematics of qubit

Ket notation

$0 \longrightarrow |0\rangle \equiv \begin{bmatrix}1\\0\end{bmatrix}$

$1 \longrightarrow |1\rangle \equiv \begin{bmatrix}0\\1\end{bmatrix}$

1-qubit $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$|\alpha|^2$

Probability that qubit to be in $|0\rangle$

$|\beta|^2$

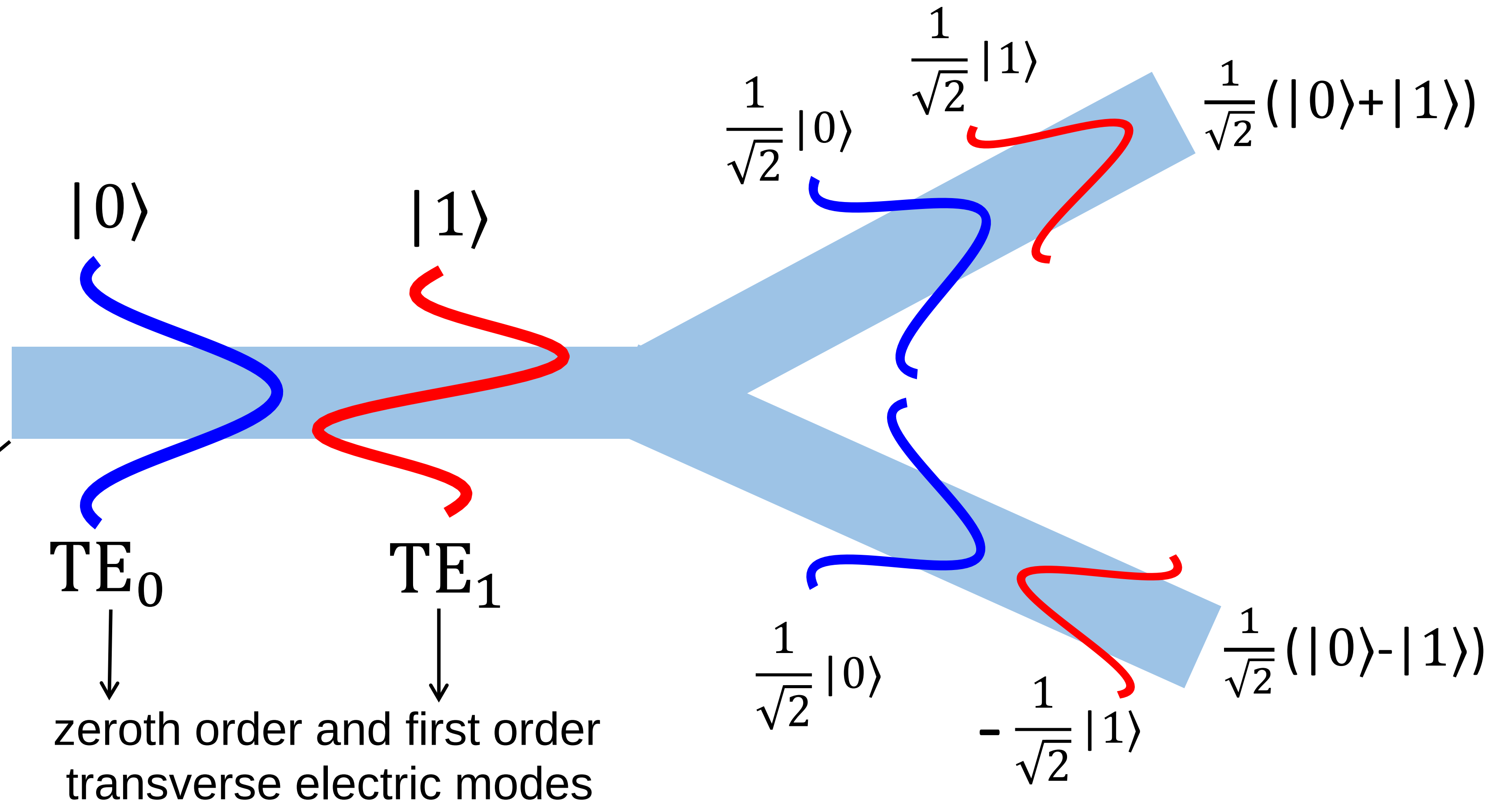
Probability that qubit to be in $|1\rangle$

Quantum computing

Quantum gate

Hadamard gate

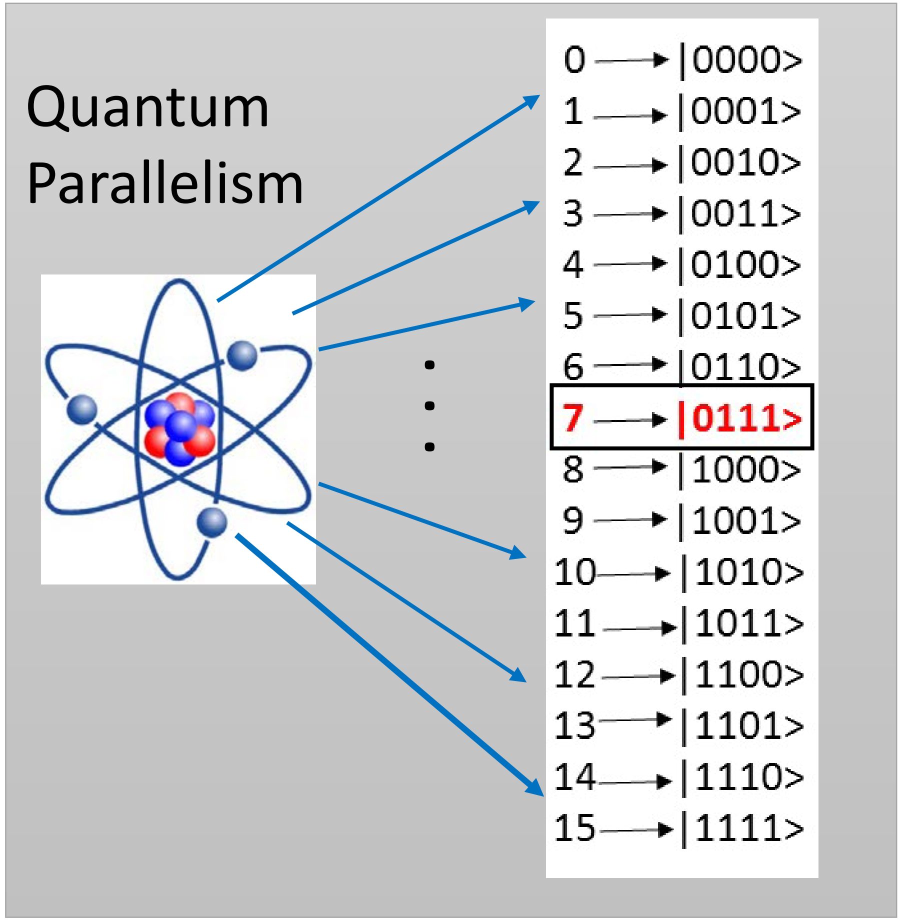
$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

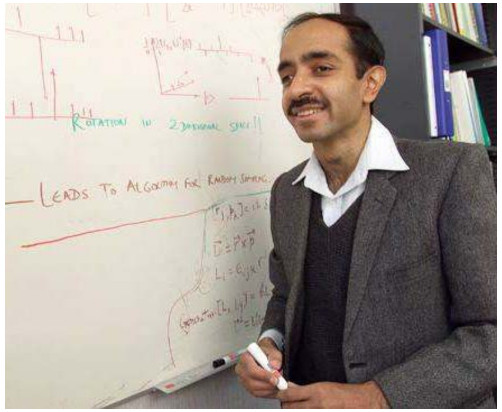


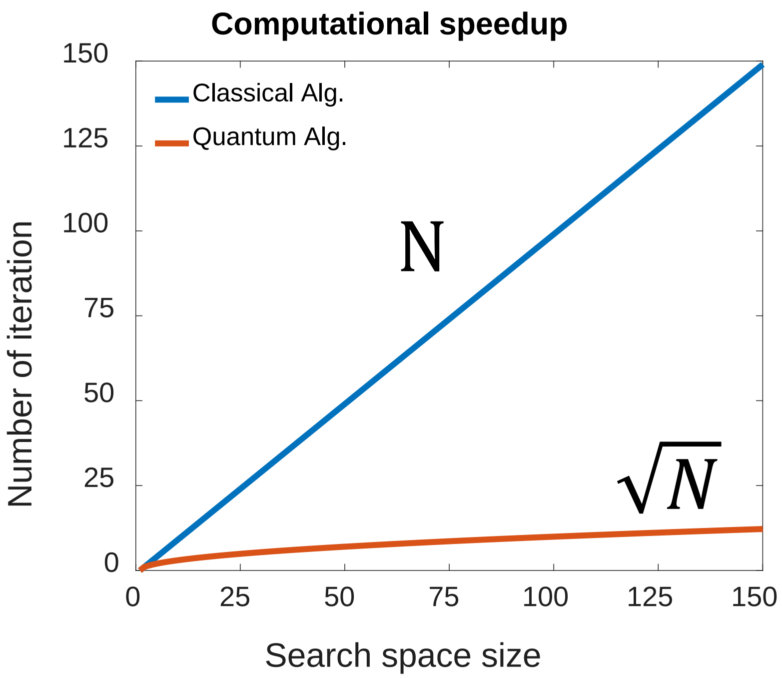
Applications

Quantum search algorithm

Quantum mechanics helps in searching for a needle in a haystack- Grover-PRL, 1997





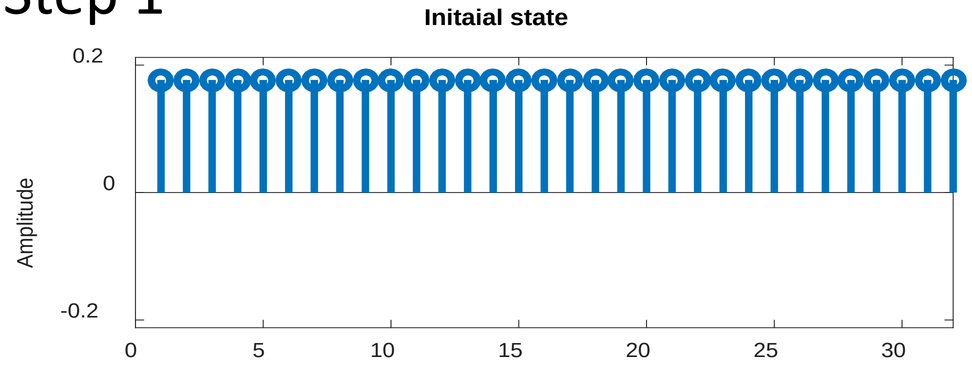


Applications

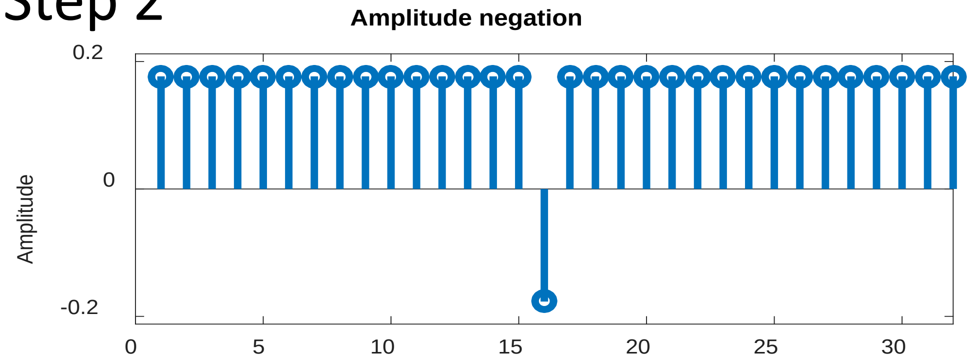
Quantum search algorithm

Numerical modeling $N=32=2^5$, simulation with 5 qubits

Step 1

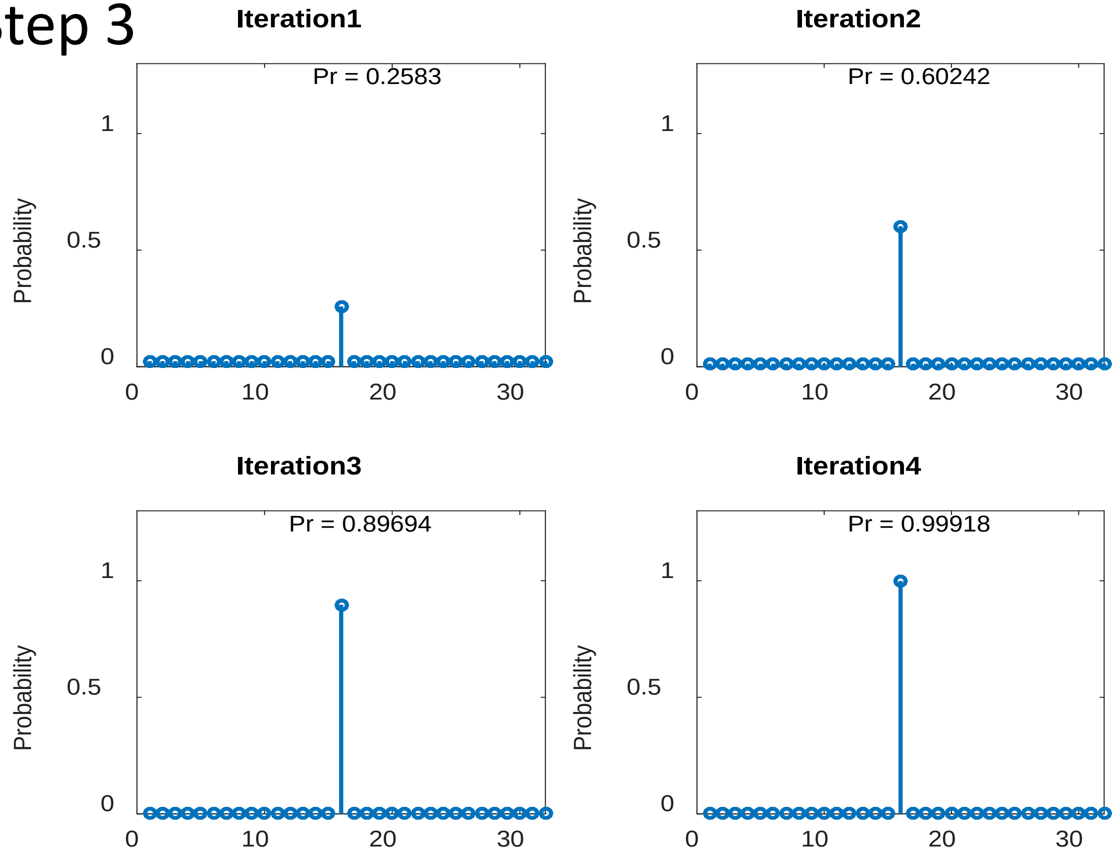


Step 2



Search space (N)

Step 3



Search space (N)

Search space (N)

Applications

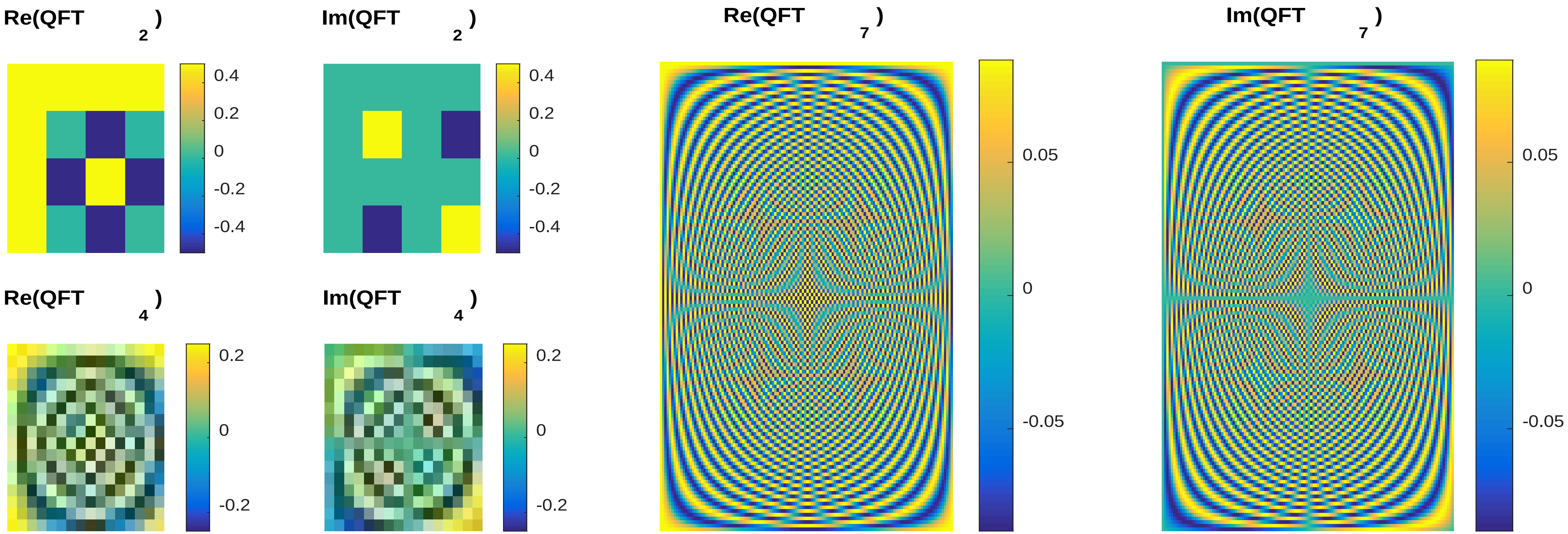
Quantum Fourier Transform

DFT	QFT
DFT: $(x_0, \dots, x_{N-1}) \mapsto (y_0, \dots, y_{N-1})$	QFT: $\sum_{j=1}^{N-1} x_j \vert j\rangle \mapsto \sum_{k=1}^{N-1} y_k \vert k\rangle$
$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j e^{i2\pi jk/N}$	$\vert j\rangle \mapsto \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{i2\pi jk/N} \vert k\rangle$
DFT on $N = 2^n$ elements requires $O(n2^n)$ gates	QFT on $N = 2^n$ elements requires only $O(n^2)$ gates (Exponential speed up!)

Applications

Quantum Fourier Transform

Numerical modeling



Applications

Quantum algorithm for linear systems of equations

Harrow, et al. PRL, 2009

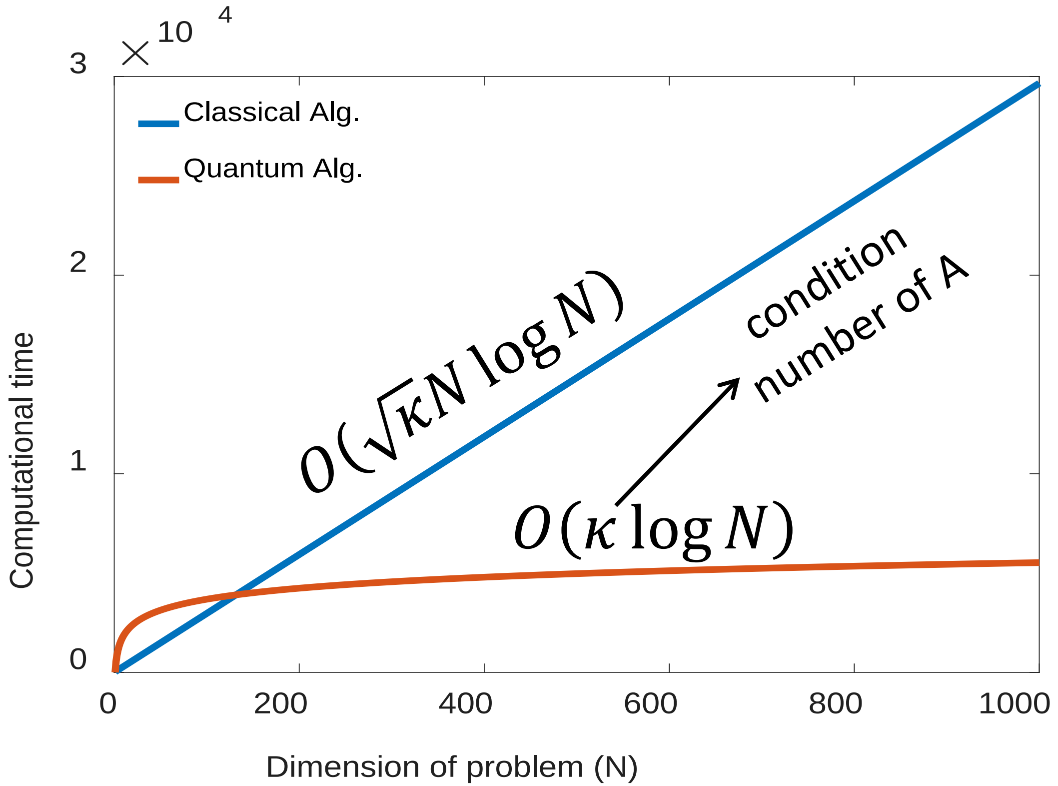
$$A\,x = b$$

A $N \times N$ Matrix

$\vec{x}$ Unknown vector

$\vec{b}$ Unit vector

$$|x\rangle = A^{-1}|b\rangle_{\text{N-qubit}}$$



Applications

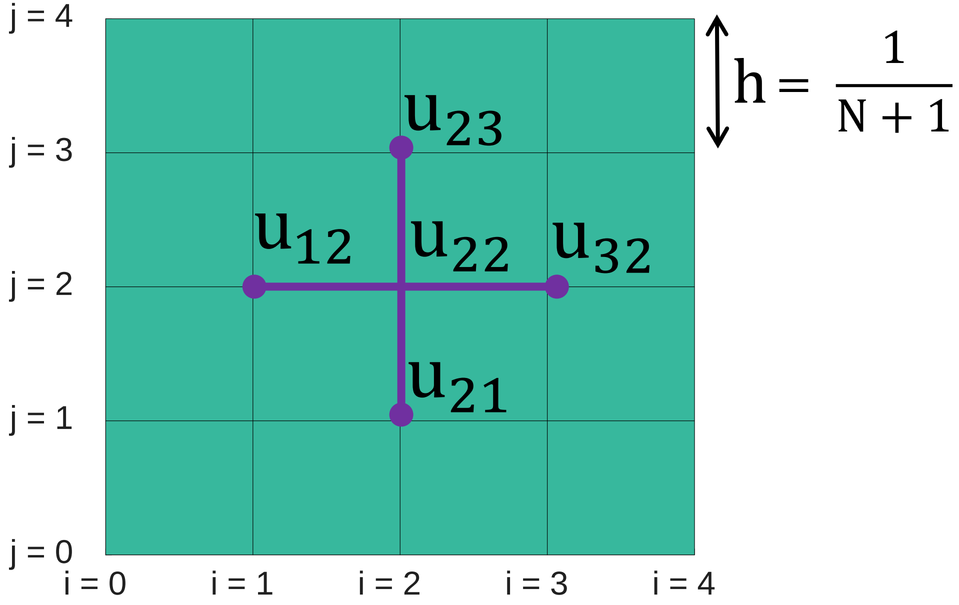
Finite Difference Modeling

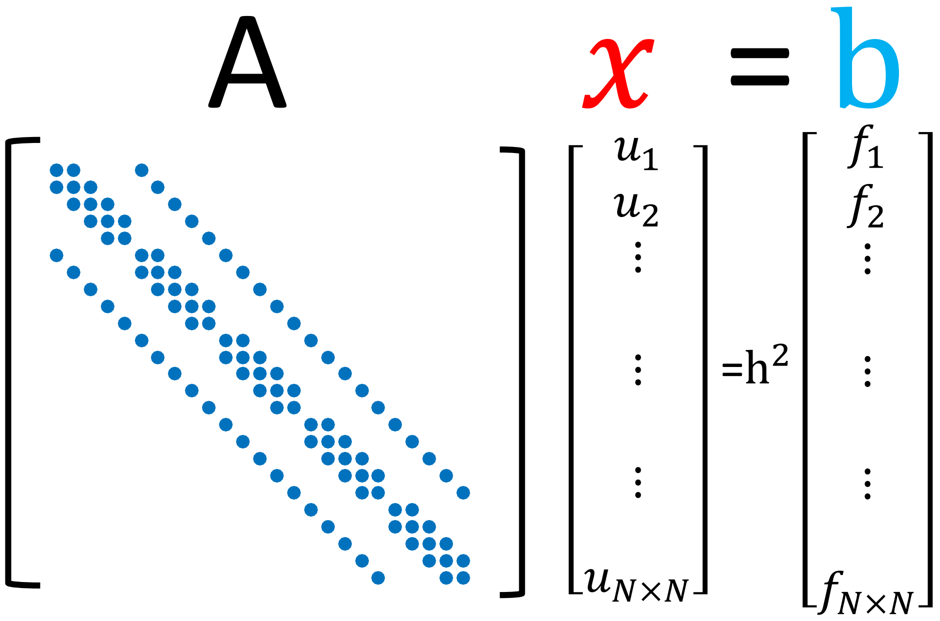
Modeling in the frequency domain

$$\nabla^2 u(\vec{r},\omega) + \frac{\omega^2}{c^2} u(\vec{r},\omega) = f(\vec{r},\omega)$$

Discretization

$$u_{ij} = u(ih, jh)$$
$$f_{ij} = f(ih, jh)$$





Applications

Finite Difference Modeling

Classical algorithm running time

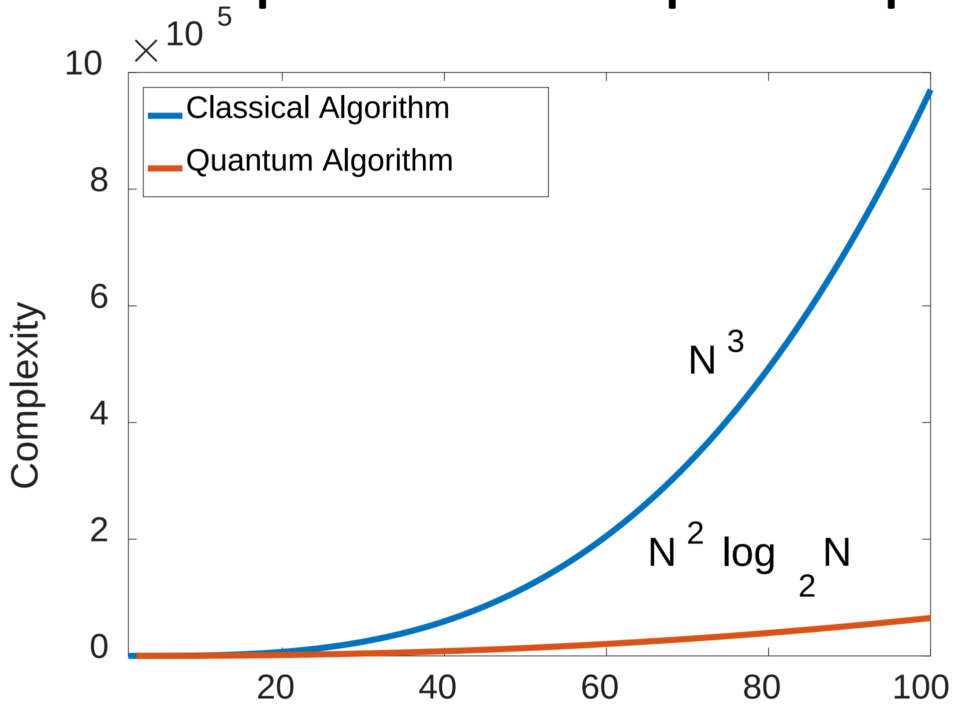
$$O(n_s n_t N^3)$$

N Number of grids
n_s Number of shots
n_t Number of time steps

Quantum Algorithm running time

$$O(\text{poly}[\log(n_s), \log(n_t)]\ N^2 \log N\ )$$

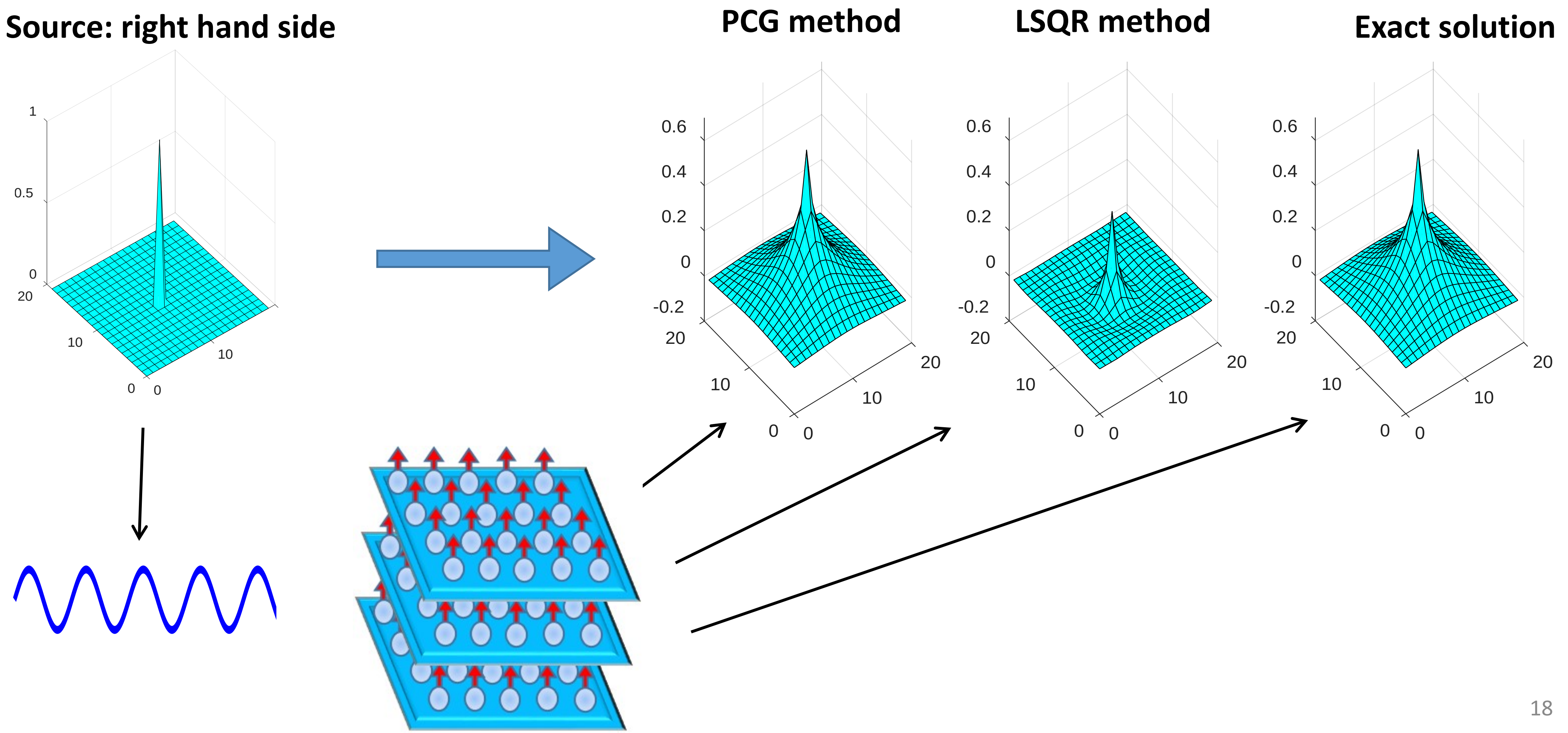
Computational speedup



Number of grids (N)

Applications

Quantum Simulator for Modeling



Summary

- Quantum computing offers the exponential speedup in terms of number of grids, time steps and number of shots in seismic wave modeling
- Develop a quantum computing software package for seismic applications

Future work

- Quantum algorithms for seismic migration and inversion (imaging)
- Design and propose a quantum simulator for modeling and inversion

Acknowledgement

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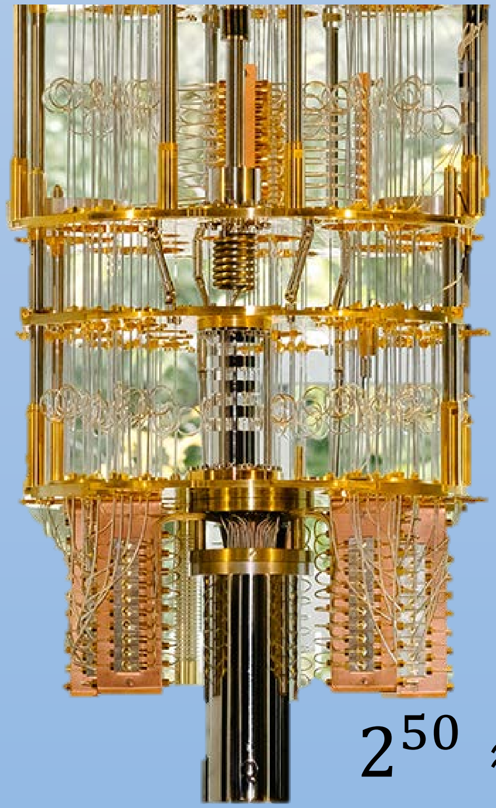
Thank you

Quantum computer

IBMQ (2017)

50 qubit quantum computer



$2^{50} \approx 10^{17}$ Flops

Blue Gene (IBM)

most powerful supercomputer

$2^{45} \approx 10^{15}$ Flops

Quantum computer

Quantum computation must be done on a time-scale less than the **decoherence time**

What is important ⟶ (decoherence time/gate operation time)

decoherence time $\sim 10^{-6}\mathrm{s}$
Typical gate operation time $\sim 10^{-12}\mathrm{s}$

$$\frac{10^{-6}\mathrm{s}}{10^{-12}\mathrm{s}} = 10^{6}$$

Number of operation that can be executed before the Quantum state decays

To factor $21 = 7 \times 3$ we need 10^{5} gates