Pure P- and S-wave elastic reverse time migration with adjoint state method imaging condition

Jorge E. Monsegny* and Daniel O. Trad

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NSERC CRSNG
CMC Containment & Monitoring Institute
UNIVERSITY OF CALGARY FACULTY OF SCIENCE Department of Geoscience

Outline

1. **Elastic RTM Introduction.**
2. Non pure wave modes RTM imaging conditions.
3. Pure wave modes RTM imaging conditions.
4. Computational complexity.
5. Numerical Experiments.
6. Conclusions.

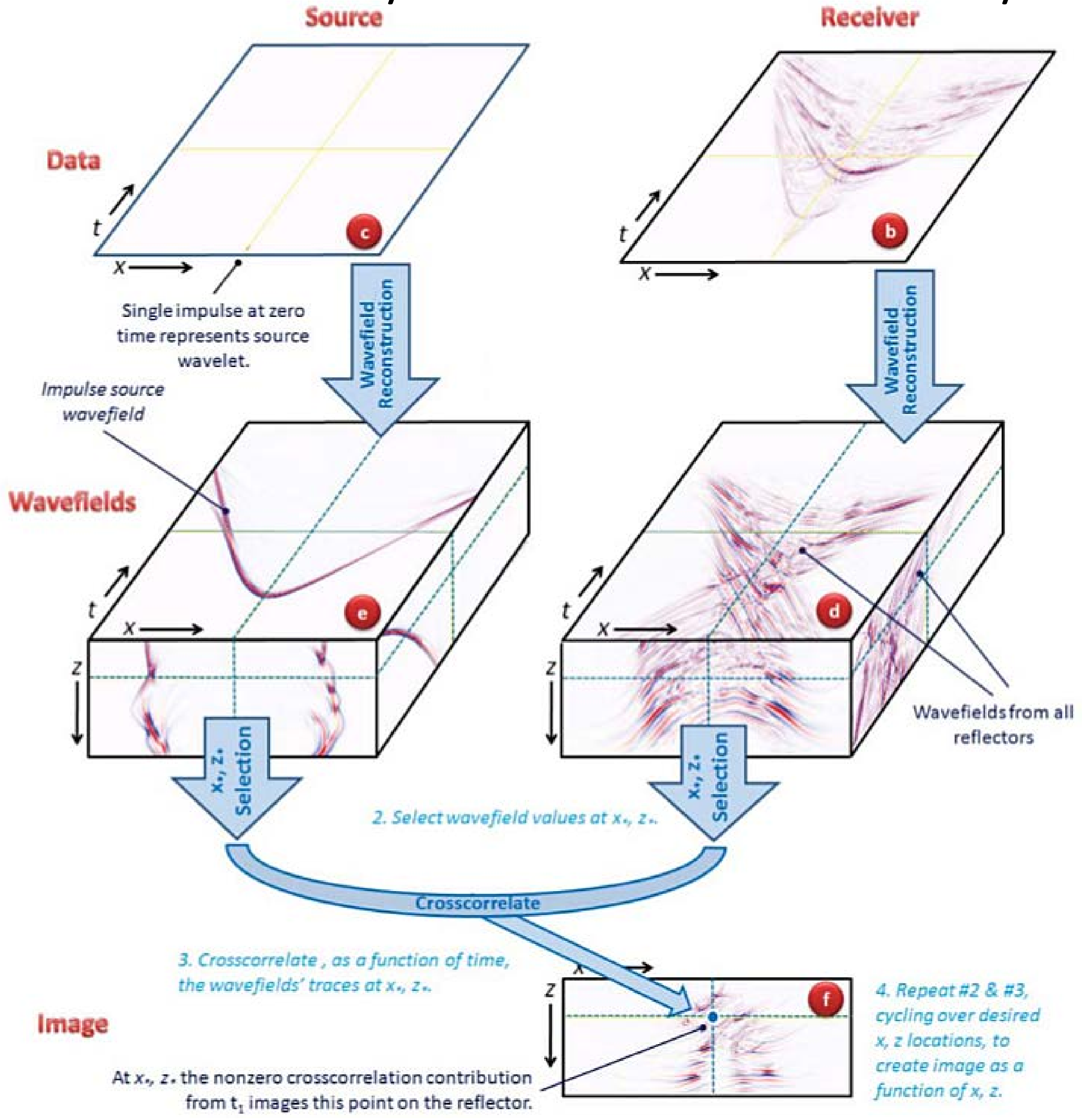
Acoustic RTM (Scalar)

Pressure/Vz   Pressure/Vz

Estimated source

Forward propagated wavefield

Imaging condition

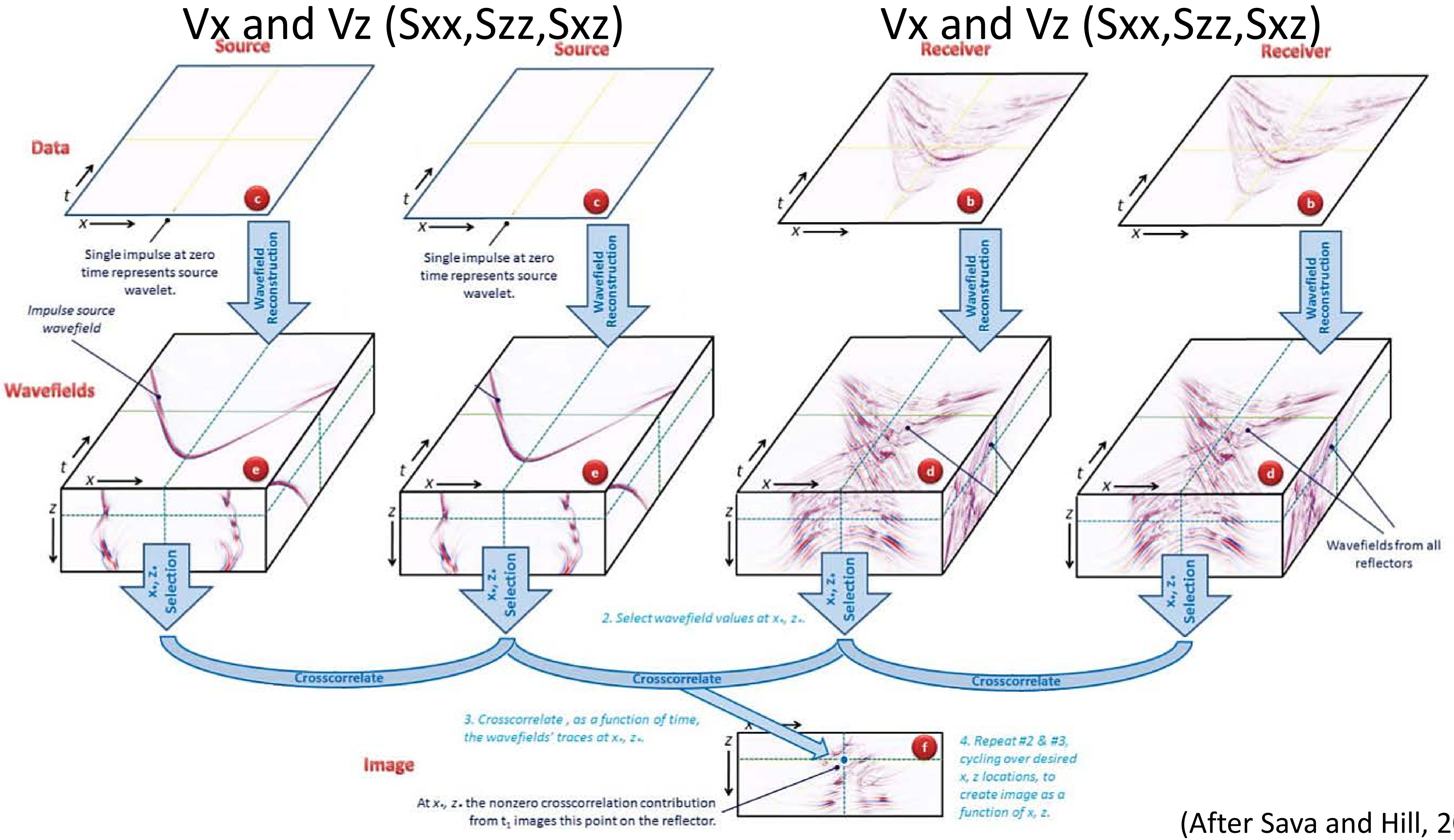


Observed data

Back propagated wavefield

(After Sava and Hill, 2009)

Elastic RTM (Vector)



(After Sava and Hill, 2009)

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FD without mode separation

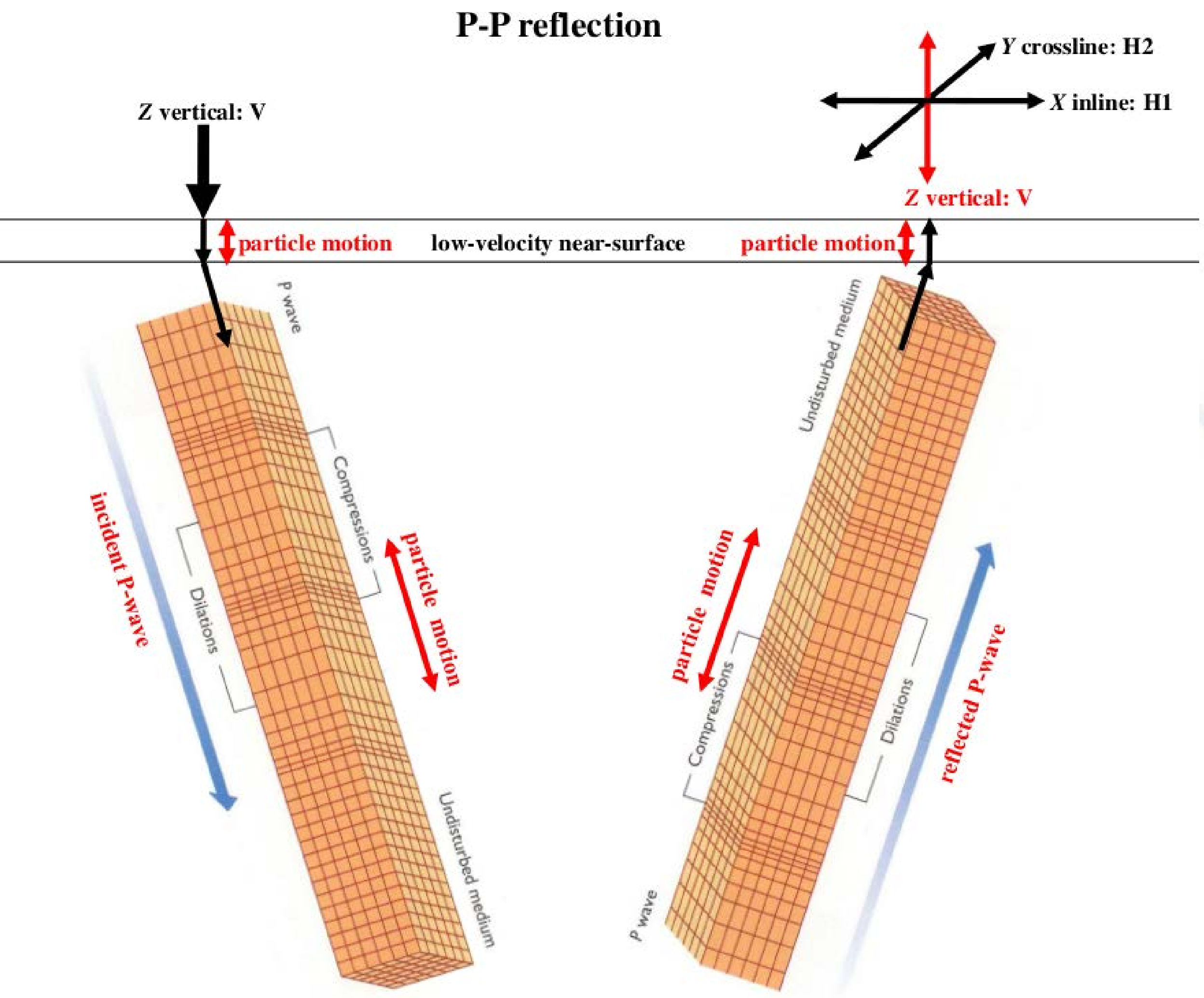
$$
\begin{aligned}
\rho \frac{\partial v_x}{\partial t} &= \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} \\
\rho \frac{\partial v_z}{\partial t} &= \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x} \\
\frac{\partial \sigma_{xx}}{\partial t} &= (\lambda + 2\mu)\frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z} + f_{\sigma 1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \sigma_{zz}}{\partial t} &= (\lambda + 2\mu)\frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x} + f_{\sigma 2} \\
\frac{\partial \sigma_{xz}}{\partial t} &= \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)
\end{aligned}
$$

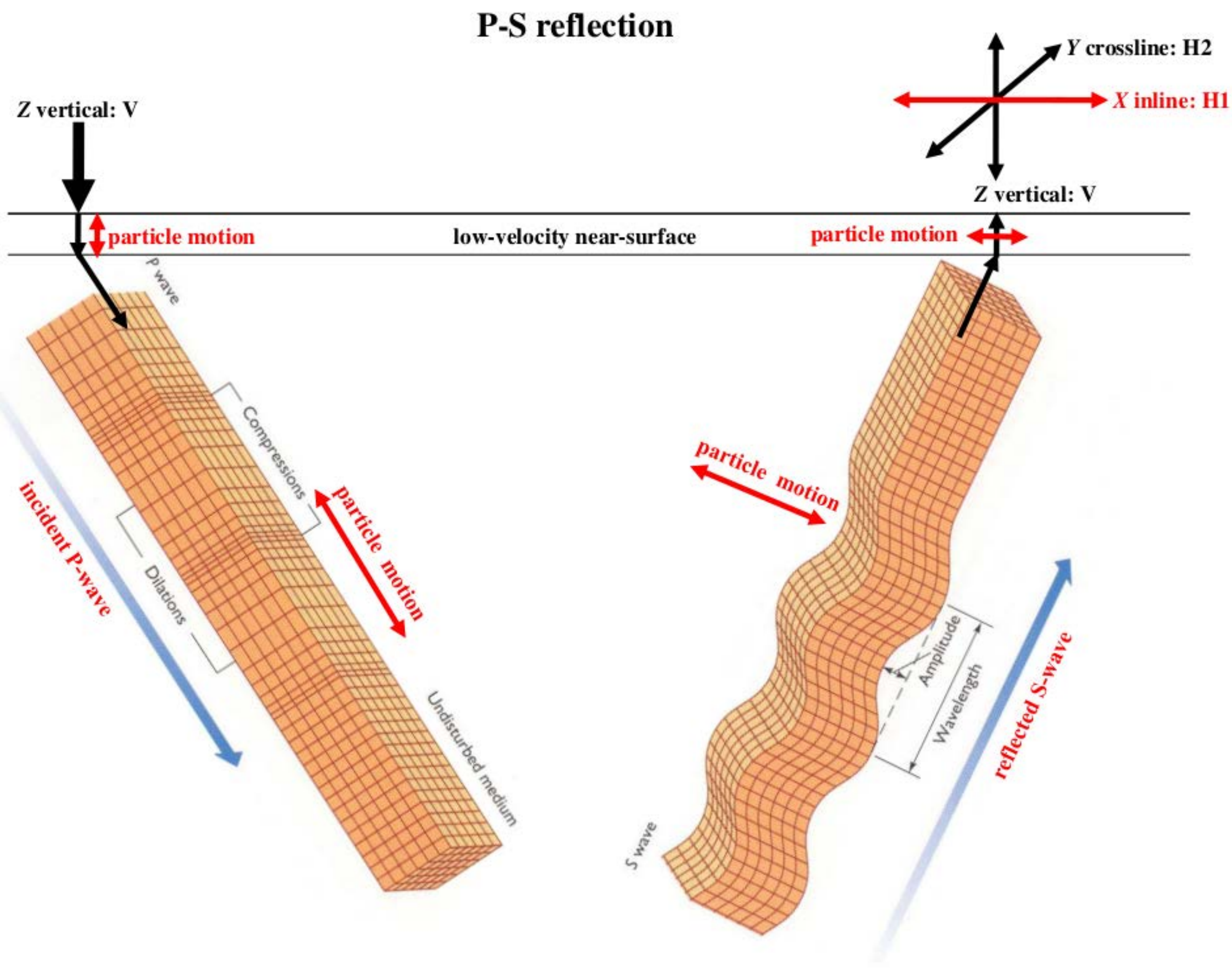
(Levander,1988)

Vx	Horizontal particle velocity
Vz	Vertical particle velocity
Sxx	Horizontal normal stress
Szz	Vertical normal stress
Sxz	Shear stress

Near surface mode filtering effect



A P-to-P reflection with an angle of incidence is recorded almost entirely by a vertical-component geophone.



A P-to-S reflection with an angle of incidence is recorded almost entirely by an inline horizontal-component geophone.

(From Yilmaz, 2018)

Imaging conditions without mode separation

$$\rho \frac{\partial v_x}{\partial t} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z}$$
$$\rho \frac{\partial v_z}{\partial t} = \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{xz}}{\partial x}$$
$$\frac{\partial \sigma_{xx}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z} + f_{\sigma 1}$$
$$\frac{\partial \sigma_{zz}}{\partial t} = (\lambda + 2\mu)\frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x} + f_{\sigma 2}$$
$$\frac{\partial \sigma_{xz}}{\partial t} = \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$$

(Levander,1988)

Modes	Forward	Backward	Concern
PP	Vz	Vz	Mode crosstalk
PP	Sxx+Szz	Sxx+Szz	?
PP	Div(Vx,Vz)	Div(Vx,Vz)	Amp. and phase changes
PS	Vz	Vx	Mode crosstalk & polarity rev.
PS	Div(Vx,Vz)	$Curl(Vx,0,Vz)_2$	Amp. and phase changes, polarity reversal & no straightforward 3D

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FD with mode separation

$$v_x = \frac{\partial u}{\partial t} \qquad v_z = \frac{\partial w}{\partial t}$$

$$v_x = v_{px} + v_{sx} \qquad v_z = v_{pz} + v_{sz}$$

$$\frac{\partial v_{px}}{\partial t} = \alpha^2 \frac{\partial A}{\partial x} \qquad \frac{\partial v_{pz}}{\partial t} = \alpha^2 \frac{\partial A}{\partial z}$$

$$\frac{\partial v_{sp}}{\partial t} = \beta^2 \frac{\partial B}{\partial z} \qquad \frac{\partial v_{sz}}{\partial t} = -\beta^2 \frac{\partial B}{\partial x}$$

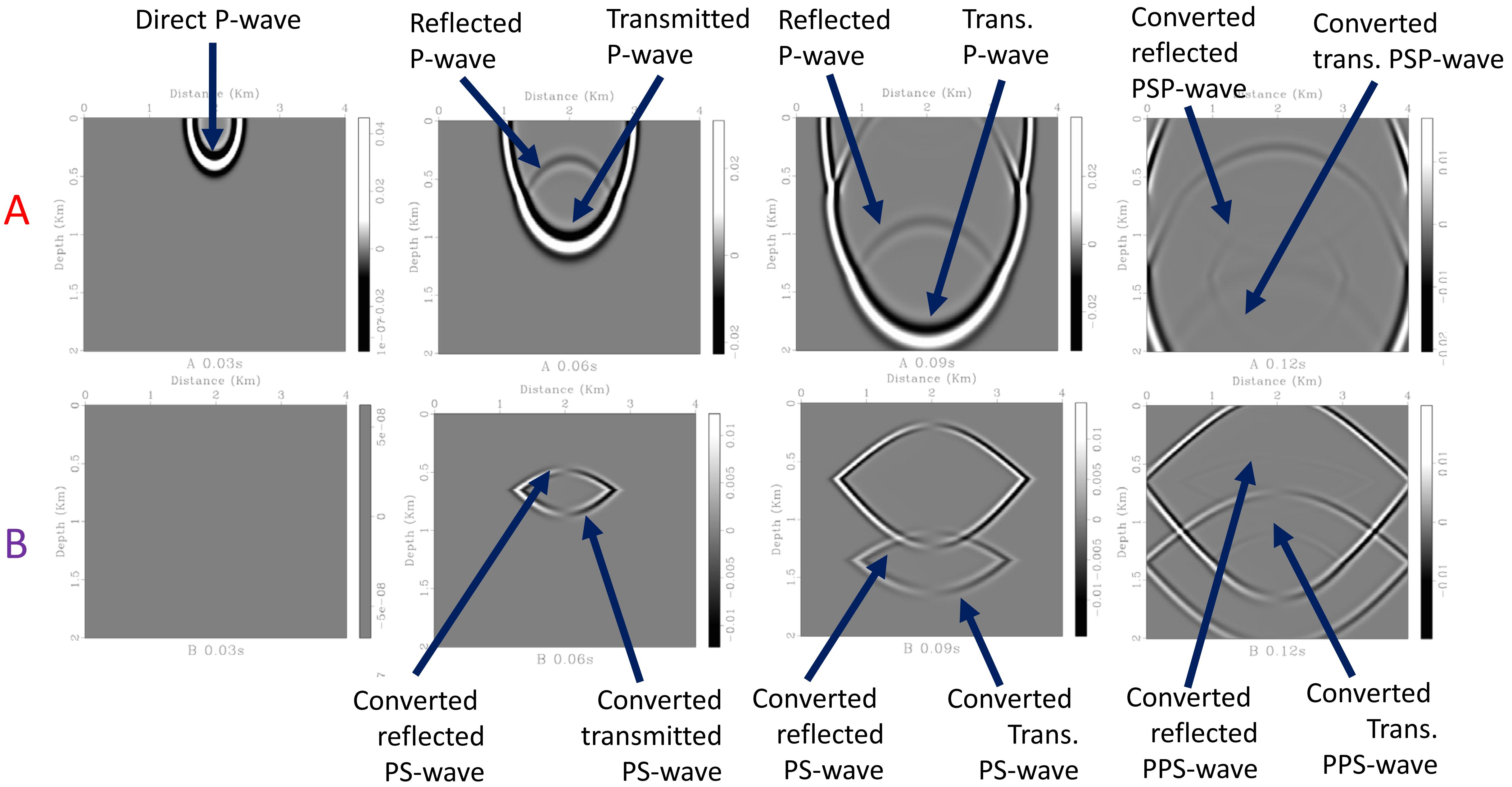
$$A = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \qquad B = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial z}$$

$$\frac{\partial A}{\partial t} = \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} + f_A \qquad \frac{\partial B}{\partial t} = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} + f_B$$

(Chen,2014)

Vpx	**Horizontal P-wave particle velocity**
Vpz	**Vertical P-wave particle velocity**
Vsx	**Horizontal S-wave particle velocity**
Vsz	**Vertical S-wave particle velocity**
A	**Displacement divergence**
B	**Displacement curl***

FD with mode separation



Imaging conditions with mode separation

$$v_x = \frac{\partial u}{\partial t} \qquad v_z = \frac{\partial w}{\partial t}$$

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(Chen,2014)

Mode	Forward	Backward	Concern
PP	(Vpx,Vpz)	(Vpx,Vpz)	Polarity reversal
PP*	**2α(Dx(A),Dz(A))**	**(Vpx,Vpz)**	
PS	A	B	Polarity reversal
PS	(Vpx,Vpz)	(Vsx,Vsz)	
PS*	**2β(Dx(B),-Dz(B))**	**(Vsx,Vsz)**	

*Obtained using adjoint state method

Adjoint state method

$$v_x = \frac{\partial u}{\partial t} \qquad v_z = \frac{\partial w}{\partial t}$$

$$v_x = v_{px} + v_{sx} \qquad v_z = v_{pz} + v_{sz}$$

$$\frac{\partial v_{px}}{\partial t} = \alpha^2 \frac{\partial A}{\partial x} \qquad \frac{\partial v_{pz}}{\partial t} = \alpha^2 \frac{\partial A}{\partial z}$$

$$\frac{\partial v_{sp}}{\partial t} = \beta^2 \frac{\partial B}{\partial z} \qquad \frac{\partial v_{sz}}{\partial t} = -\beta^2 \frac{\partial B}{\partial x}$$

(Chen,2014)

$$A = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \qquad B = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial z}$$

$$\frac{\partial A}{\partial t} = \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} + f_A \qquad \frac{\partial B}{\partial t} = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} + f_B$$

$$\begin{pmatrix} \frac{\partial}{\partial t} & 0 & 0 & 0 & -\alpha^2\frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial t} & 0 & 0 & -\alpha^2\frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial t} & 0 & 0 & -\beta^2\frac{\partial}{\partial z} \\ 0 & 0 & 0 & \frac{\partial}{\partial t} & 0 & \beta^2\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial x} & -\frac{\partial}{\partial z} & -\frac{\partial}{\partial x} & -\frac{\partial}{\partial z} & \frac{\partial}{\partial t} & 0 \\ -\frac{\partial}{\partial z} & \frac{\partial}{\partial x} & -\frac{\partial}{\partial z} & \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial t} \end{pmatrix} \begin{pmatrix} v_{px} \\ v_{pz} \\ v_{sx} \\ v_{sz} \\ A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ f_A \\ f_B \end{pmatrix}$$

$$\boldsymbol{Sw} = \boldsymbol{F}$$

Adjoint state
method gradient

$$\frac{\partial \epsilon}{\partial \boldsymbol{m}} = -\langle \frac{\partial \boldsymbol{S}}{\partial \boldsymbol{m}} \boldsymbol{w}, \boldsymbol{w}^* \rangle$$

(Feng and Schuster 2014)

$$\frac{\partial \epsilon}{\partial \alpha} = -\langle \frac{\partial}{\partial \alpha} \begin{pmatrix} \frac{\partial}{\partial t} & 0 & 0 & 0 & -\alpha^2 \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial t} & 0 & 0 & -\alpha^2 \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial t} & 0 & 0 & -\beta^2 \frac{\partial}{\partial z} \\ 0 & 0 & 0 & \frac{\partial}{\partial t} & 0 & \beta^2 \frac{\partial}{\partial x} \\ -\frac{\partial}{\partial x} & -\frac{\partial}{\partial z} & -\frac{\partial}{\partial x} & -\frac{\partial}{\partial z} & \frac{\partial}{\partial t} & 0 \\ -\frac{\partial}{\partial z} & \frac{\partial}{\partial x} & -\frac{\partial}{\partial z} & \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial t} \end{pmatrix} \begin{pmatrix} v_{px} \\ v_{pz} \\ v_{sx} \\ v_{sz} \\ A \\ B \end{pmatrix}, \begin{pmatrix} \hat{v}_{px} \\ \hat{v}_{pz} \\ \hat{v}_{sx} \\ \hat{v}_{sz} \\ \hat{A} \\ \hat{B} \end{pmatrix} \rangle$$

$$= 2\alpha \left(\frac{\partial A}{\partial x} \hat{v}_{px} + \frac{\partial A}{\partial z} \hat{v}_{pz} \right)$$

$$\frac{\partial \epsilon}{\partial \beta} = 2\beta \left(\frac{\partial B}{\partial z} \hat{v}_{sx} - \frac{\partial B}{\partial x} \hat{v}_{sz} \right)$$

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1. Elastic RTM Introduction.
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3. Pure wave modes RTM imaging conditions.
4. **Computational complexity.**
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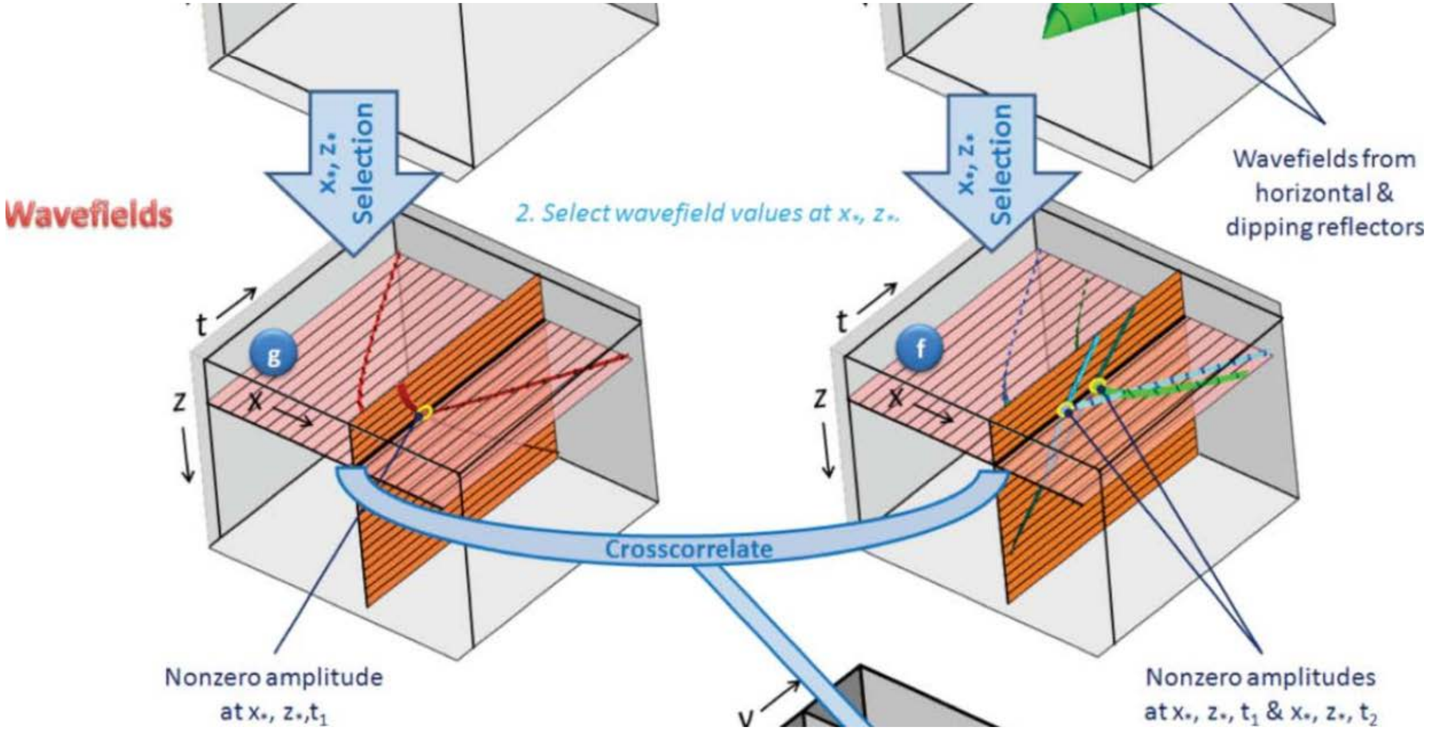
Computational complexity

For each shot gather;
for $s \leftarrow 1$ **to** N_s **do**
Do a complete forward propagation;
$\boldsymbol{F} \leftarrow \texttt{FD}(\boldsymbol{m}, s)$;
;
Reverse time loop;
for $t \leftarrow t_{max}$ **to** 0 **do**
Do one step backwards using shot gathers as sources;
$\boldsymbol{B} \leftarrow \texttt{OneStepFD}(\boldsymbol{m}, S(:,:,t))$;
Apply imaging conditions at current time;
for *each type of imaging condition* **do**
$I_w(:,:) \leftarrow I_w(:,:) + \texttt{ImagCond}_w(\boldsymbol{F}, \boldsymbol{B})$
end
end
end

For one shot only

Nt: Time steps
Nw: Number of wavefields
NxNz: Model size

Operations Θ(Nt **Nw** Nx Nz)
Storage Θ(Nt **Nw** (Nx + Nz)) Saving only borders



(Sava and Hill, 2009)

Computational complexity

```
For each shot gather;
for s ← 1 to N_s do
    Do a complete forward propagation;
    F ← FD(m,s);
    ;
    Reverse time loop;
    for t ← t_max to 0 do
        Do one step backwards using shot gathers
        as sources;
        B ← OneStepFD(m,S(:,:,t));
        Apply imaging conditions at current time;
        for each type of imaging condition do
            I_w(:,:) ← I_w(:,:) + ImagCond_w(F,B)
        end
    end
end
```

For one shot only

Nt: Time steps

Nw: Number of wavefields

NxNz: Model size

Operations Θ(2 Nt **Nw** Nx Nz)

Storage Θ(**Nw** Nx Nz) + Θ(Nt **Nw** (Nx + Nz))

Using a staggered grid implementation with PML

Case	Nw	Wavefields
Acoustic	4(3)	P**h**, P**v**, V**h**, V**v**
Non pure mode elastic	10(5)	Vx**h**,Vx**v**,Vz**h**,Vz**v**, Sxx**h**,Sxx**v**,Szz**h**,Szz**v**,Sxz**h**,Sxz**v**
Pure mode elastic	8(6)	Vpx,Vpz,Vsx,Vsz, A**h**,A**v**,B**h**,B**v**

Computational complexity

```
For each shot gather;
for s ← 1 to N_s do
    Do a complete forward propagation;
    F ← FD(m,s);
    ;
    Reverse time loop;
    for t ← t_max to 0 do
        Do one step backwards using shot gathers
        as sources;
        B ← OneStepFD(m,S(:,:,t));
        Apply imaging conditions at current time;
        for each type of imaging condition do
            I_w(:,:) ← I_w(:,:) + ImagCond_w(F,B)
        end
    end
end
```

+: sum

x: multiplication

*:uses extrabuffers

Imaging conditions

Forward	Backward	Operations per point per timestep
Vz	Vz	0+,1x
Vz	Vx	
A	B	
Sxx+Szz	Sxx+Szz	2+,1x
Div(Vx,Vz)*	Div(Vx,Vz)*	14+,17x
Div(Vx,Vz)*	Curl(Vx,0,Vz)*2	
(Vpx,Vpz)	(Vpx,Vpz)	1+,2x
(Vpx,Vpz)	(Vsx,Vsz)	
2α(Dx(A),Dz(A))*	**(Vpx,Vpz)**	7+,11x
2β(Dx(B),-Dz(B))*	**(Vsx,Vsz)**	

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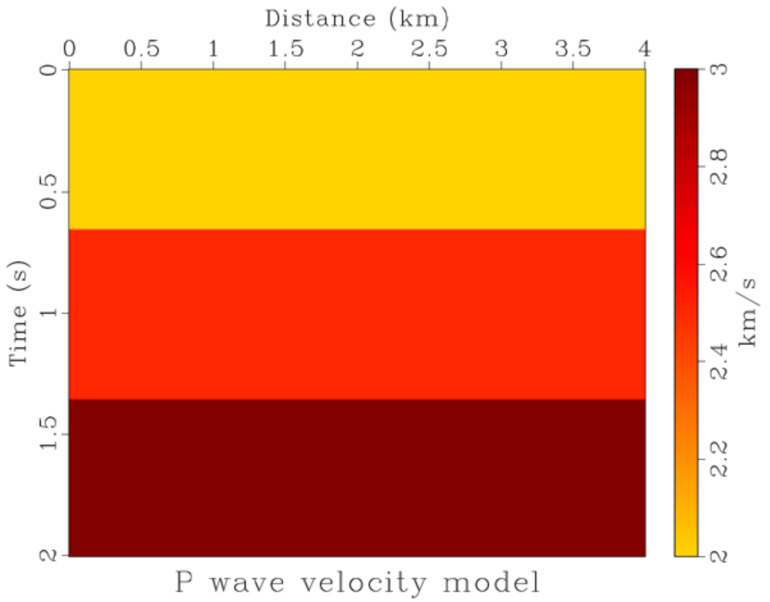
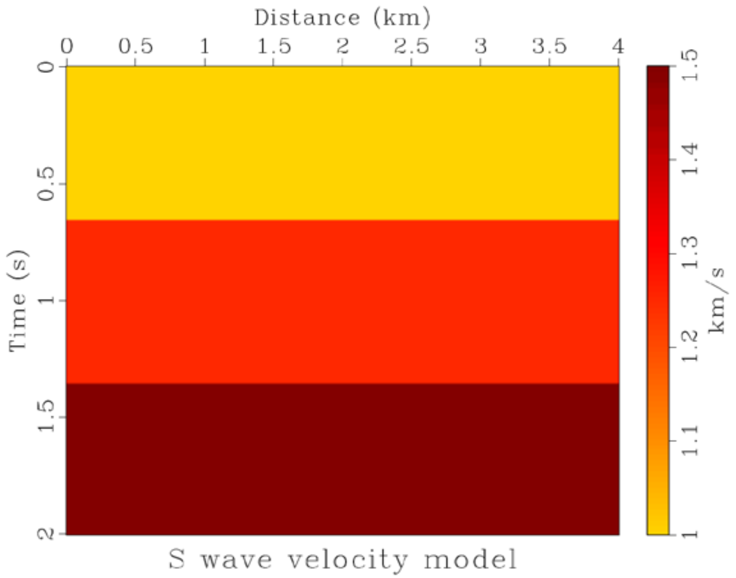
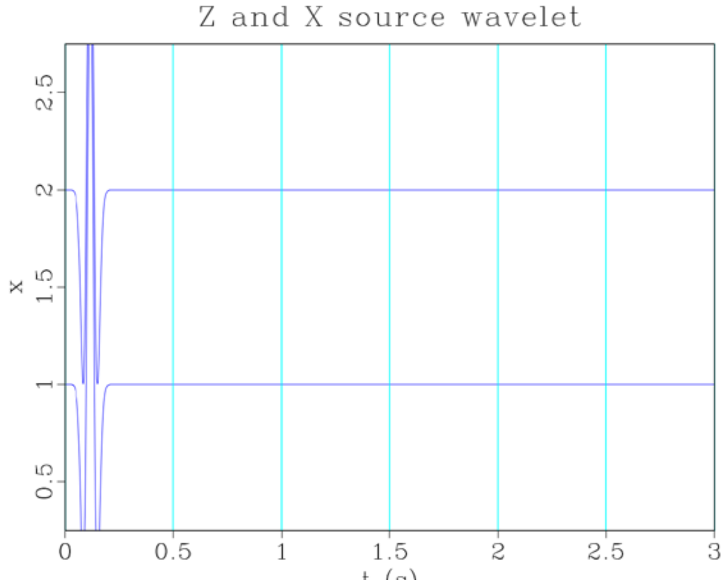
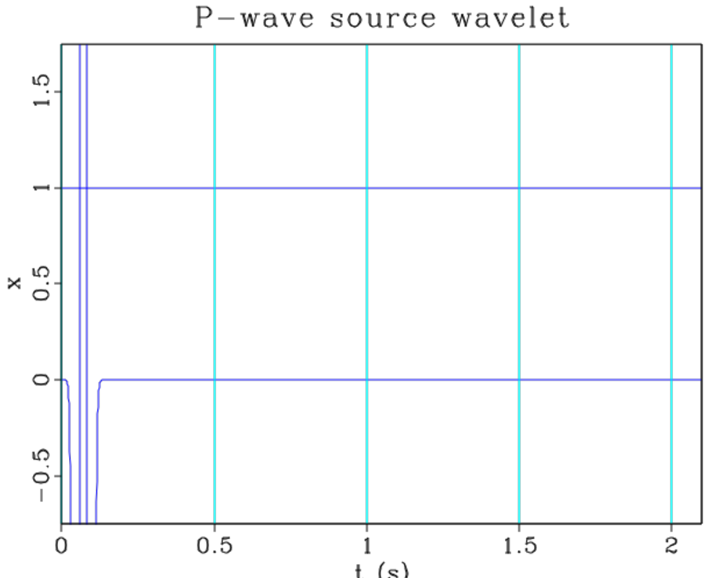
Numerical experiment 1: Models



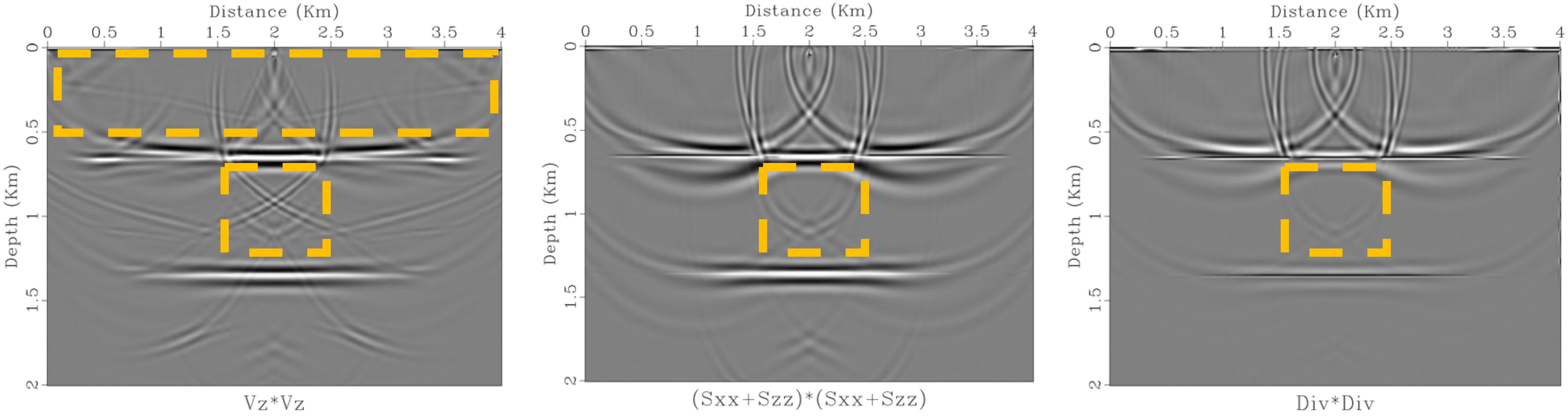




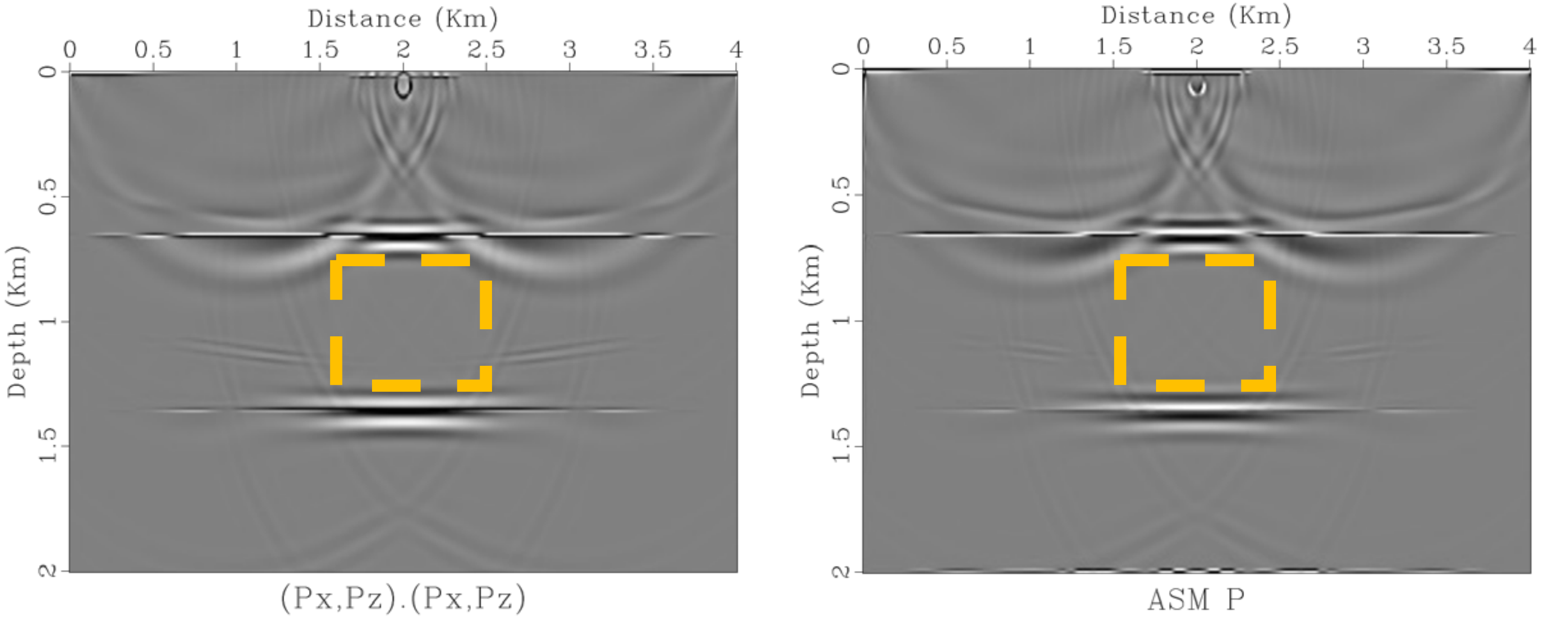


Numerical experiment 1: PP migrations

Non pure modes



Pure modes

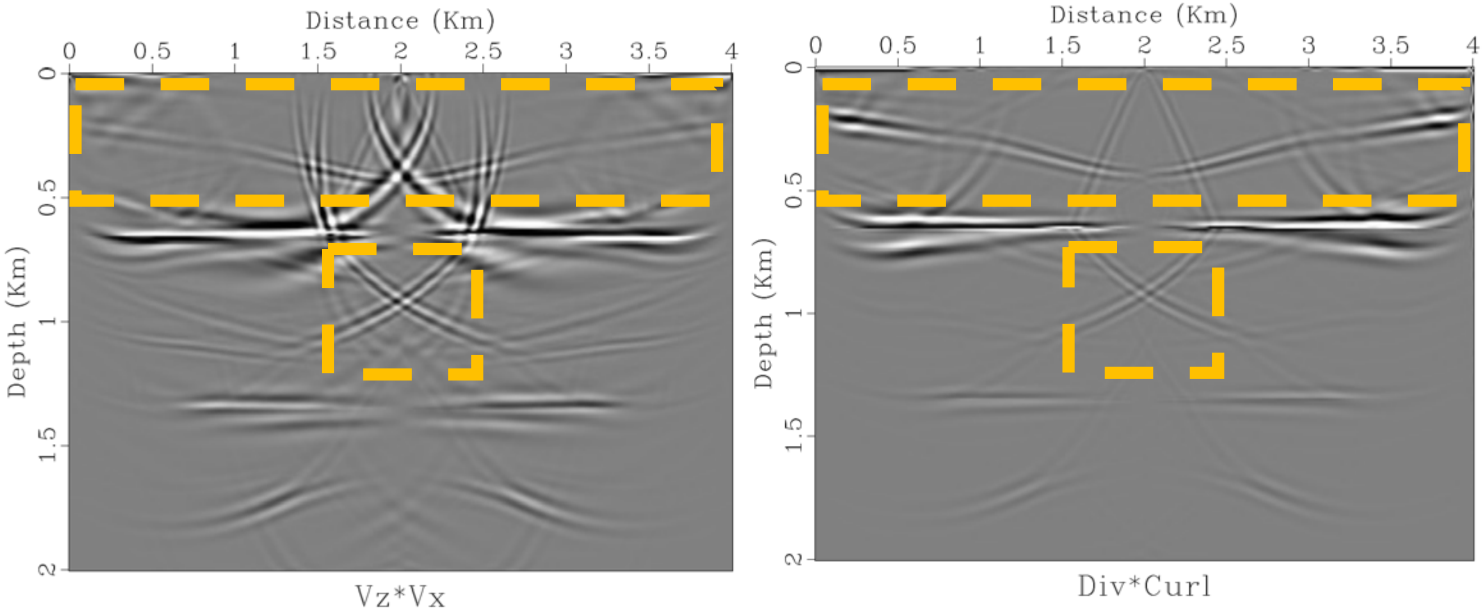


*Laplacian Applied

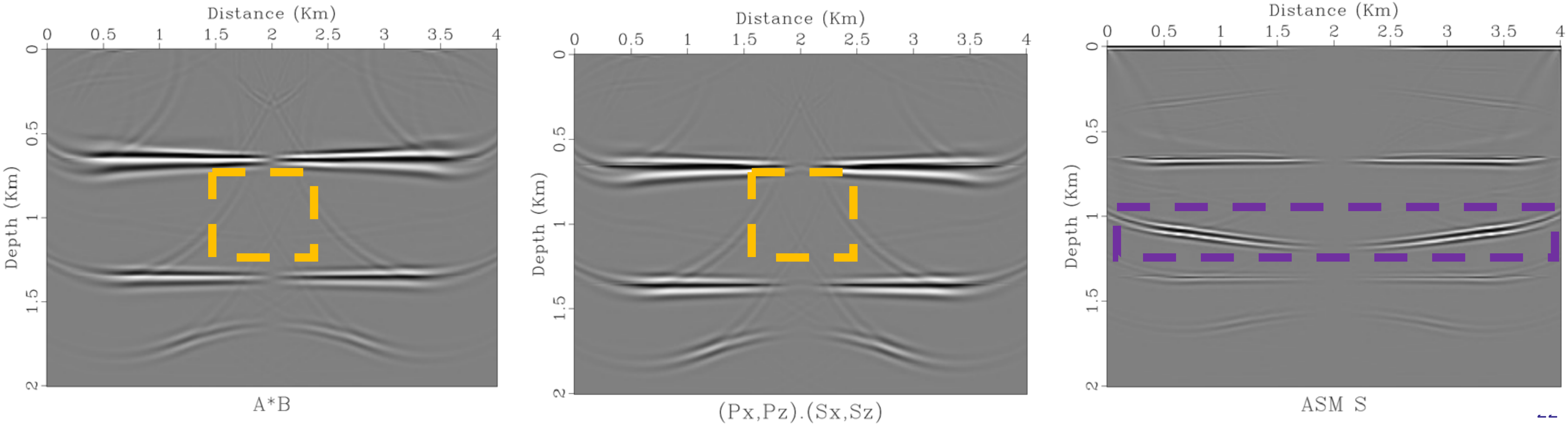
Numerical experiment 1: PS migrations

*Laplacian applied

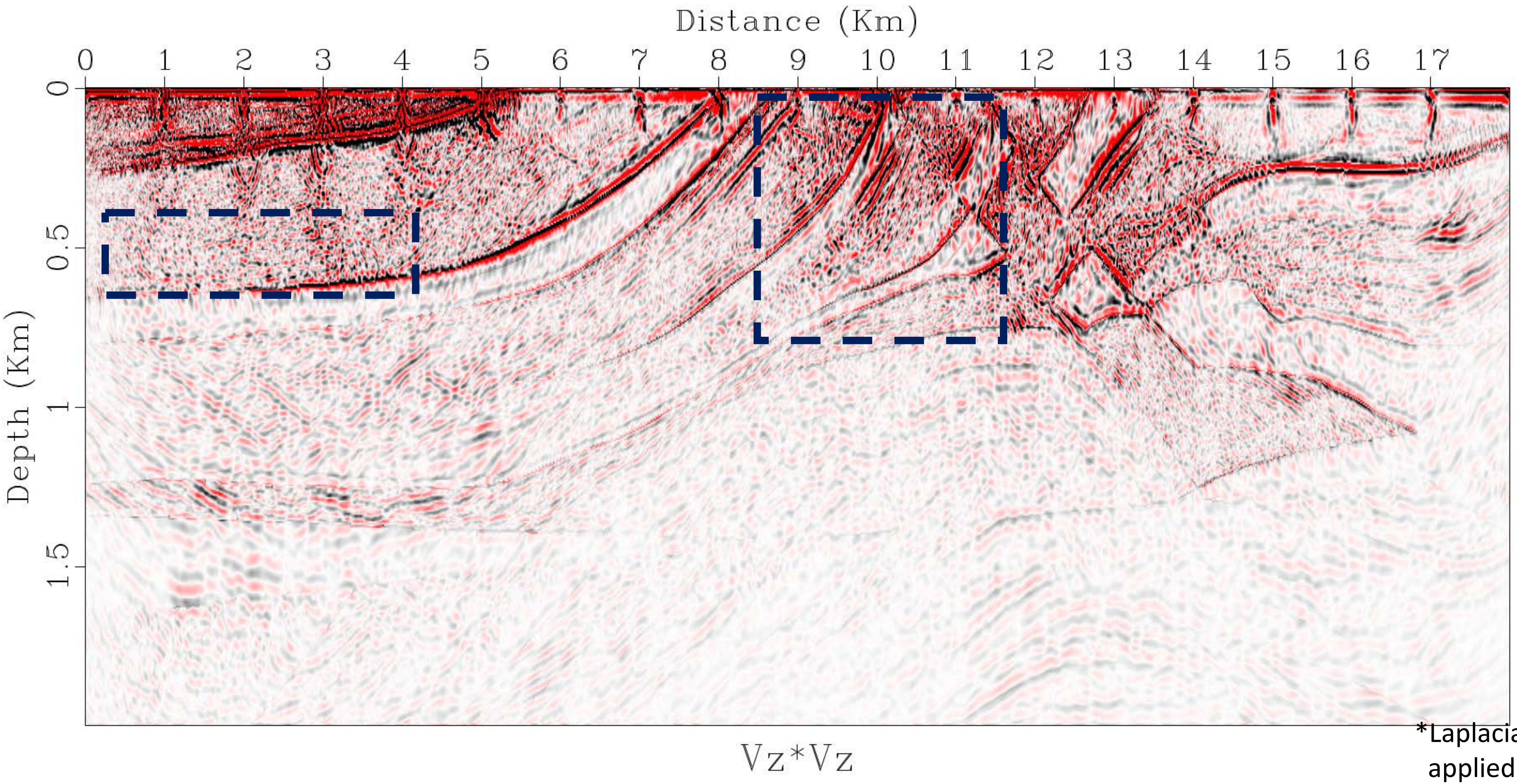
Non pure modes



Pure modes

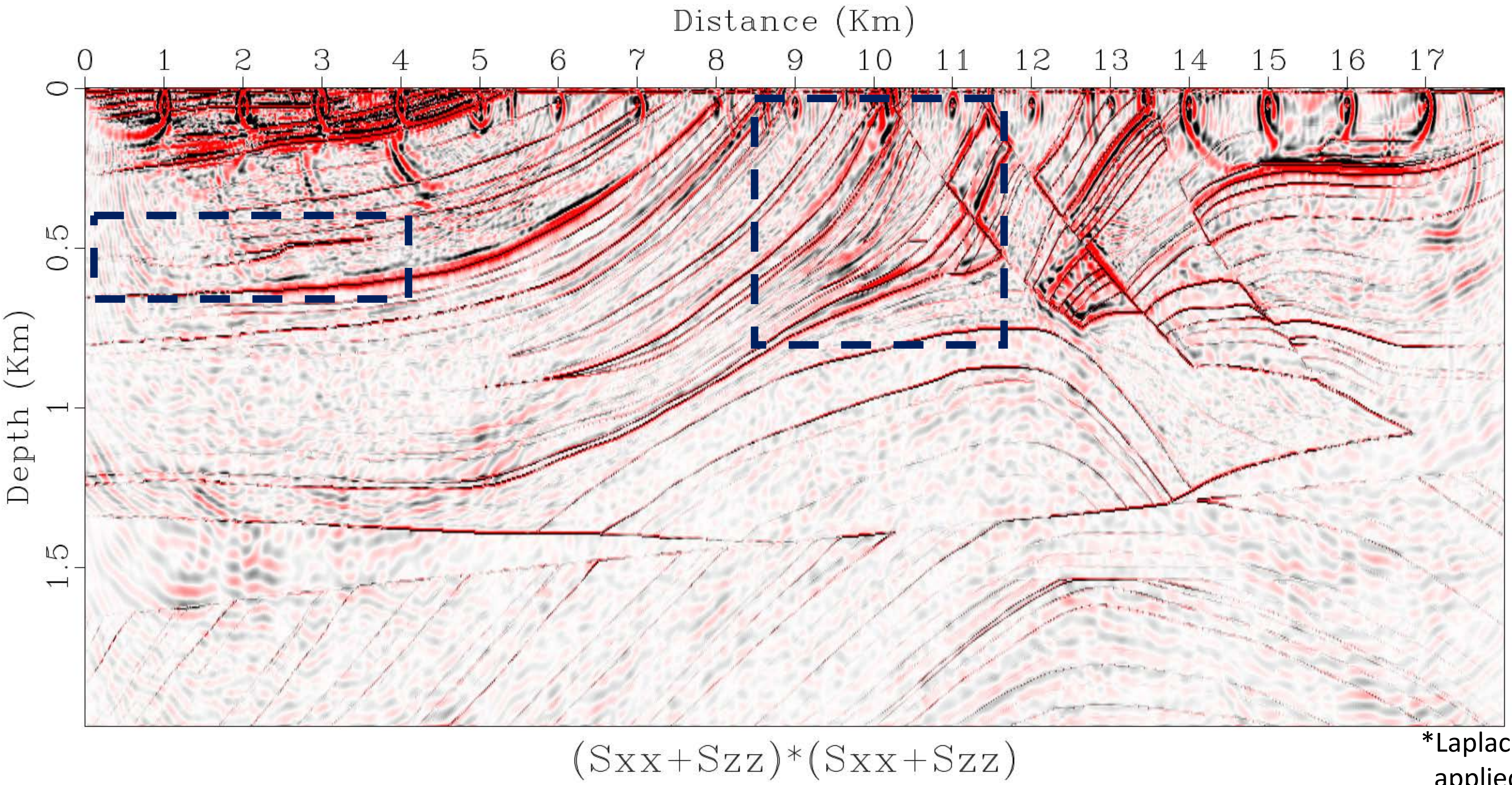


Numerical experiment 2: PP migration VzVz



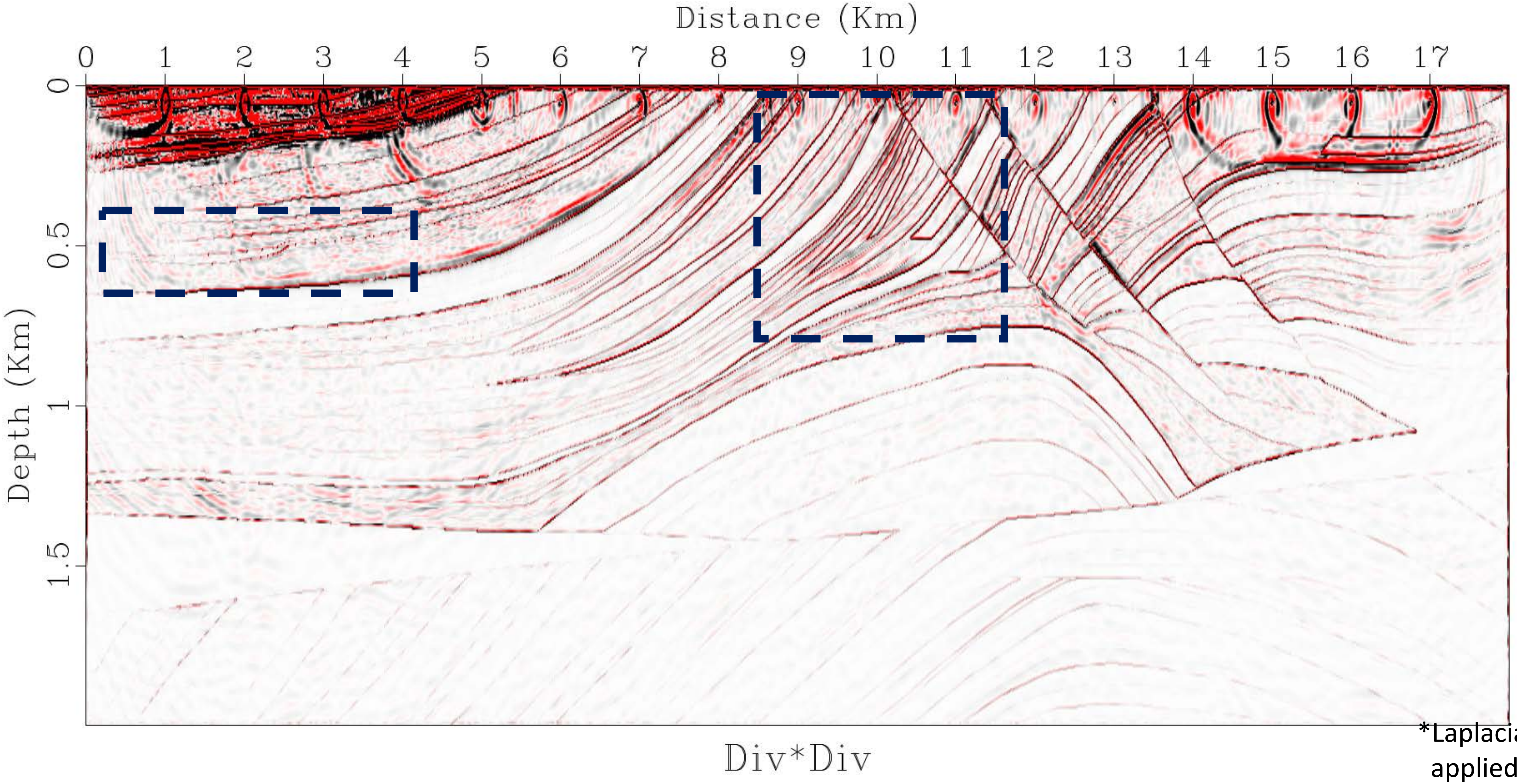
*Laplacian applied

Numerical experiment 2: PP migration (Sxx+Szz)(Sxx+Szz)



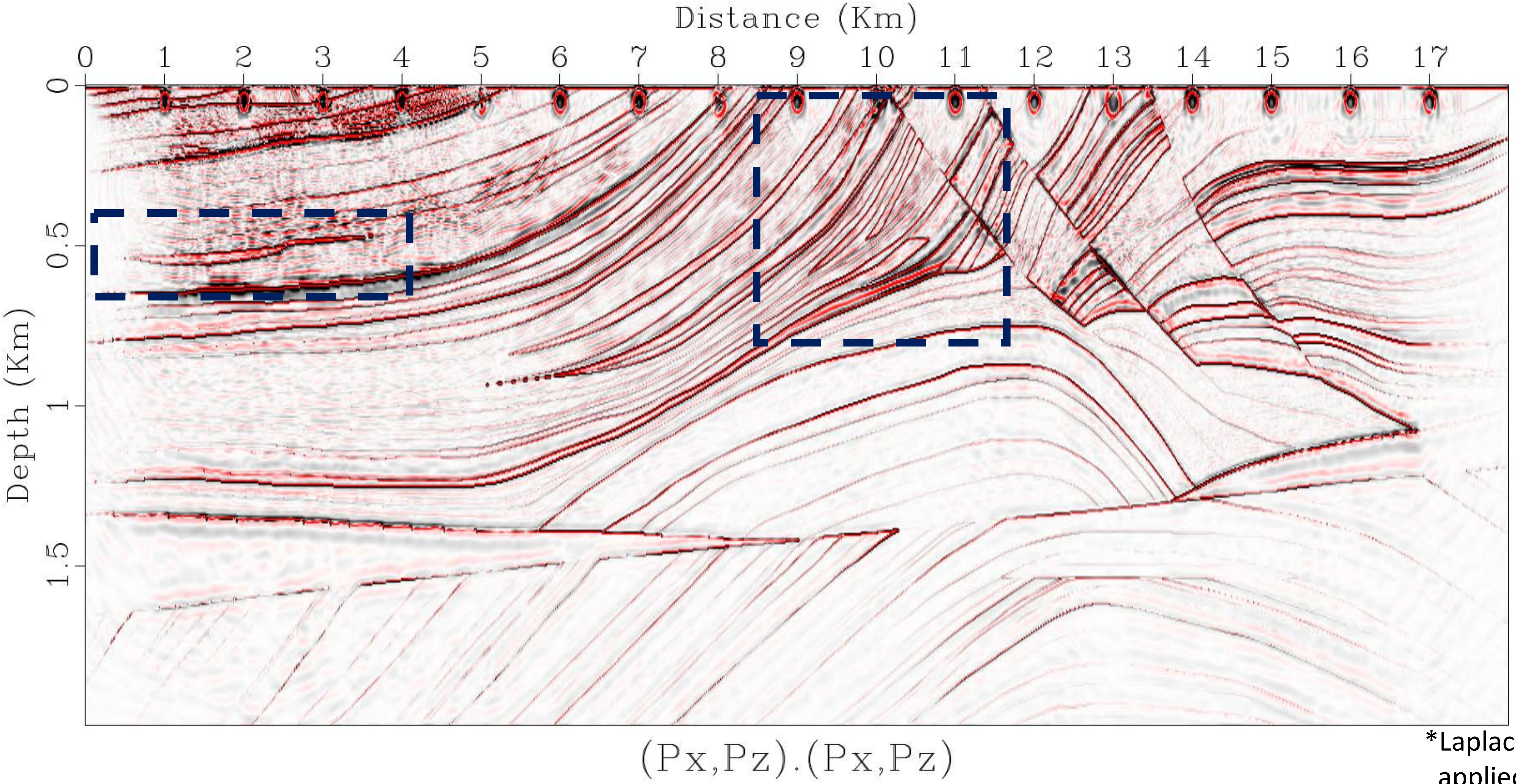
*Laplacian applied

Numerical experiment 2: PP migration div*div



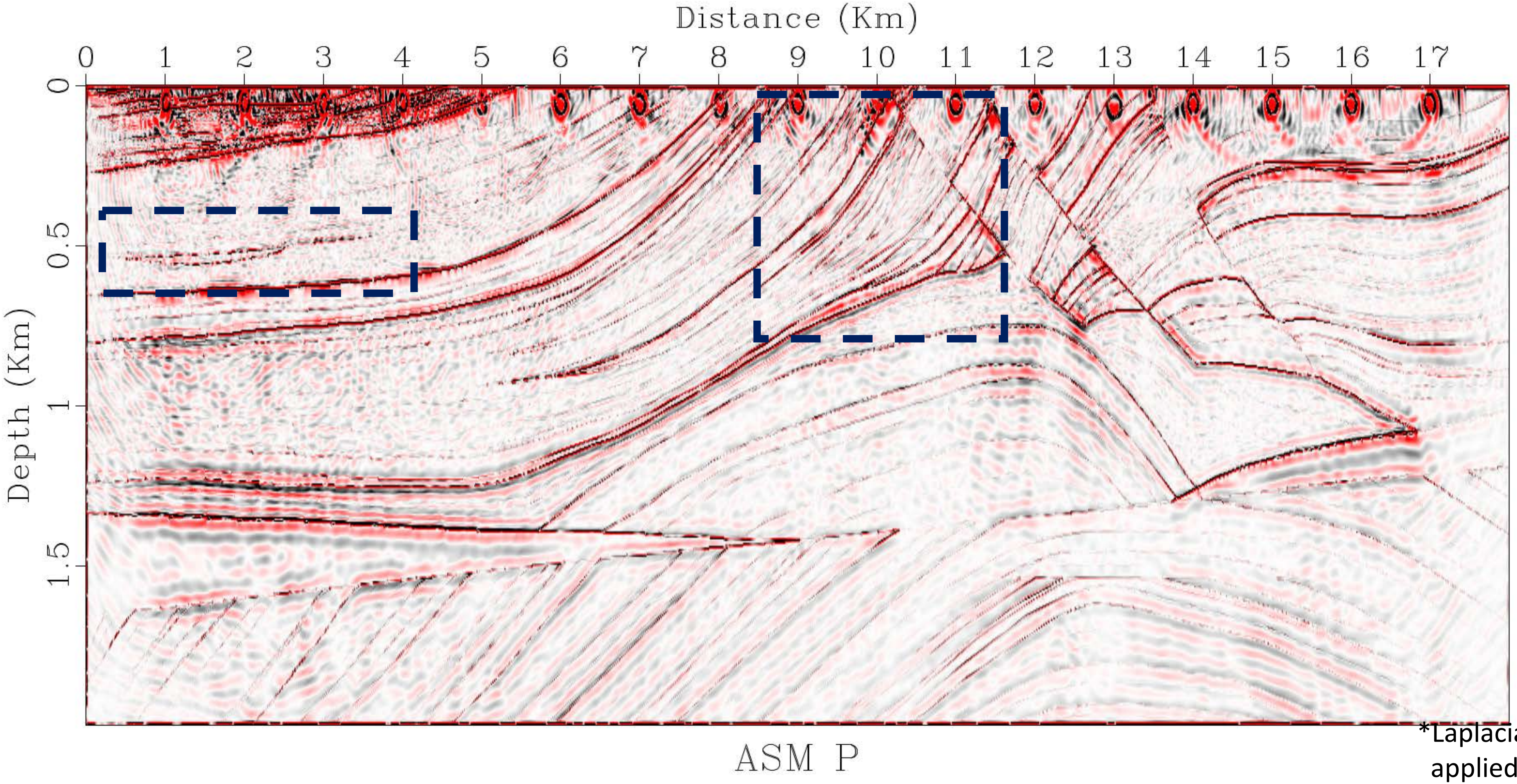
*Laplacian applied

Numerical experiment 2: PP migration (Vpx,Vpz)(Vpx,Vpz)



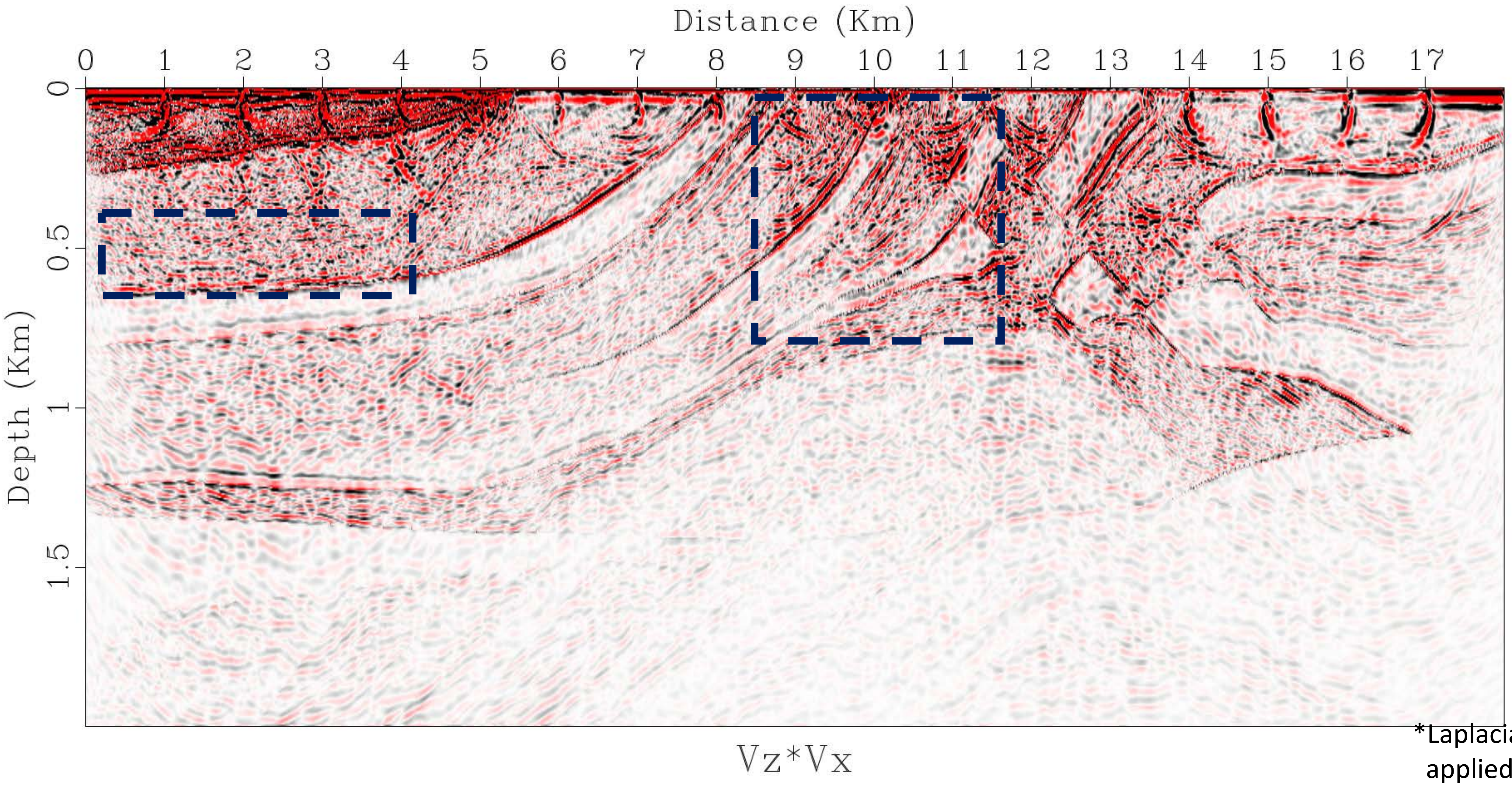
*Laplacian applied

Numerical experiment 2: PP migration ASM P



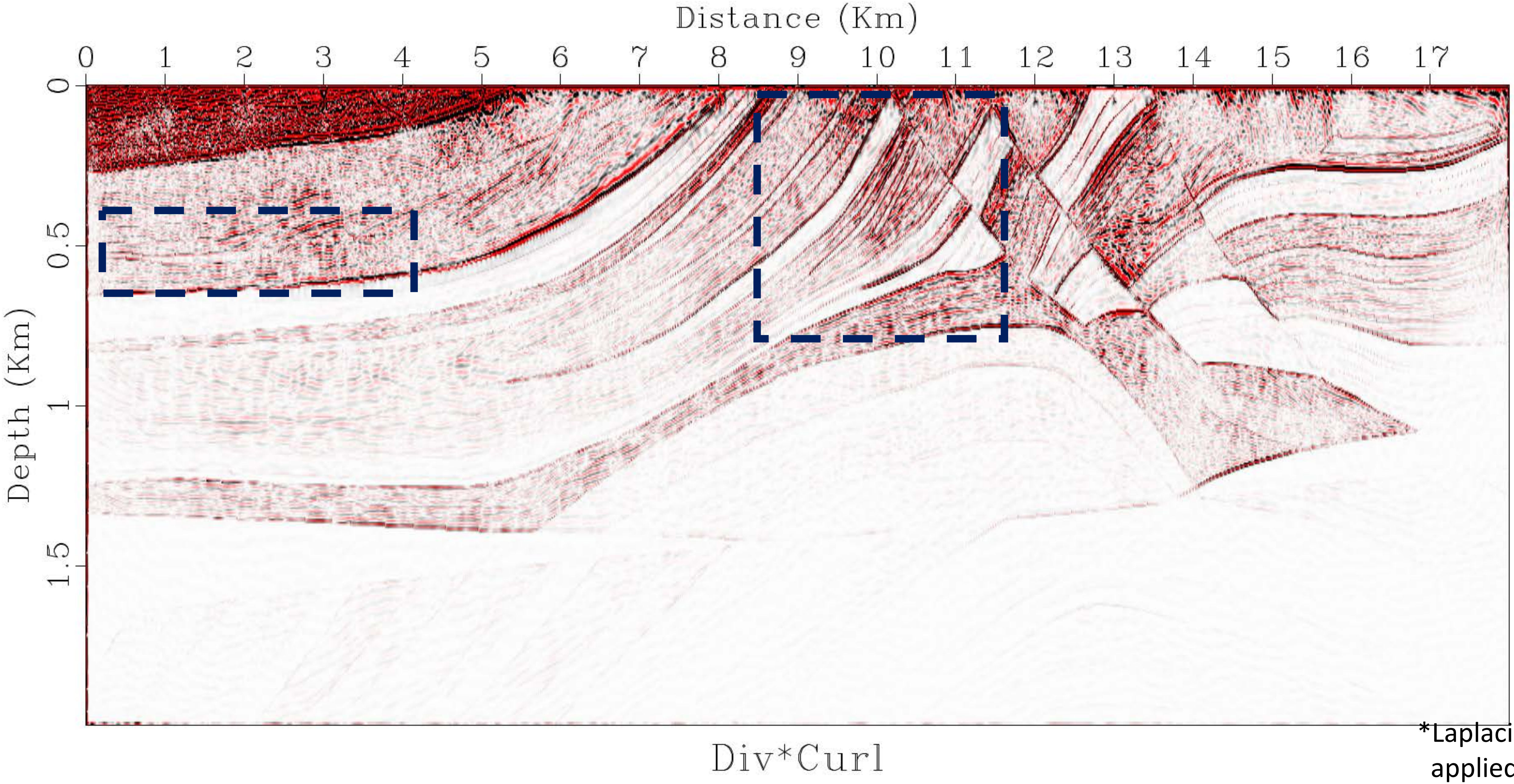
*Laplacian applied

Numerical experiment 2: PS migration VzVx



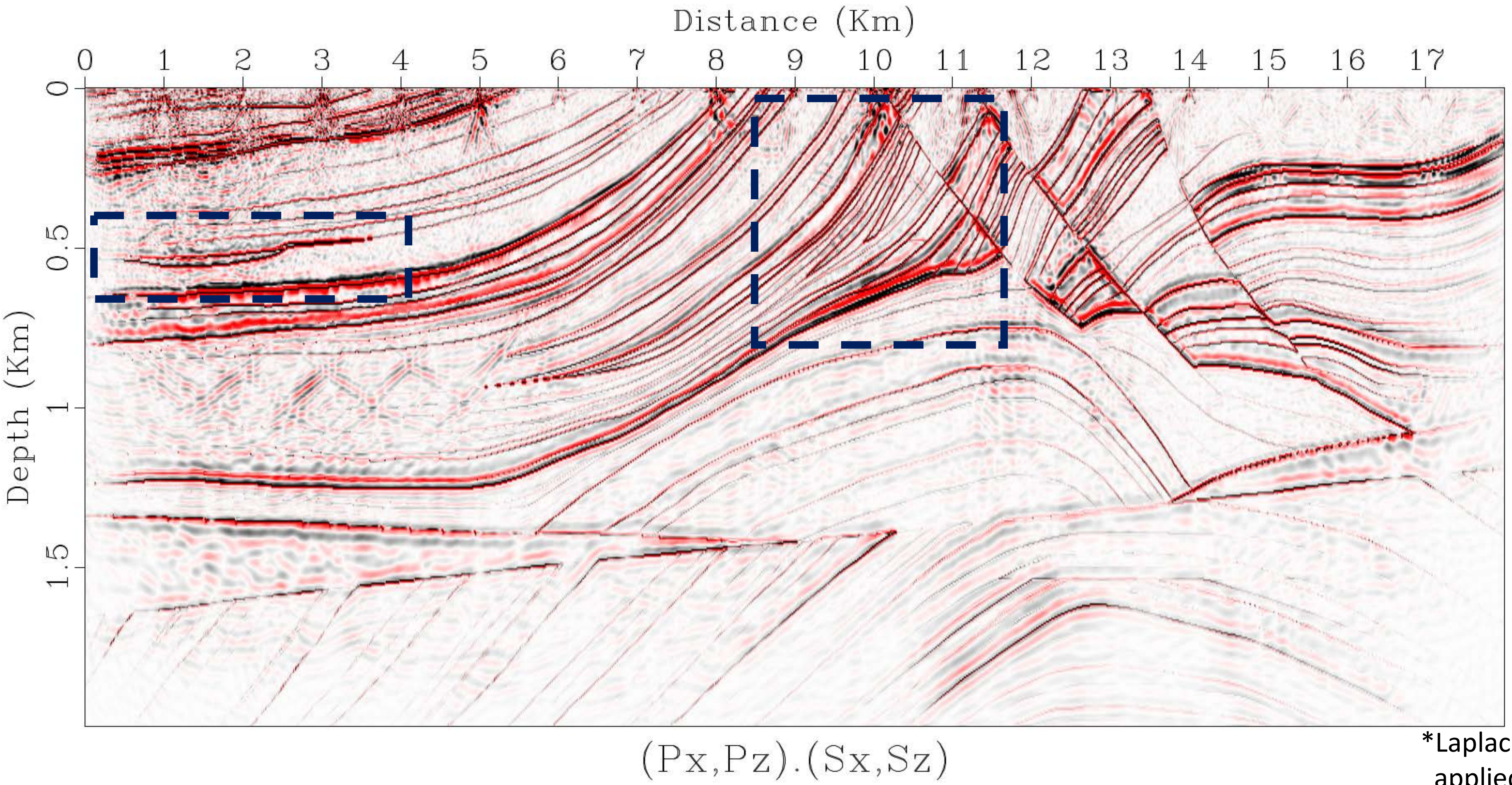
*Laplacian applied

Numerical experiment 2: PS migration div*curl



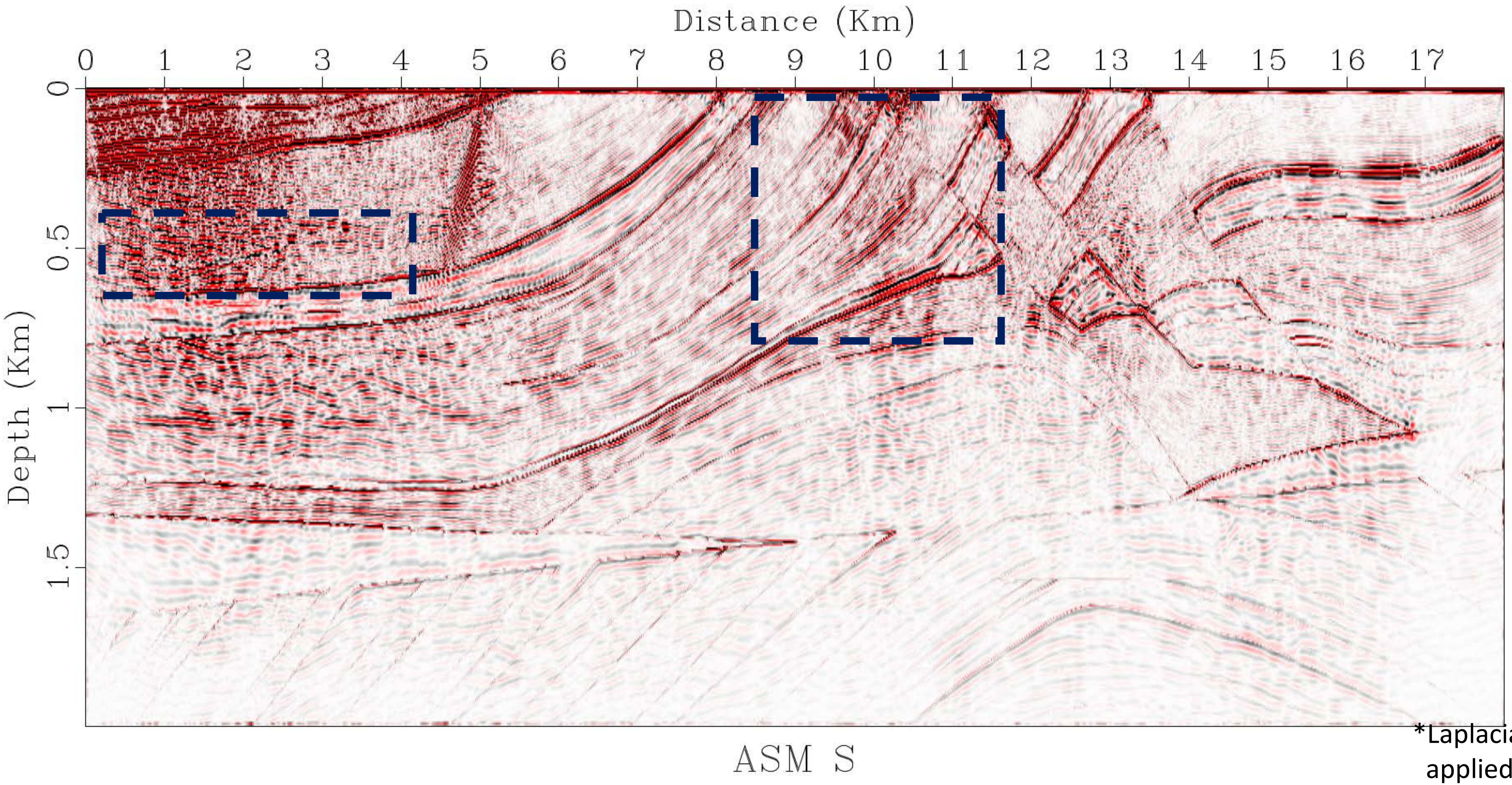
*Laplacian applied

Numerical experiment 2: PS migration (Vpx,Vpz)(Vsx,Vsz)



*Laplacian applied

Numerical experiment 2: PS migration ASM S



*Laplacian applied

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Conclusions

Regarding the pure P- and S-wave RTM imaging conditions, the PP and PS dot product generated the best images.

The adjoint state imaging conditions were second place. However, they performed much better than the classical non-pure modes imaging conditions and they do not suffer from PS polarity reversal.

Regarding the non-pure modes RTM imaging conditions, the sum of stresses and the divergence also produced very good migrated images. Although they are limited to only PP imaging.

Dot product and ASM imaging conditions are more expensive than cross-correlations but cheaper than Helmholtz imaging conditions.

Thanks

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RIPED, PetroChina

Saudi Aramco

Sinopec

TGS

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