Quantum computers in oil and gas exploration

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What is quantum computer? Why it is important?

Quantum computer works based on the laws of quantum mechanics such as superposition. For some specific problems it is **exponentially** faster than classical computers!!

"…Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical..."

Simulation of Chemistry

Richard Feynman

(**1982**)

Factoring large integers: given an n-digit integer N find prime numbers p and q such that $N = p \times q$.

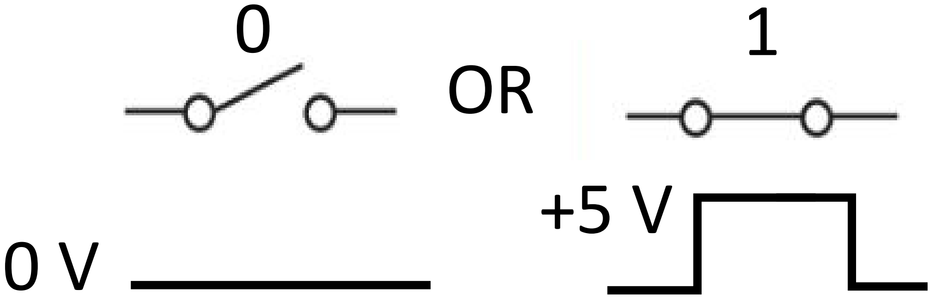
Breaking encryption system that widely used for secure data transmission

Peter Shor

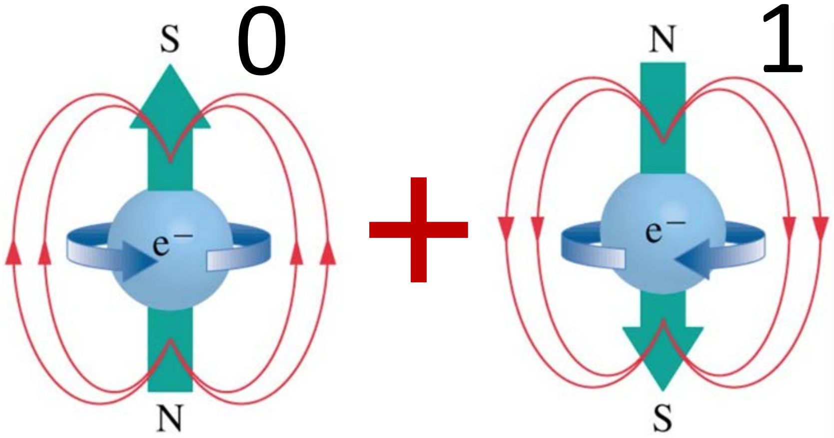
(1994)

What is quantum computer? Why it is important?

Classical binary digit: exists in either of two possible states



Quantum binary digit (qubit) exists in superposition of two possible states



Axiom 1: superposition

$$|\psi\rangle = \sqrt{0.2}|0\rangle + \sqrt{0.8}|1\rangle$$

Axiom 2: measurement

When we measure the qubit it collapses to either 0 or 1

The probability of outcome 0

$$\left|\sqrt{0.2}\right|^2 \times 100 = 20\%$$

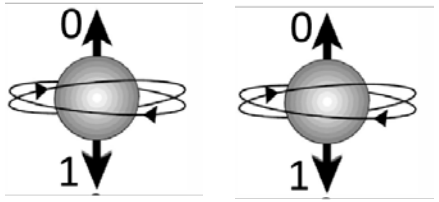
The probability of outcome 1

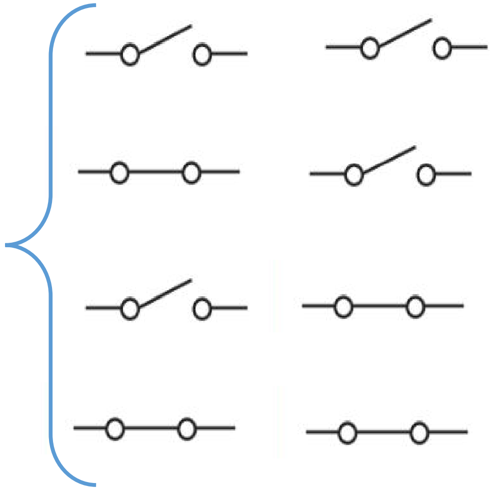
$$\left|\sqrt{0.8}\right|^2 \times 100 = 80\%$$

What is quantum computer? Why it is important?

Adding one qubit doubles the power of computation!!!

2-qubit





00 10%

10 20%

01 40%

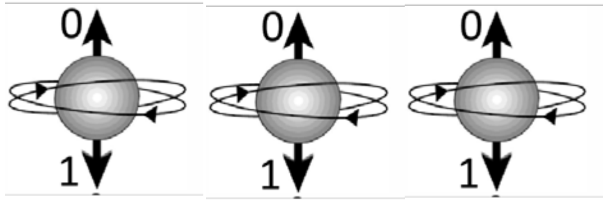
11 30%

$2 \times 2 \sim 4$

possible states that 2-qubit might be in

$$|\psi\rangle = \sqrt{0.1}|00\rangle + \sqrt{0.4}|10\rangle + \sqrt{0.3}|01\rangle + \sqrt{0.2}|11\rangle$$

3-qubit



$2 \times 2 \times 2 \sim 8$

possible states that 3-qubit might be in

72-qubit $2^{72} \sim 10^{36} = 1000000000000000000000000000000000000$

Exponential grow!!

What is quantum computer? Why it is important?

NISQ era

Noisy Intermediate-Scale Quantum

(50- to few-hundred-qubits)

Google 72 qubit

IBM 50 qubit

Near term applications (up to 5 years)

- Material design
- Oil and gas
- Carbon dioxide capture
- PDE's with machine leaning
- Drug discovery

Universal Quantum Computer
(Fault tolerant quantum computation)

(Millions of qubits are required!!)

Long term applications (>10)

- Breaking crypto systems,
- PDEs, Regression, Machine Learning,
- Seismic modeling/inversion

Application for seismic wave modeling

Quantum algorithm for linear systems of equations (Harrow, 2009)

$$A\,p = s \qquad A \text{ is } 2^n \times 2^n \text{ Matrix}$$

1. Initial state preparation

Load the 2^n-dimensional vector $s = (s_1, \dots s_{2^n})$ into quantum computer's memory with $\boldsymbol{n}$ qubits (axiom 1: superposition)

$$|s\rangle = s_1|000\dots0\rangle + s_2|001\dots0\rangle + s_3|001\dots0\rangle + \cdots + s_{2^n}|111\dots111\rangle$$

2. Matrix inversion: $|p\rangle = A^{-1}|s\rangle$ With $O(n)$ numbers of operations (best classical algorithms solve this with $O(2^n)$ number of operations Exponential speed-up!!

3. Quantum Solution: solution is a quantum state $|p\rangle$ rather than p= $(p_1, \dots p_{2^n})$ (axiom 1: superposition)

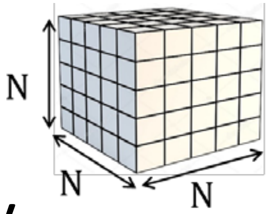
$$|p\rangle = p_1|000\dots0\rangle + p_2|001\dots0\rangle + p_3|001\dots0\rangle + \cdots + p_{2^n}|111\dots111\rangle$$

By measuring the quantum solution you get only one output (axiom 2: measurement)

Application for seismic wave modeling

How running time scales with different parameters?

$$\left(-\nabla^2 - \frac{\omega^2}{c^2(\mathbf{x})}\right) p(\mathbf{x}, \omega) = s(\mathbf{x}, \omega) \xrightarrow{\text{Discretization}} A_{N \times N}\, P = S$$



highly sparse
ill-posed

Runtime complexity

$$\underbrace{\hspace{10em}}$$

Classical algorithm $O(s\kappa n)$		
1D	$O(N^3)$	$n = N$
2D	$O(N^4)$	$n = N^2$
3D	$O(N^5)$	$n = N^3$

Quantum algorithm $O(s^2\kappa \log n)$	
1D	$O(N^2 \log N)$
2D	$O(N^2 \log N)$
3D	$O(N^2 \log N)$

n is the dimension of the system

s is the sparsity of the matrix A

$N^2 = \kappa$ is the condition number of matrix A, the ratio of its largest to smallest eigenvalue, i.e. $\kappa = \lambda_{max}/\lambda_{min}$

Application for seismic wave modeling

Quantum Fourier Transform for seismic wave modeling

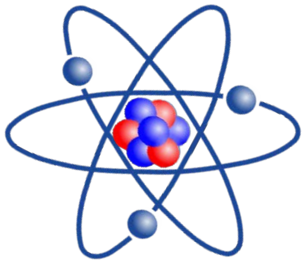
Classical algorithm (Y Zhang, 2009)

$$\left(\frac{\partial}{\partial t} + i\underset{\downarrow\ \sqrt{-\nabla^2}}{\mathrm{H}}\right)\hat{\mathrm{P}}(\mathbf{x}, \mathrm{t}) = 0$$

$$\underset{\downarrow\ \text{Number of time steps=N}}{\mathrm{N_t}}\overbrace{\mathrm{O}(\mathrm{N}^3\log\mathrm{N})}^{\text{FFT}} \sim \mathbf{O}\left(\mathbf{N^4 \log N}\right)$$

FFT on $n = \mathrm{N}^3$ elements costs $\mathrm{O}(\mathrm{n}\log\mathrm{n})$

Quantum algorithm



Schrödinger equation

$$i\frac{\partial}{\partial t}\hat{\mathrm{P}}(\mathbf{x}, \mathrm{t}) = \mathrm{H}\hat{\mathrm{P}}(\mathbf{x}, \mathrm{t})$$

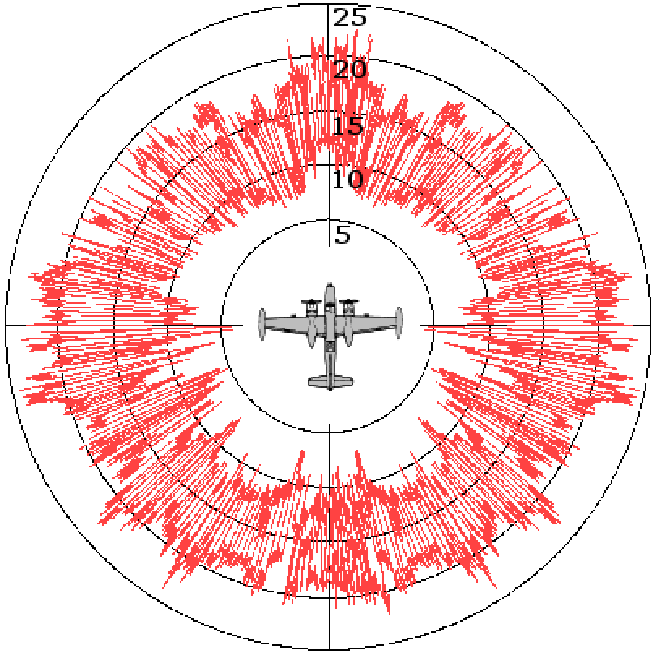
$$\mathrm{N_t}\overbrace{\mathrm{O}(\log\mathrm{N}^3)}^{\text{QFT}} \sim \mathbf{O(N\log N)}$$

QFT on $n = \mathrm{N}^3$ elements costs $\mathrm{O}(\log\mathrm{n})$

Application for seismic wave modeling

Don’t ask for the model, use it!

Radar cross-section (**RCS**) is a measure of how detectable an object is by radar. A larger RCS indicates that an object is more easily detected (Wikipedia)



Classical

$$\sigma = \lim_{R\to\infty} 4\pi R^2 \frac{|E_s|^2}{|E_i|^2}$$

- R: distance between radar and target
- E_s: scattered field strength at radar
- E_i: incident field strength at target

Quantum

$$\sigma = \lim_{R\to\infty} 4\pi R^2 \frac{\langle E_s|E_s\rangle}{\langle E_i|E_i\rangle}$$

Ket $|x\rangle = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Bra $\langle x| = \begin{bmatrix} x_1^* & x_2^* \end{bmatrix}$

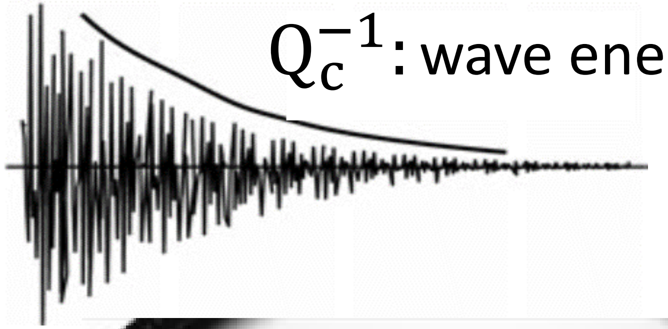
Inner product

$$\langle x|x\rangle = |x_1|^2 + |x_2|^2$$

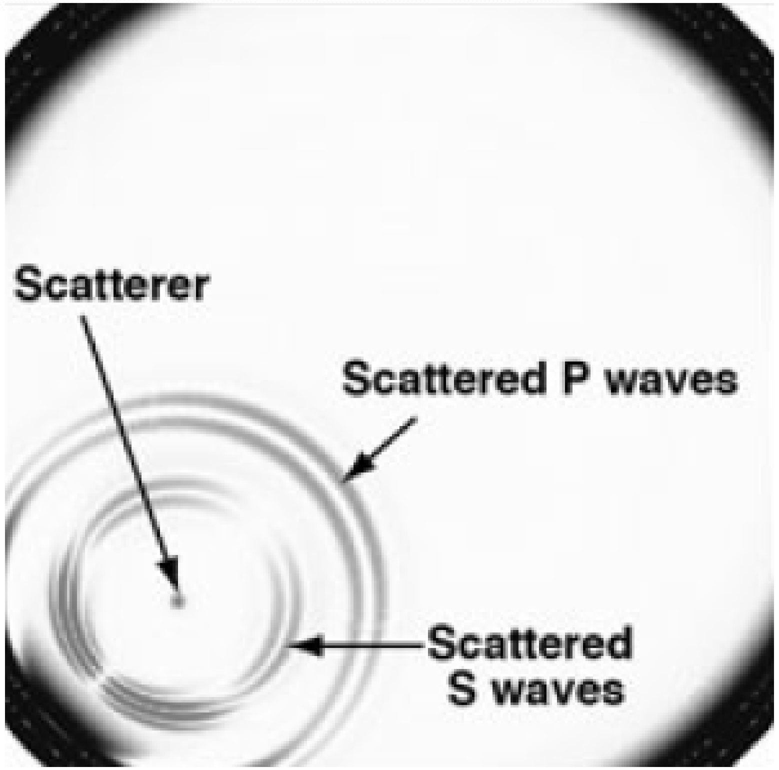
We don’t need to explore the components of the scattered field. We just need the scattered field strength! (Clader, 2013)

Application for seismic wave modeling

Don't ask for the model, use it!



Q_c^{-1}: wave energy decreases with increasing travel distance due to scattering



(Sato et al., 2012)

F_{PP} Scattered **P-wave** from the localized inhomogeneity for the incidence **P-wave**

F_{PS} Scattered **P-wave** from the localized inhomogeneity for the incidence **S-wave**

$$Q_P^{-1} \sim \langle F_{PP} | F_{PP} \rangle + \langle F_{PS} | F_{PS} \rangle$$

$$Q_S^{-1} \sim \langle F_{SP} | F_{SP} \rangle + \langle F_{SS} | F_{SS} \rangle$$

Summary

- ✓ Quantum computation has the potential to simulate the seismic wave propagation faster than classical computers
- ✓ Limited access to the quantum computer output due to the nature of quantum information
- ✓ What features of the quantum ‘solution’ can be used in exploration seismology? **Don't ask for the model, Use it!**

Acknowledgement

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NSERC
CRSNG

Thank you

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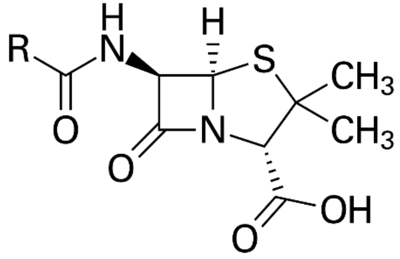
  Canadian Journal of Exploration Geophysics, vol. 44, no. 1 December 2019

What is quantum computer? Why it is important?

- Exact representation of the molecular energy states of a single Caffeine molecule requires 10^{48} bits!!!
- We can represent and store this much information with only 160 quantum bits!!

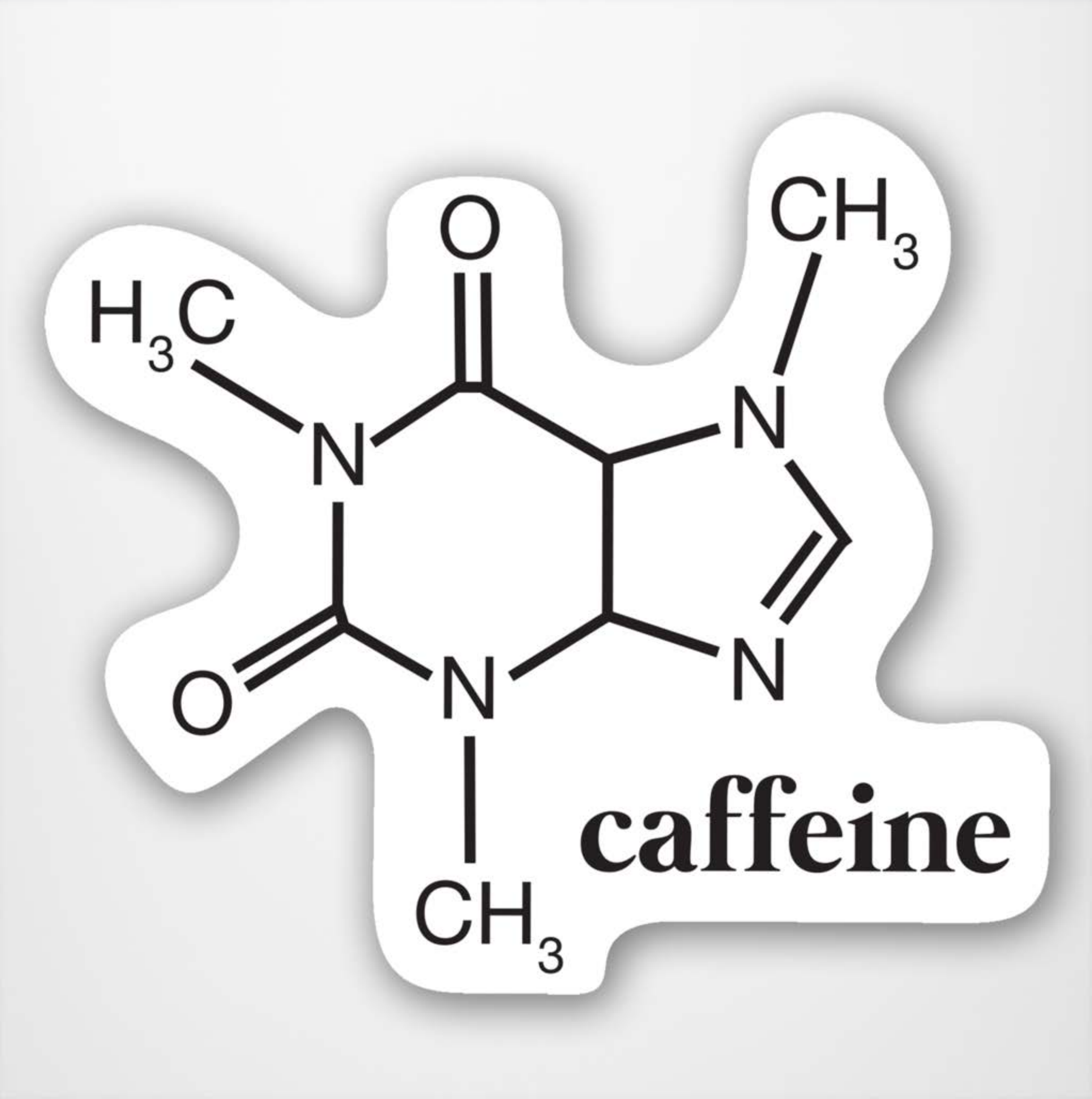
$$10^{48} \sim (10^3)^{16} \sim (2^{10})^{16} \sim 2^{160}$$

Penicillin

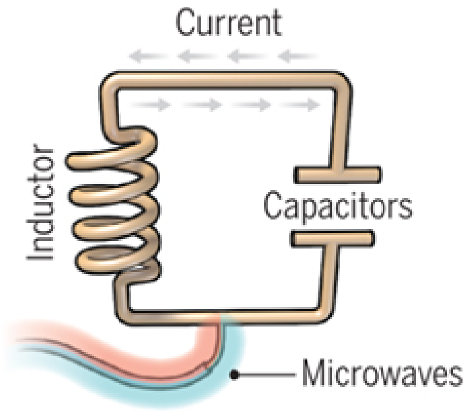


Classical bits 10^{86}

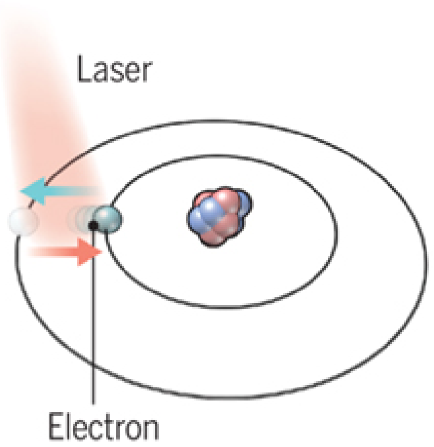
Quantum bits 280



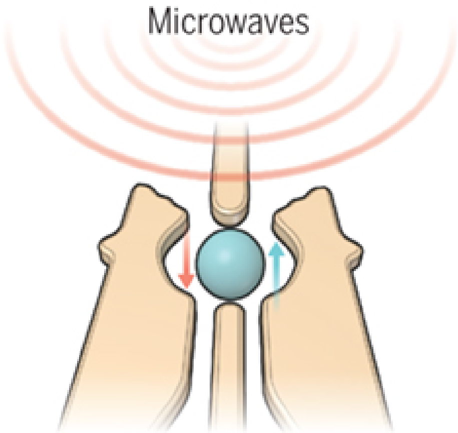
What is quantum computer? Why it is important?



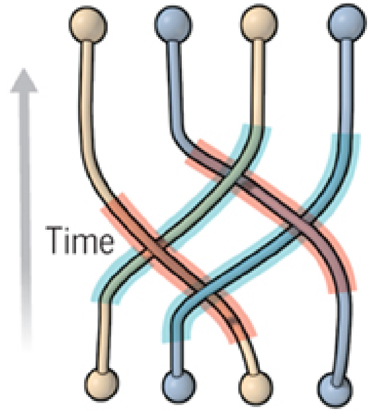
Superconducting loops
A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.



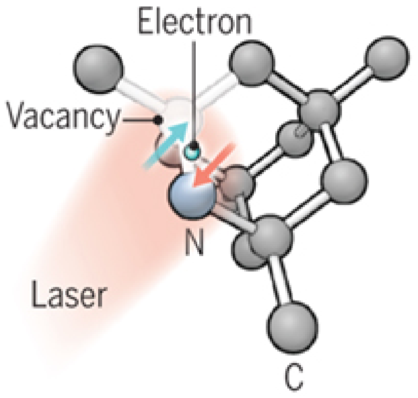
Trapped ions
Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.



Silicon quantum dots
These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.



Topological qubits
Quasiparticles can be seen in the behavior of electrons channeled through semi-conductor structures. Their braided paths can encode quantum information.



Diamond vacancies
A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.

IBM
Google
D:Wave
The Quantum Computing Company™

IONQ

intel

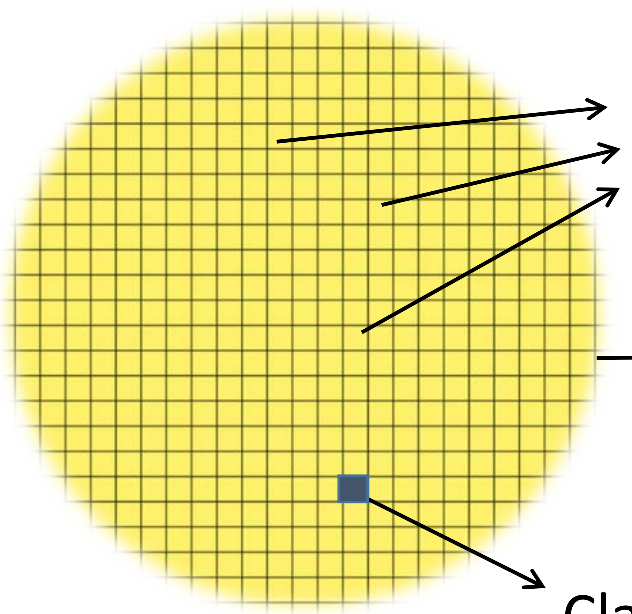
Microsoft

QDTI Quantum Diamond Technologies

Modeling of seismic wave propagation

Output of quantum computer is not fully accessible!

Quantum computer process considerable amounts of information at a rate that cannot be matched in real time by any classical means, yet at the same time, most of this processed information is kept hidden from us!



Quantum information
(Inaccessible information)

2^N cells each contains
N bits of information

Classical information
(Accessible information)

Holevo's theorem

A quantum state with 2^N information content: at most $N = \log 2^N$ classical bits of information can be extracted

We can take advantages of the output if we are interested in some statistical features of it.