How to QC FWI: uncertainty analysis with null-space shuttles

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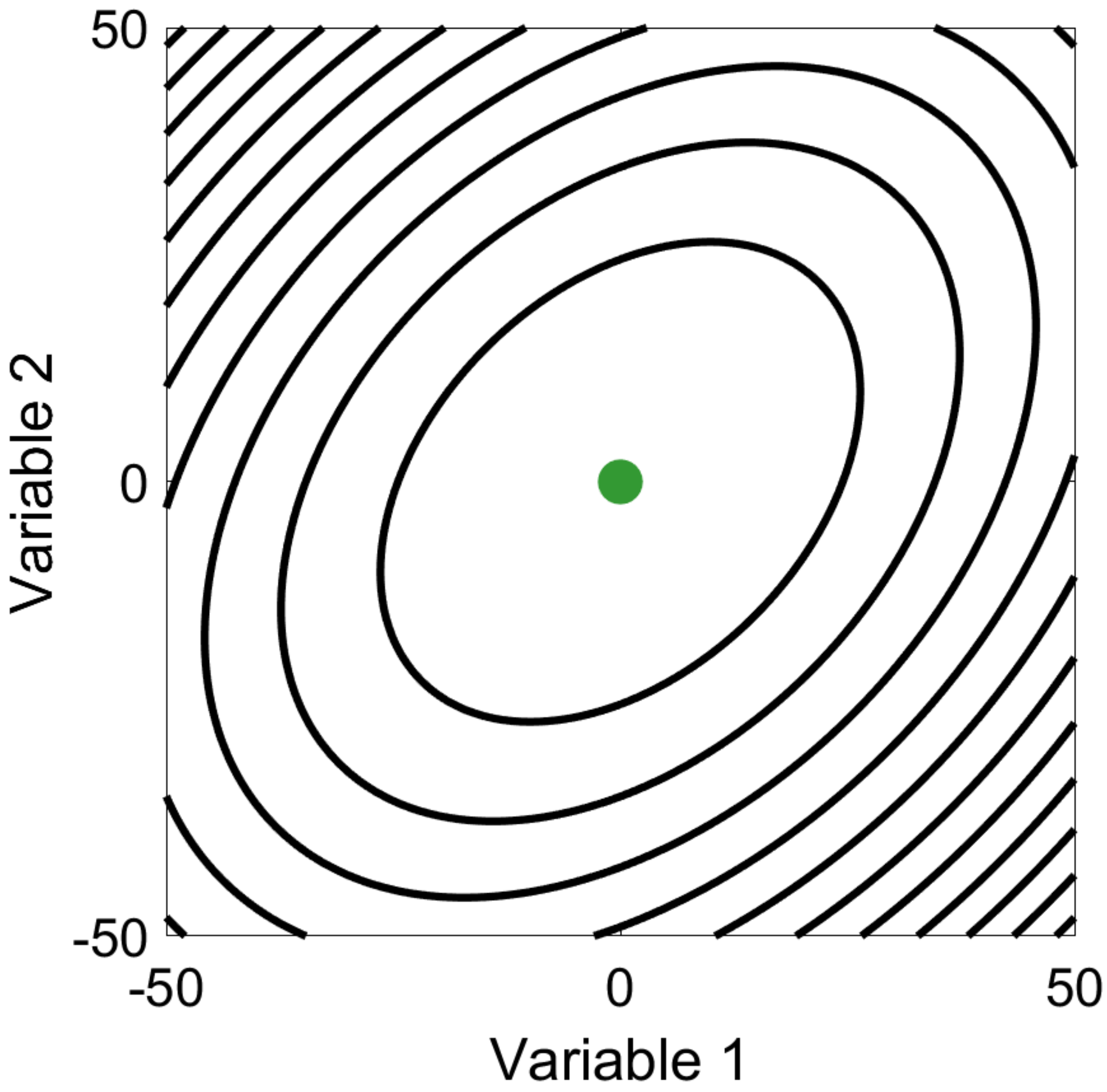
The FWI problem: find the subsurface model which

a) reproduces the measured data, and
b) agrees with prior information

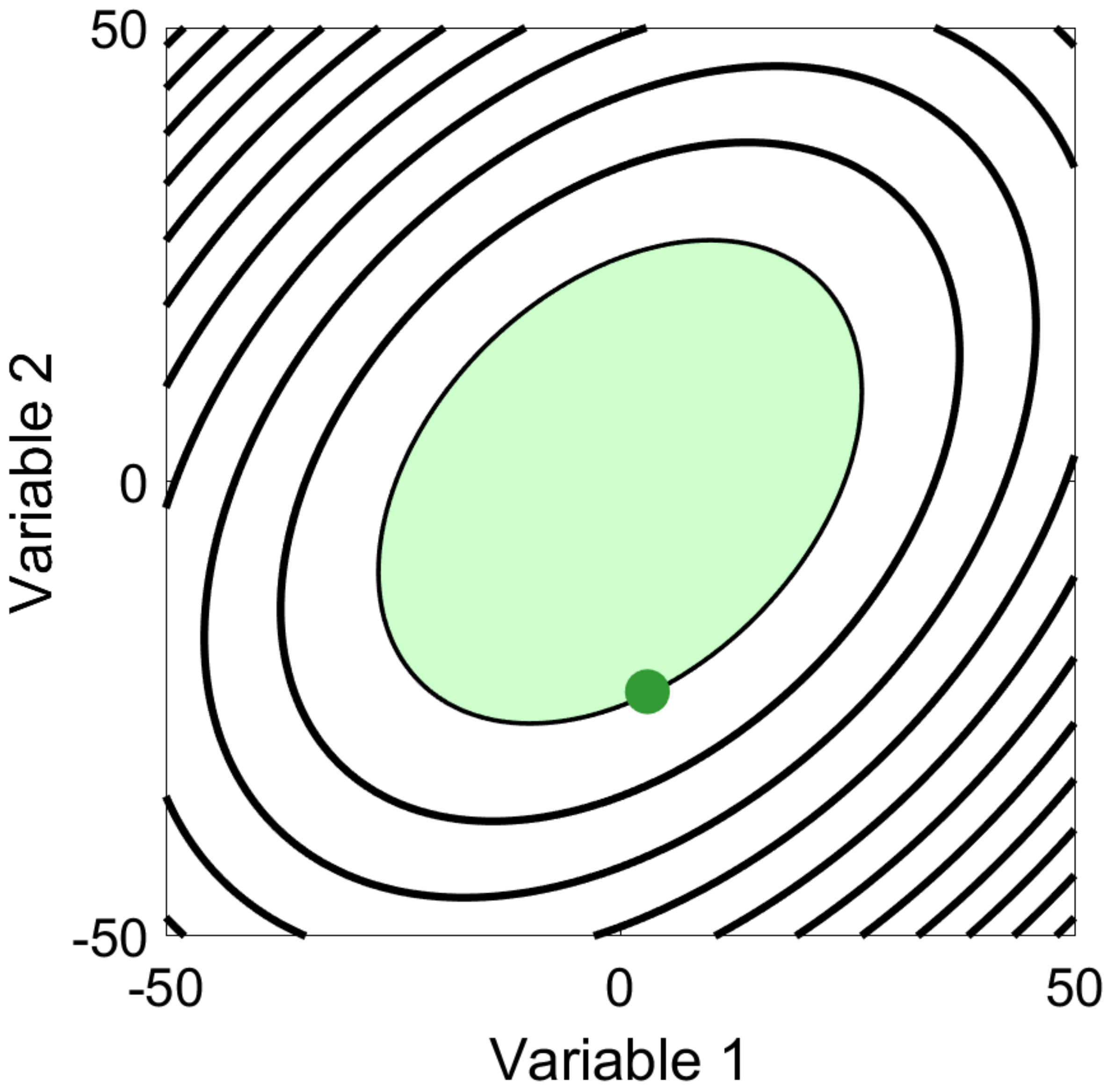
We do this by minimizing an objective function



In reality, we stop at the first acceptable model we find…

… but this is just one of many!

The acceptable models not chosen represent uncertainty

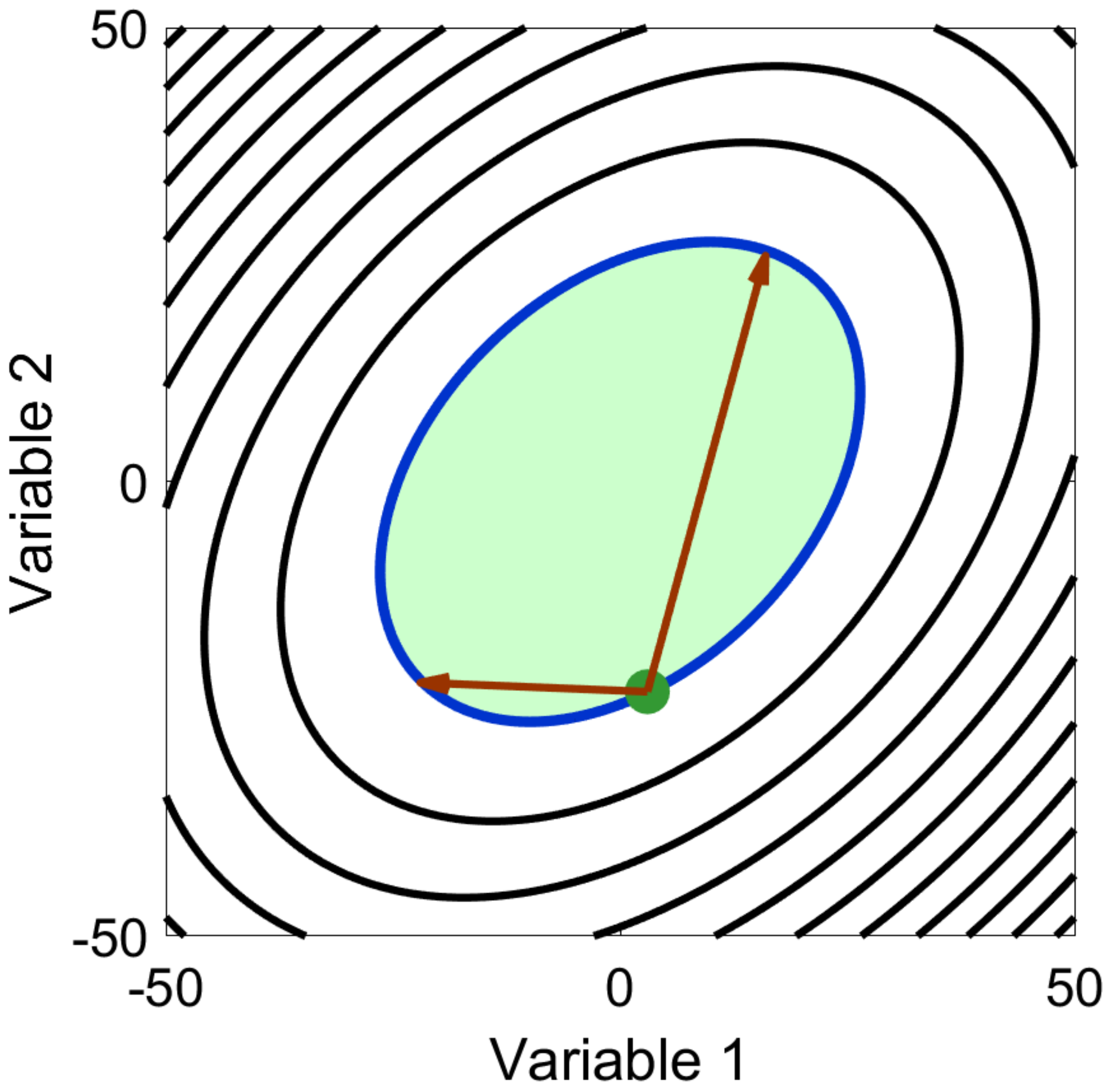


Nullspace shuttles

- Nullspace shuttle – operator that moves from one solution to another without changing objective

- How long a step could we take in a given direction without increasing the objective?

- Long steps indicate high uncertainty directions



Focused uncertainty

Calculating **one** nullspace shuttle is computationally inexpensive

Calculating **all** nullspace shuttles is infeasible

This means answering **focused questions** about the inversion result is easier than generally characterizing uncertainty

Which directions have important uncertainty?

Focused uncertainty

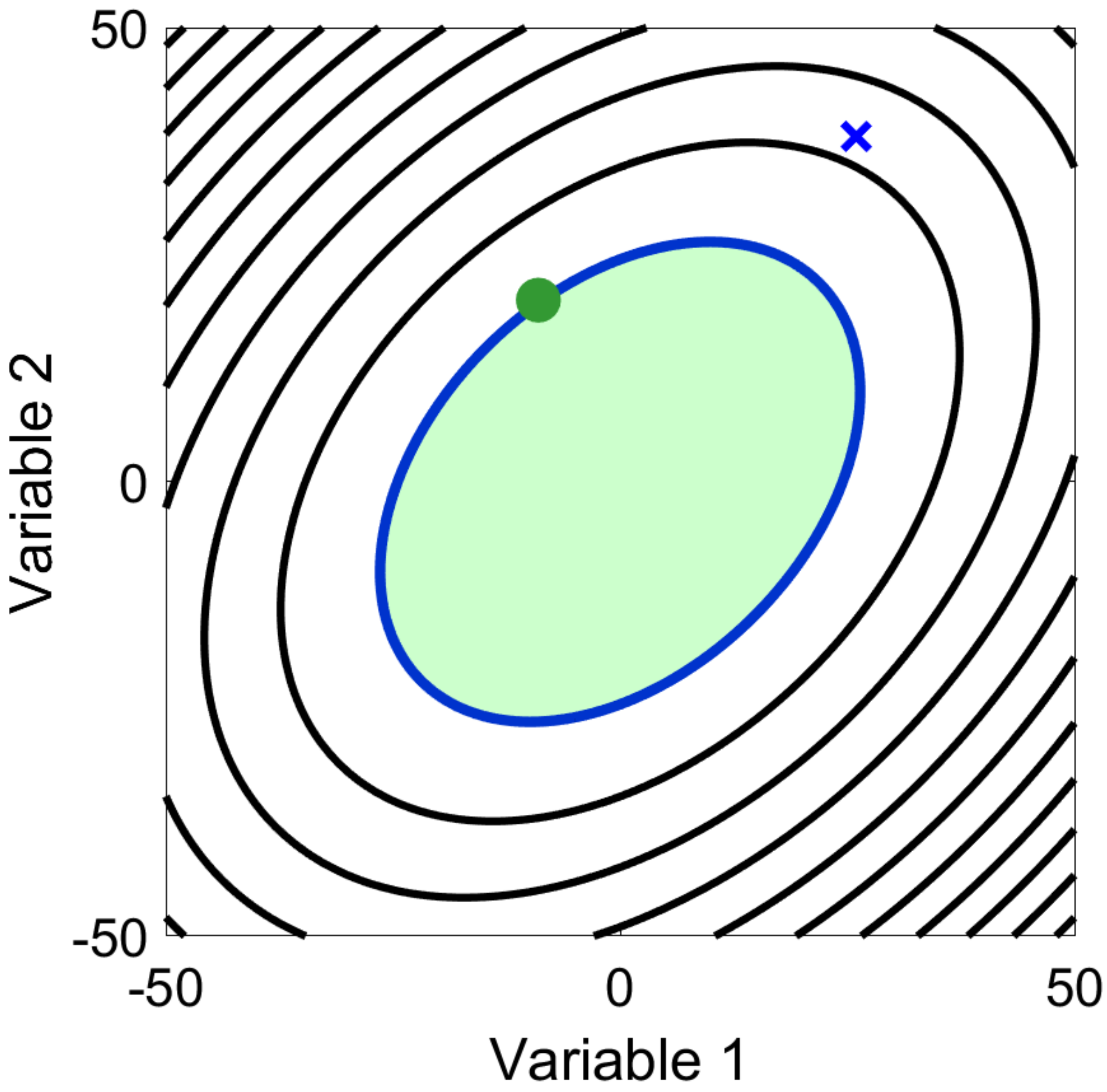
Not every aspect of an inversion output affects decision making

We can use nullspace shuttles to treat uncertainty in the key inversion features that affect how the result is used

Approach

1. Define a function ψ which captures an important feature of the inversion output

We will try to determine how much smaller ψ could be without changing the objective function



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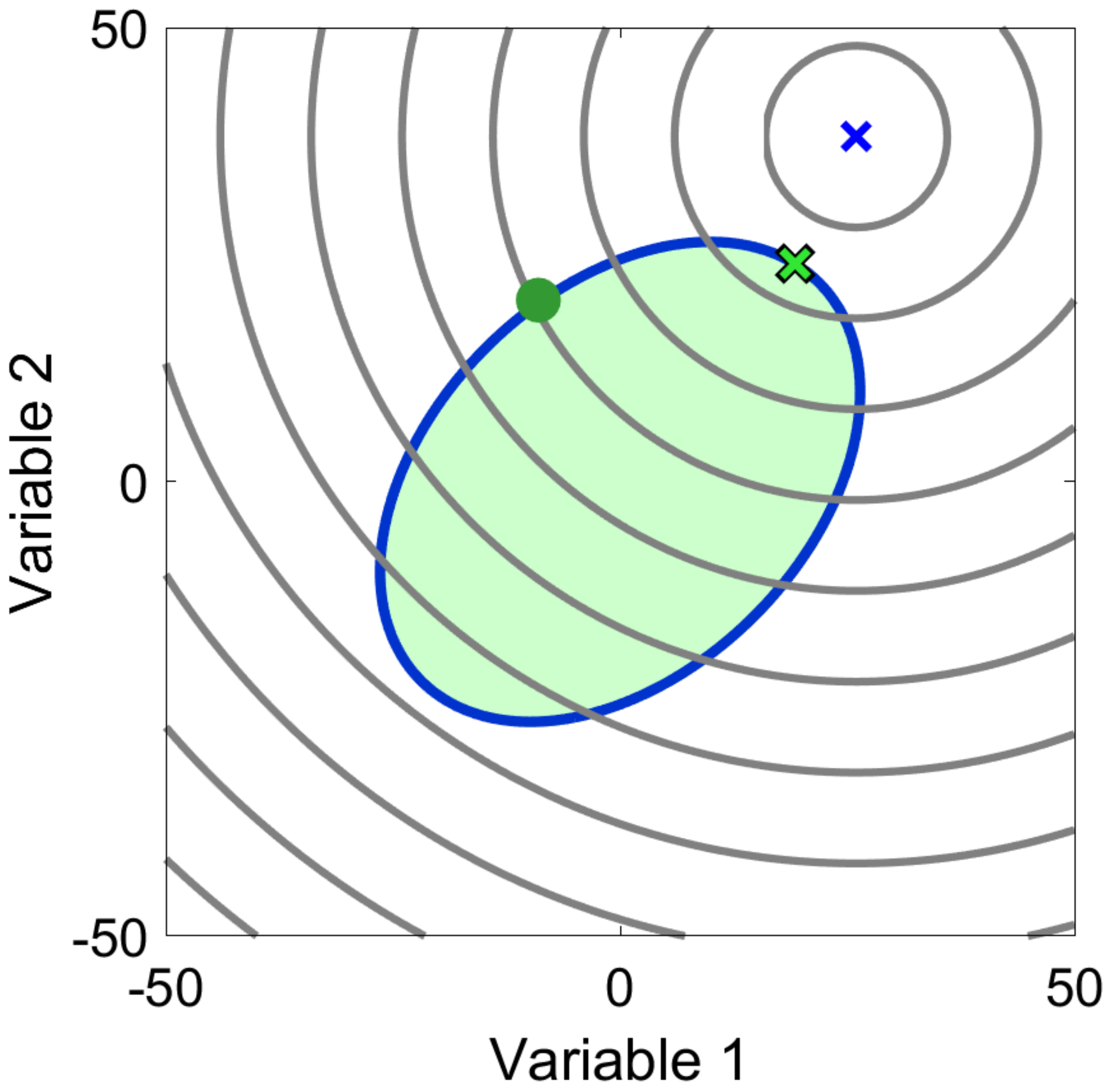


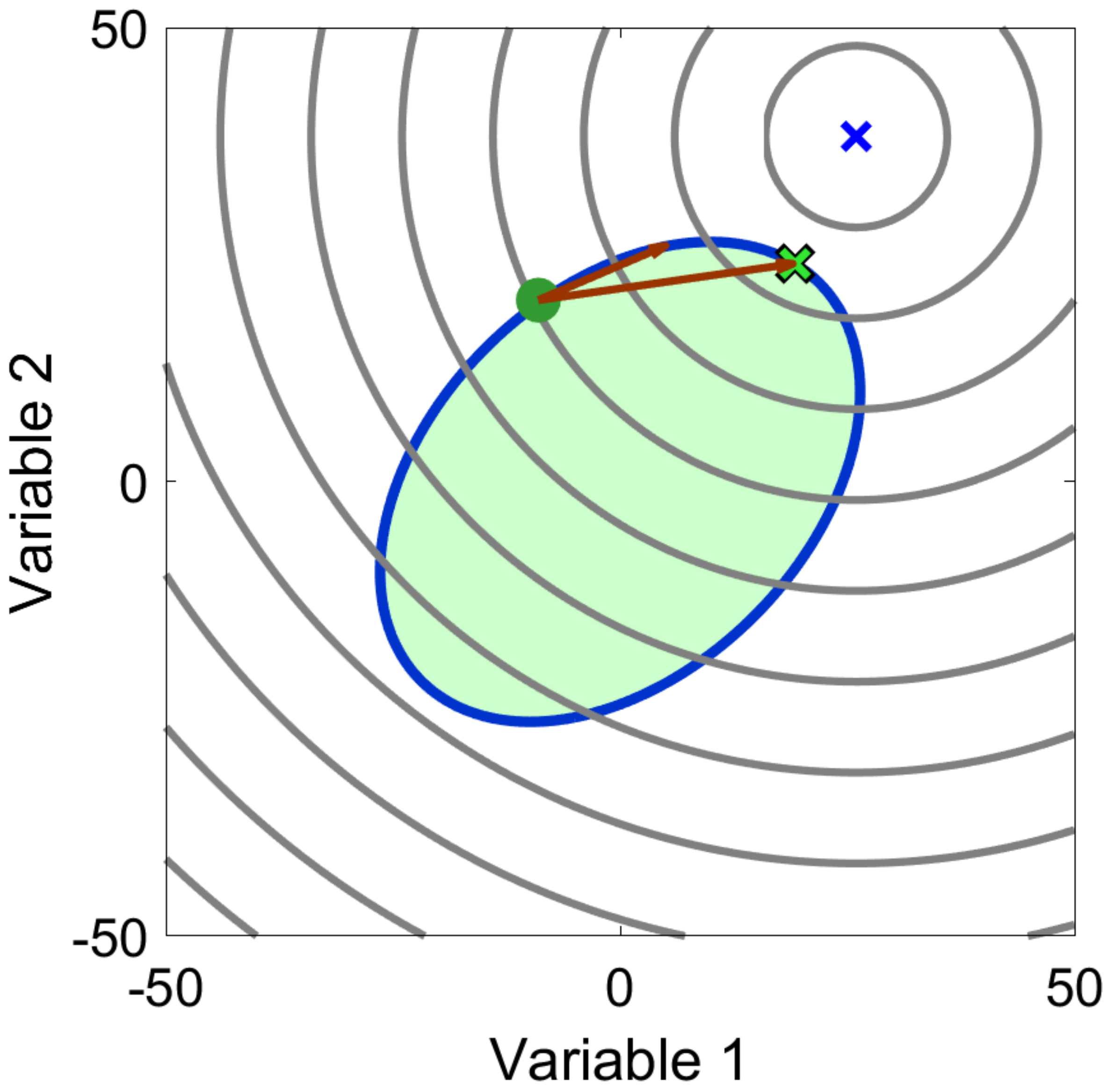
Approach

2. Solve: $\min_{\delta\hat{m}} \psi(m_0 + \alpha\delta\hat{m})$ through L-BFGS

$$\Phi(\alpha\delta\hat{m}) \approx \Phi_0 + \frac{\alpha^2}{2}\delta\hat{m}^T H \delta\hat{m} + \alpha\delta\hat{m}^T g$$

$$\alpha = \frac{-2\delta\hat{m}^T g}{\delta\hat{m}^T H \delta\hat{m}}$$

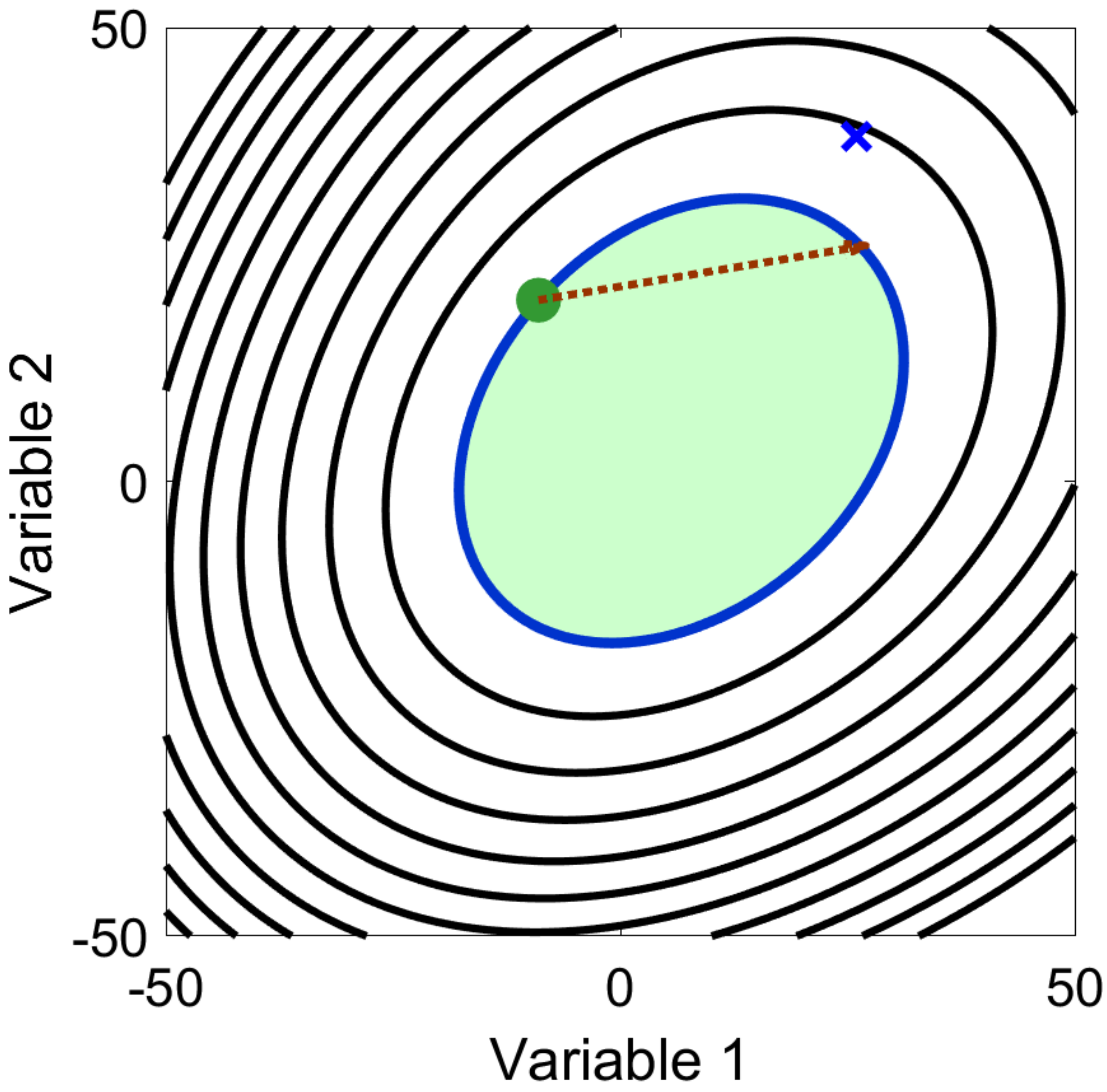
This requires two Hessian-vector product calculations per iteration



Approach

3. Use line-search to find α^* such that FWI objective
$\phi(m_0 + \alpha^* \delta\widehat{m}) = \phi(m_0)$

Very few objective function evaluations should be needed



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3. Use line-search to find α^* such that $\phi(m_0 + \ \alpha^*\delta\hat{m}) = \phi(m_0)$

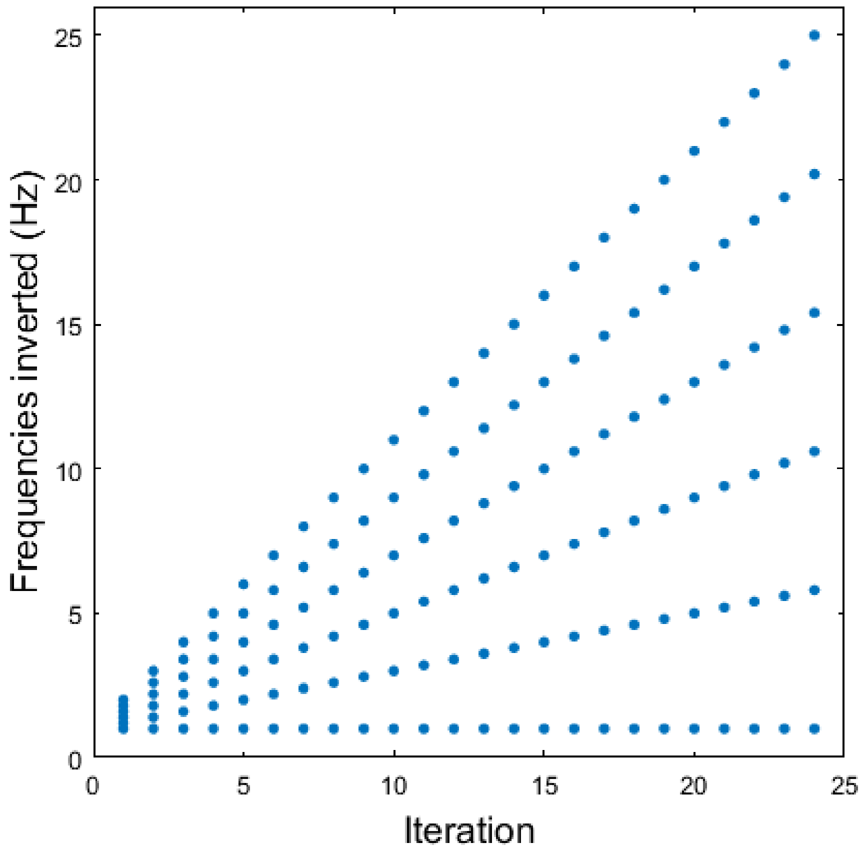
4. Repeat 2 & 3 if necessary

We consider viscoelastic inversion in five parameters

- Truncated Gauss-Newton optimization is used for update calculation

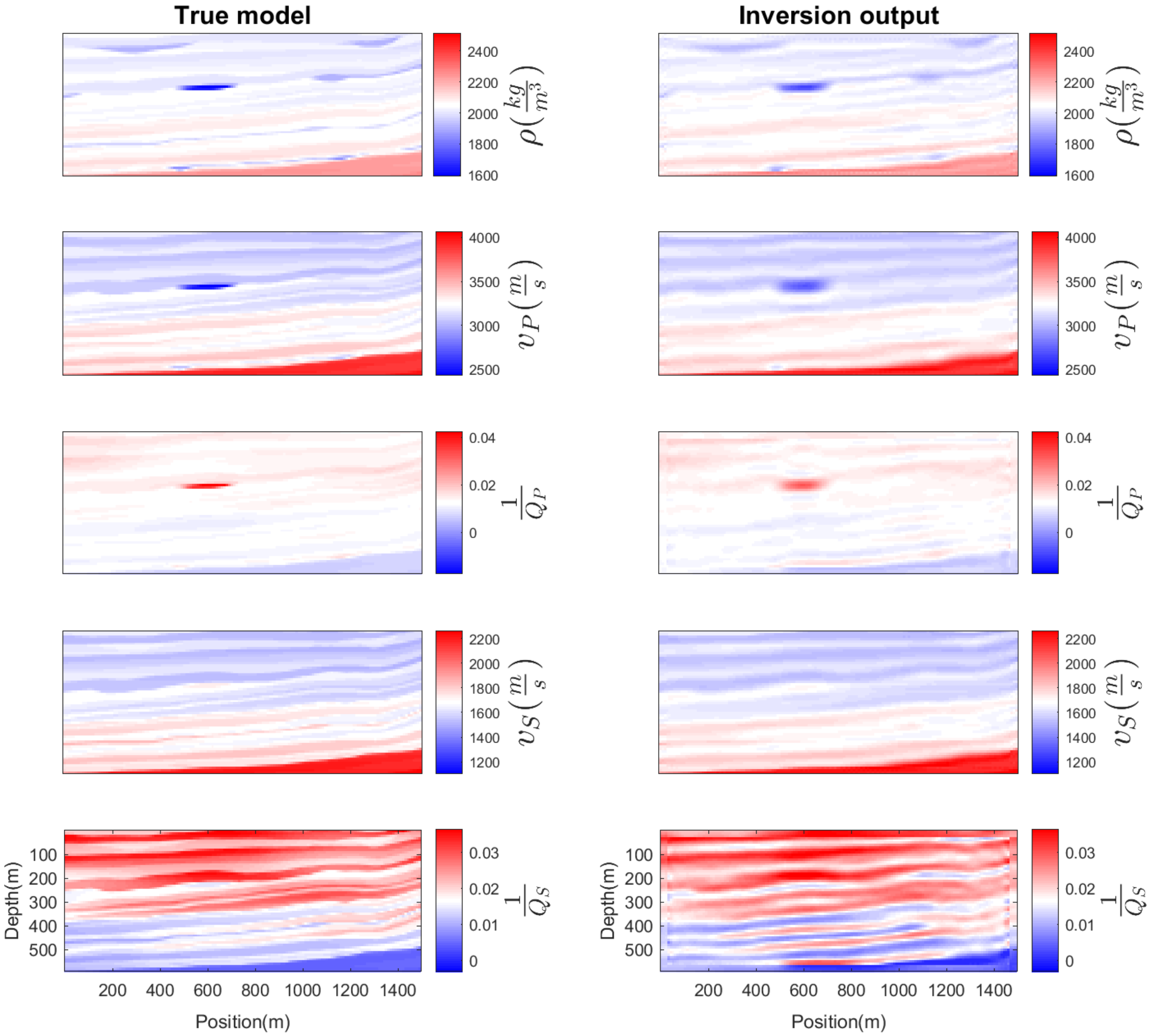
- Only explosive sources here

- Multi-scale approach in frequency domain



Layer model example

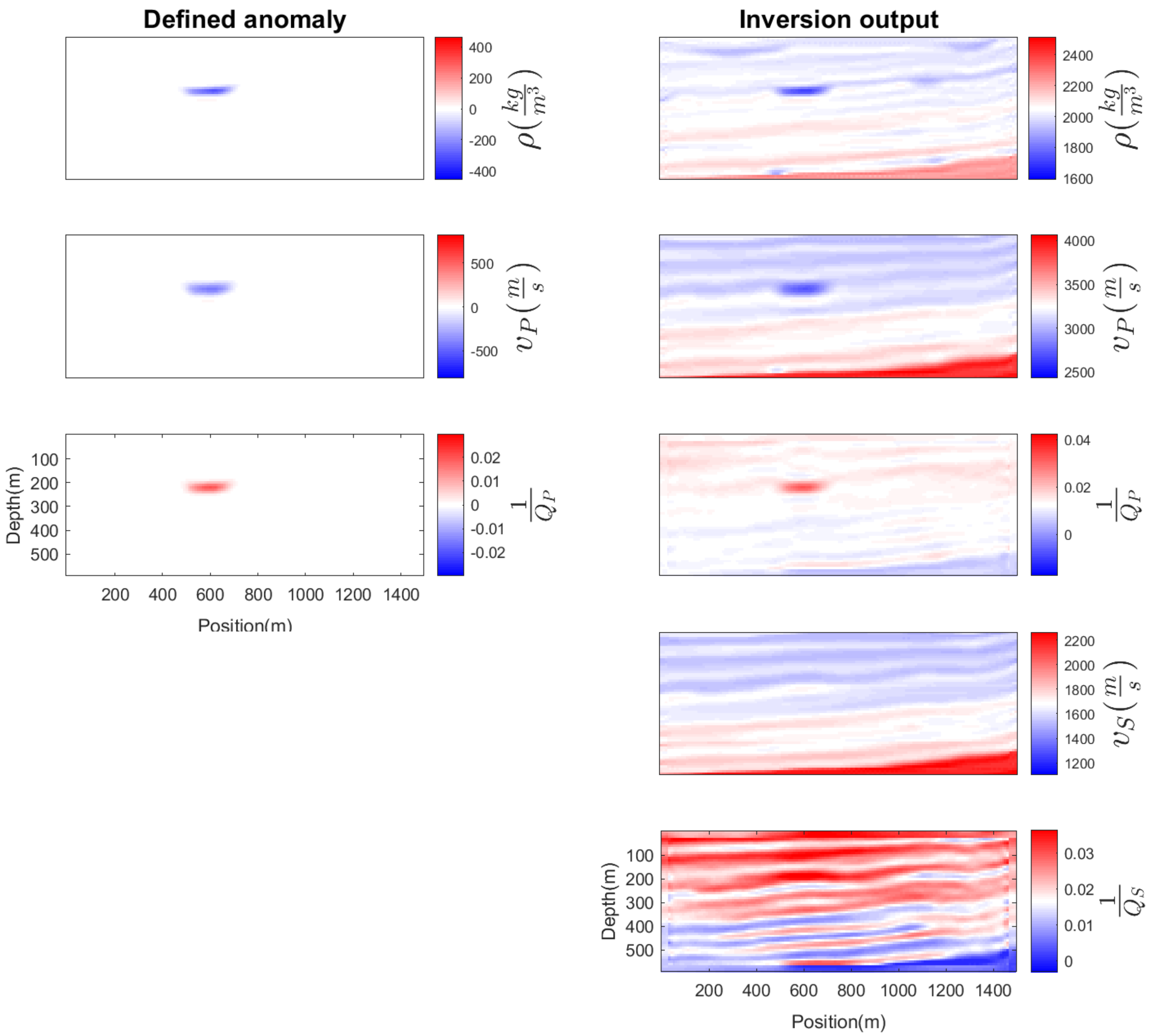
- Model based on subset of Marmousi



Anomaly penalty

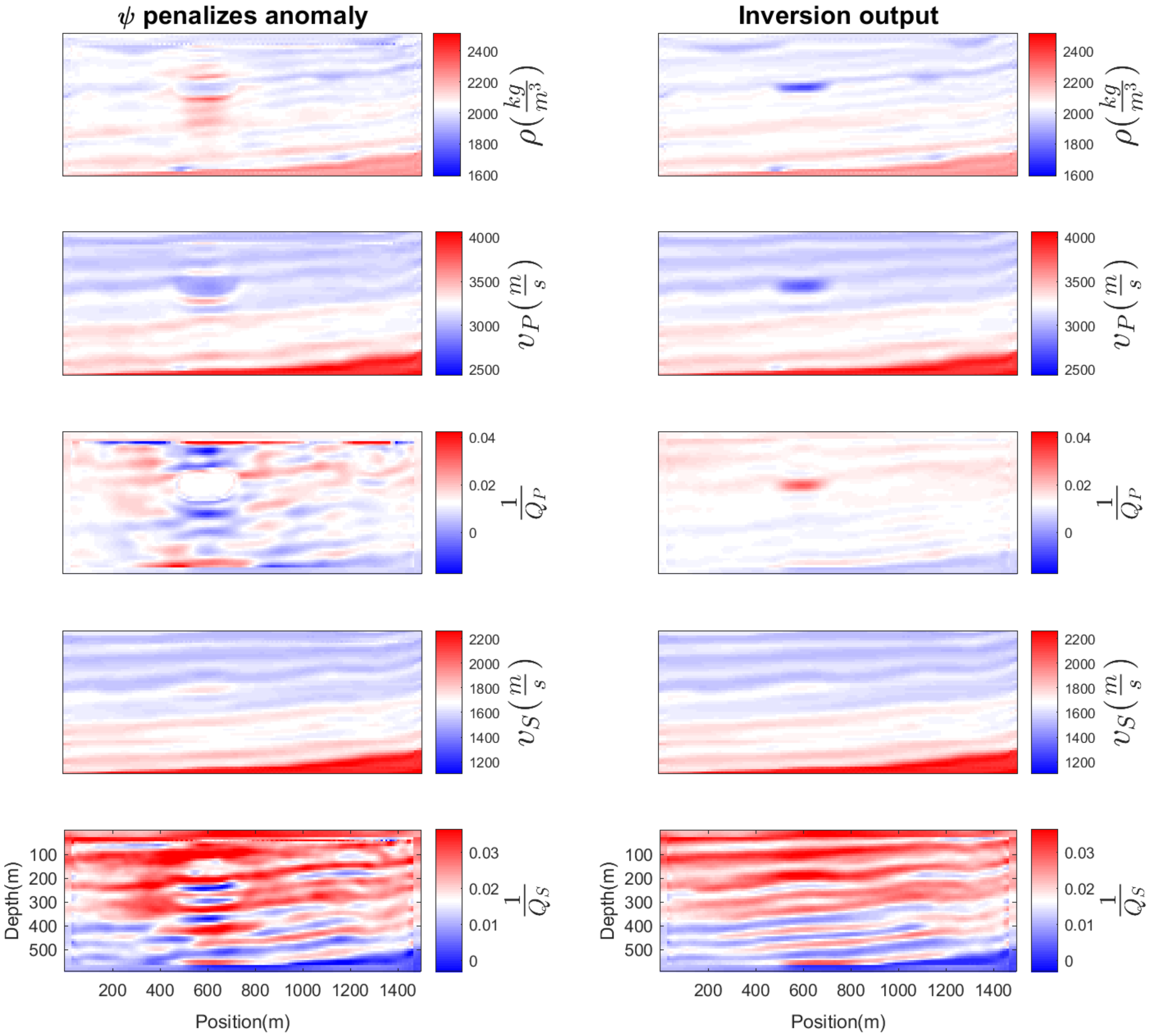
- How sure are we that there really is a v_P / Q_P / ρ anomaly at this location?

- To what extent can we remove the observed anomaly without changing the objective function?



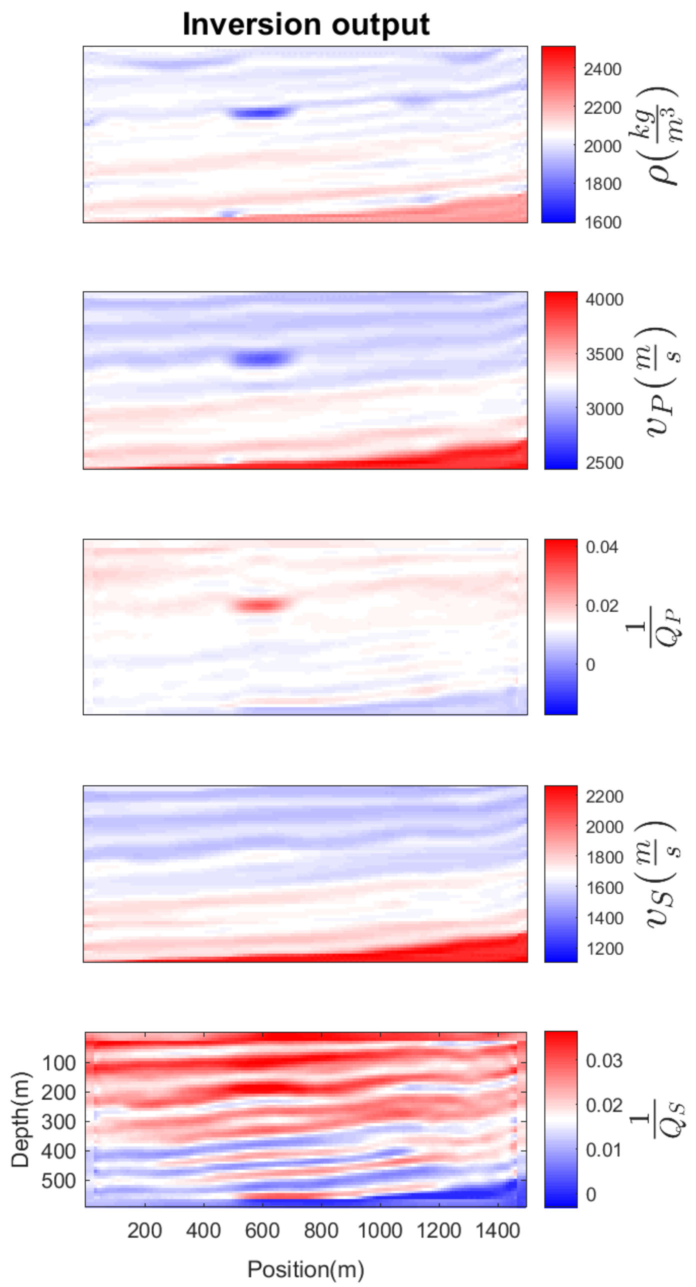
Anomaly penalty

- Density contrast is preserved, but confidence in the anomaly is low
- V_P anomaly is relatively high-confidence
- Q_P anomaly is very uncertain



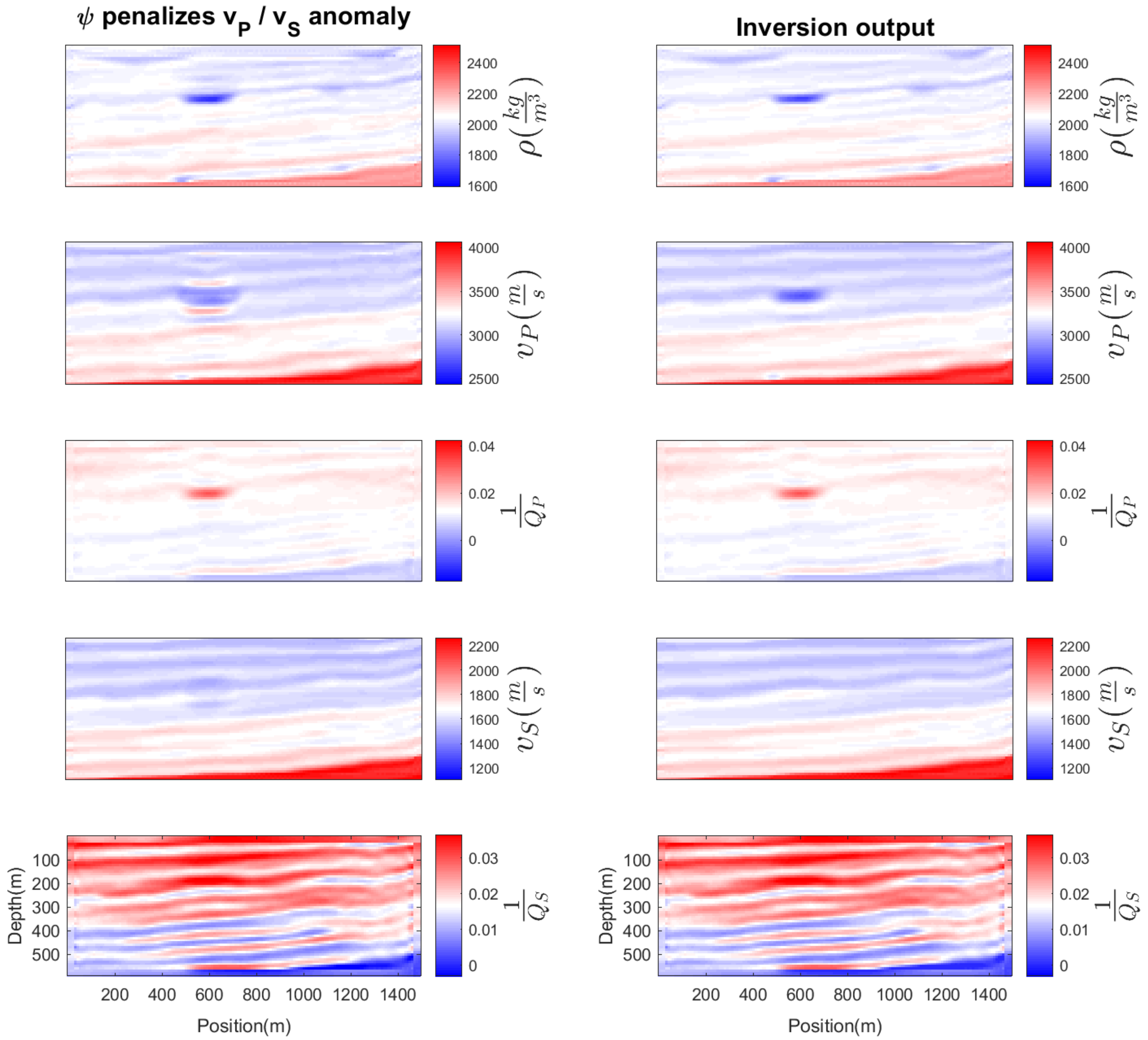
v_P / v_S penalty

Could the v_P / v_S ratio be larger at the anomaly?



v_P / v_S penalty

Objective function is relatively resistant to changes in v_P and v_S



Conclusions

Nullspace shuttles find models with equivalent objective functions

Nullspace shuttles can help with **targeted** quantification of uncertainty

The nullspace shuttle which maximally changes an interpretation metric represents an uncertainty in that metric

Optimal shuttles can be determined through a minimization procedure

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