# VTI and TTI full waveform inversion based on a theory-guided neural network

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# Outline

(1) The recurrent neural network (RNN)

(2) Viscoelastic VTI full waveform inversion based on RNN

(3) TTI Full waveform inversion based on RNN

(4) Conclusions and future study

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# Part ONE: The recurrent neural network

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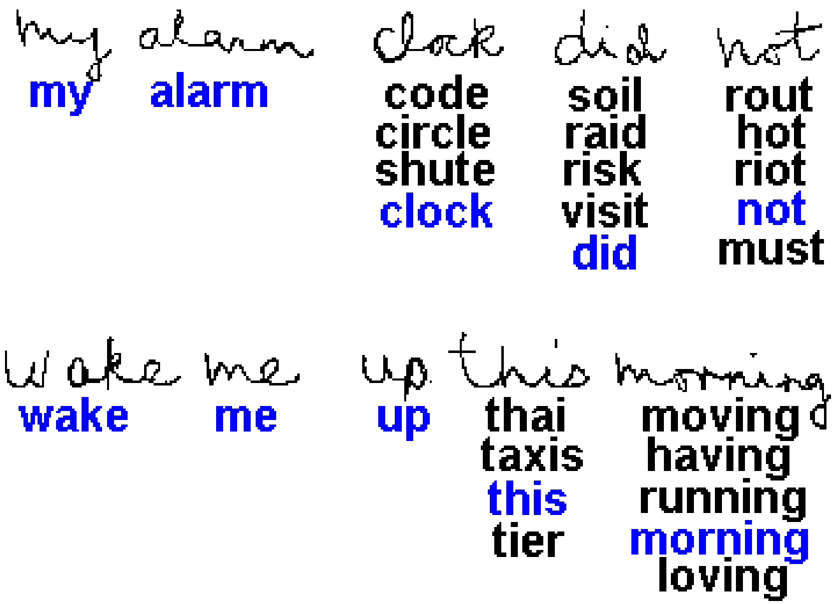
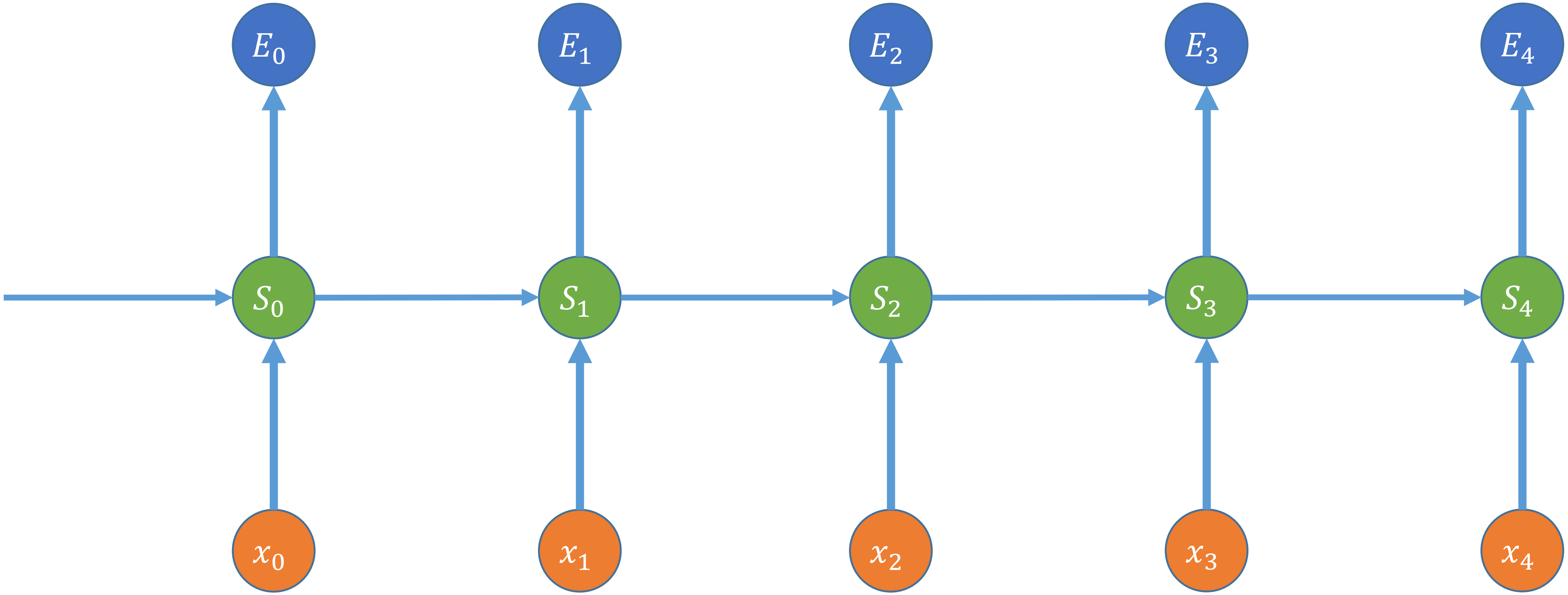
***A recurrent neural network (RNN)*** is a class of artificial neural networks where connections between nodes form a directed graph along a temporal sequence. This allows it to exhibit temporal dynamic behavior.



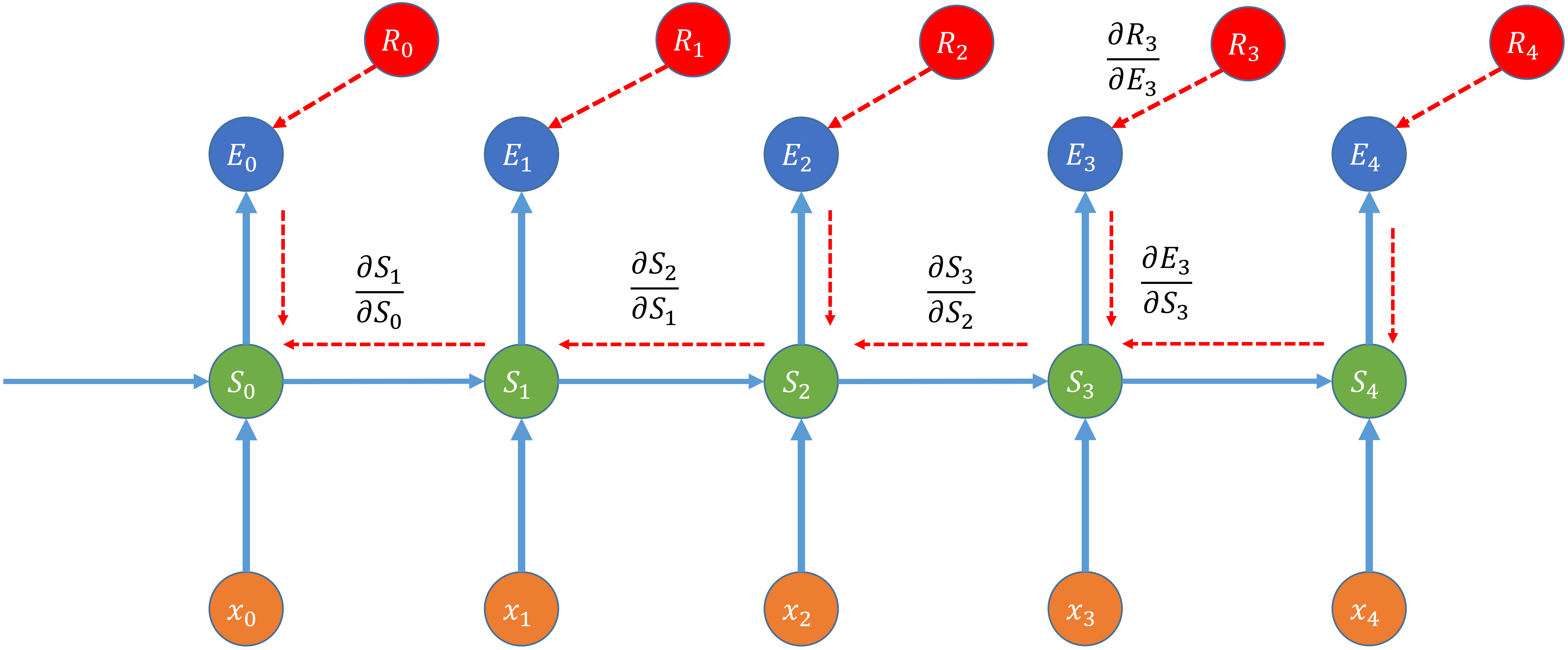

Hand writing recognition


Speech recognition

# Part One: The recurrent neural network

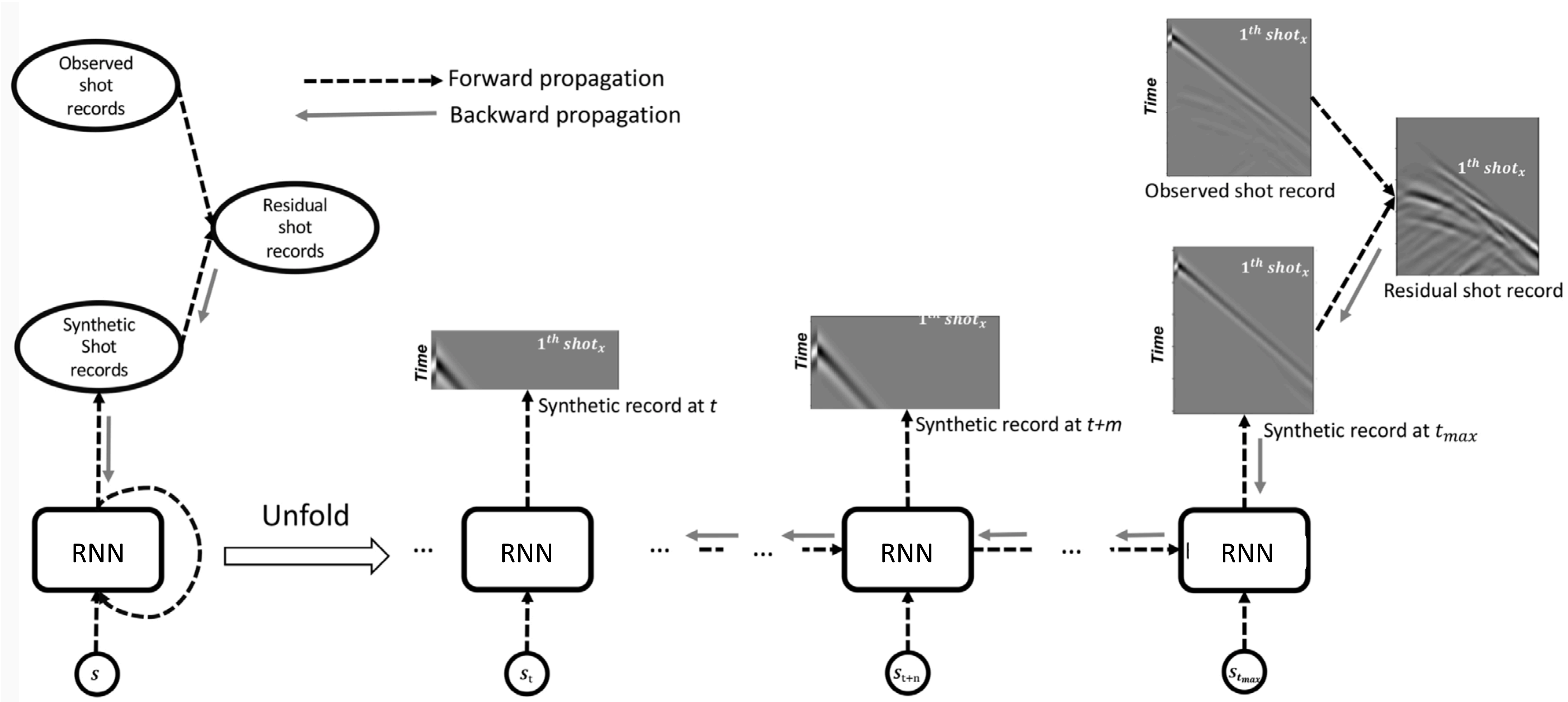



**Forward network**




**Backpropagation of the recurrent neural network**

# Part TWO: Viscoelastic VTI full waveform inversion based on RNN




Wave propagation based on RNN format

# Part TWO:
# Viscoelastic VTI full waveform inversion based on RNN

# Part TWO: Viscoelastic VTI full waveform inversion based on RNN

The constitutive relationship for the VTI viscoelastic media can be expressed as formula (1):

$$\sigma_{ij} = C_{ijkl} * \dot{\epsilon}_{kl} = \dot{C}_{ijkl} * \epsilon_{kl}, \tag{1}$$

The expression for the viscoelastic stiffness parameter based on the GSLS framework can be expressed as formula (2):

$$C(t) = C\left(1 - \sum_{l=1}^{L}\left(1 - \frac{\tau_{\varepsilon l}^{C}}{\tau_{\sigma l}^{C}}\right) e^{-t/\tau_{\sigma l}^{C}}\right)\theta(t), \tag{2}$$

,where $C$ is the elastic modulus, $L$ is the number of the relaxation scheme for the viscoelastic media.

The time derivative of the relaxed stiffness parameter can be expressed as formula (3):

$$\dot{C} = C\left(\frac{1}{\tau_{\sigma l}^{C}} \sum_{\ell=1}^{L}\left(1 - \frac{\tau_{e\ell}^{C}}{\tau_{\sigma\ell}^{C}}\right) \mathrm{e}^{-t/\tau_{\sigma l}^{C}}\right)\theta(t) + C\left(1 - \sum_{\ell=1}^{L}\left(1 - \frac{\tau_{e\ell}^{C}}{\tau_{\sigma\ell}^{C}}\right) \mathrm{e}^{-t/\tau_{\sigma l}^{C}}\right)\delta(t), \tag{3}$$

Thus the 2D viscoelastic VTI stiffness matrix is formula (4):

$$\mathbf{C}_{\mathbf{ANVTI}} = \begin{bmatrix} \dot{C}_{11} & \dot{C}_{13} & 0 \\ \dot{C}_{13} & \dot{C}_{33} & 0 \\ 0 & 0 & \dot{C}_{44} \end{bmatrix}, \tag{4}$$

The stress velocity relationship between the stress and strain in the viscoelastic media can be expressed as (5):

$$\begin{bmatrix} \partial_t \sigma_{xx} \\ \partial_t \sigma_{zz} \\ \partial_t \sigma_{xz} \end{bmatrix} = \begin{bmatrix} \dot{C}_{11} & \dot{C}_{13} & 0 \\ \dot{C}_{13} & \dot{C}_{33} & 0 \\ 0 & 0 & \dot{C}_{44} \end{bmatrix} * \begin{bmatrix} \partial_x v_x \\ \partial_z v_z \\ \partial_x v_z + \partial_z v_x \end{bmatrix}, \tag{5}$$

When $l = 1$ the stress in x direction can be expressed as equation (6):

$$\partial_t \sigma_{xx} = \dot{C}_{11} \partial_x v_x + \dot{C}_{13} \partial_z v_z = \\ C_{11} R_{xx}^{C_{11}} + \left[ C_{11} \left( \frac{\tau_{\epsilon\ell}^{C_{11}}}{\tau_{\sigma\ell}^{C_{11}}} \right) \right] \partial_x v_x + \quad C_{13} R_{zz}^{C_{13}} + \left[ C_{13} \left( \frac{\tau_{\epsilon\ell}^{C_{13}}}{\tau_{\sigma\ell}^{C_{13}}} \right) \right] \partial_z v_z, \tag{6}$$

$R_{xx}^{C_{11}}$ and $R_{xx}^{C_{13}}$ are the relaxation fields for parameter $C_{11}$ and $C_{13}$ and can be expressed as (7) and (8):

$$\partial_t R_{xx}^{C_{11}} = -\frac{1}{\tau_{\sigma l}^{C_{11}}} R_{xx}^{C_{11}} - \frac{1}{\tau_{\sigma l}^{C_{11}}} C_{11} \left( \frac{\tau_{\epsilon l}^{C_{11}}}{\tau_{\sigma l}^{C_{11}}} - 1 \right) \partial_x v_x \tag{7}$$

$$\partial_t R_{zz}^{C_{13}} = -\frac{1}{\tau_{\sigma l}^{C_{13}}} R_{zz}^{C_{13}} - \frac{1}{\tau_{\sigma l}^{C_{13}}} C_{13} \left( \frac{\tau_{\epsilon l}^{C_{13}}}{\tau_{\sigma l}^{C_{13}}} - 1 \right) \partial_z v_z \tag{8}$$

# Part TWO: Viscoelastic VTI full waveform inversion based on RNN

**Algorithm 1** Sequence of calculations in the viscoelastic VTI RNN cell

**Input:** Source: $s_x$, $s_z$; Space partial derivative convolution kernel. $\mathbf{k}_{x_2}$,$\mathbf{k}_{z_1}$,$\mathbf{k}_{x_1}$,$\mathbf{k}_{z_2}$ time step: $dt$. Stiffness parameters: $C_{11}$,$C_{13}$,$C_{33}$,$C_{44}$,

**Output:** Update velocity field at $t+\frac{1}{2}$ and stress fields at $t+1$

$\sigma_{xx}^t \leftarrow \sigma_{xx}^t + s_x$

$\sigma_{zz}^t \leftarrow \sigma_{zz}^t + s_z$

Add source into stress fields

$\partial_x \sigma_{xx}^t \leftarrow (\sigma_{xx}^t * \mathbf{k}_{x_1})/\rho$

$\partial_z \sigma_{xz}^t \leftarrow (\sigma_{xz}^t * \mathbf{k}_{z_2})/\rho$

$\partial_x \sigma_{xz}^t \leftarrow (\sigma_{xz}^t * \mathbf{k}_{x_2})/\rho$

$\partial_z \sigma_{zz}^t \leftarrow (\sigma_{zz}^t * \mathbf{k}_{z_1})/\rho$

Calculate space partial derivative for stresses

$v_x^{t+\frac{1}{2}} \leftarrow v_x^{t-\frac{1}{2}} + dt(\partial_x \sigma_{xx}^t) + dt(\partial_z \sigma_{xz}^t)$

$v_z^{t+\frac{1}{2}} \leftarrow v_z^{t-\frac{1}{2}} + dt(\partial_z \sigma_{zz}^t) + dt(\partial_x \sigma_{xz}^t)$

Update velocity fields

$\partial_x v_x^{t+\frac{1}{2}} \leftarrow v_x^{t+\frac{1}{2}} * \mathbf{k}_{x_2}$

$\partial_z v_x^{t+\frac{1}{2}} \leftarrow v_x^{t+\frac{1}{2}} * \mathbf{k}_{z_1}$

$\partial_x v_z^{t+\frac{1}{2}} \leftarrow v_z^{t+\frac{1}{2}} * \mathbf{k}_{x_1}$

$\partial_z v_z^{t+\frac{1}{2}} \leftarrow v_z^{t+\frac{1}{2}} * \mathbf{k}_{z_2}$

Calculate space partial derivative for velocity

$\sigma_{zz}^{t+1} = \sigma_{zz}^t + dt\left\{C_{13}R_{xx}^{C_{13}} + \left[C_{13}\left(\frac{\tau_{e\ell}^{C_{13}}}{\tau_{\sigma\ell}^{C_{13}}}\right)\right]\partial_x v_x + C_{33}R_{zz}^{C_{33}} + \left[C_{33}\left(\frac{\tau_{e\ell}^{C_{33}}}{\tau_{\sigma\ell}^{C_{33}}}\right)\right]\partial_z v_z\right\}$

$\sigma_{xx}^{t+1} = \sigma_{xx}^t + dt\left\{C_{11}R_{xx}^{C_{11}} + \left[C_{11}\left(\frac{\tau_{e\ell}^{C_{11}}}{\tau_{\sigma\ell}^{C_{11}}}\right)\right]\partial_x v_x + C_{13}R_{zz}^{C_{13}} + \left[C_{13}\left(\frac{\tau_{e\ell}^{C_{13}}}{\tau_{\sigma\ell}^{C_{13}}}\right)\right]\partial_z v_z\right\}$

$\sigma_{xz}^{t+1} = \sigma_{xz}^t + dt\left\{C_{44}R_{xz}^{C_{44}} + \left[C_{44}\left(\frac{\tau_{e\ell}^{C_{44}}}{\tau_{\sigma\ell}^{C_{44}}}\right)\right](\partial_z v_x + \partial_x v_z)\right\}$

Update stress fields

$\partial_t R_{xx}^{C_{11}} = -\frac{1}{\tau_{\sigma l}^{C_{11}}}R_{xx}^{C_{11}} - \frac{1}{\tau_{\sigma l}^{C_{11}}}C_{11}\left(\frac{\tau_{\epsilon l}^{C_{11}}}{\tau_{\sigma l}^{C_{11}}} - 1\right)\partial_x v_x$

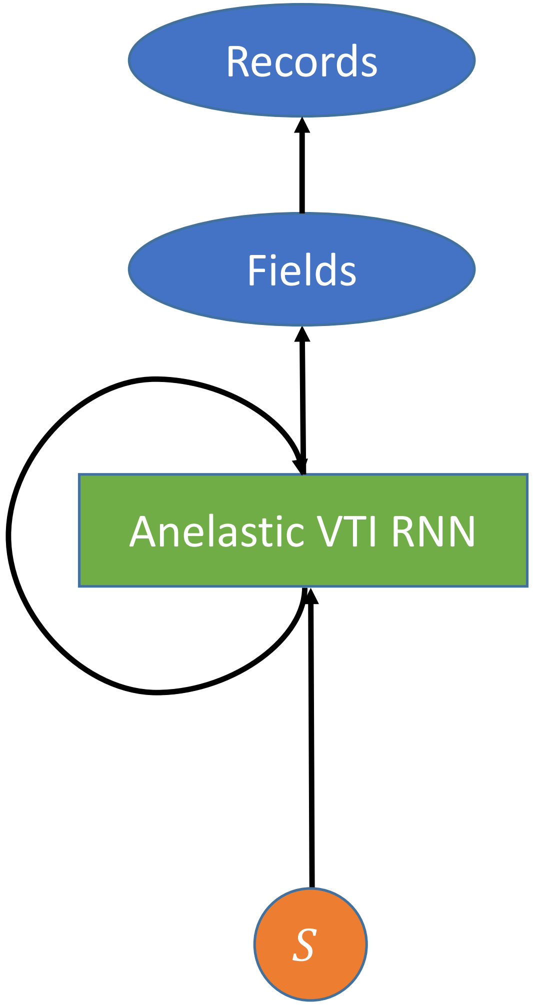
$\partial_t R_{xx}^{C_{13}} = -\frac{1}{\tau_{\sigma l}^{C_{13}}}R_{xx}^{C_{13}} - \frac{1}{\tau_{\sigma l}^{C_{13}}}C_{13}\left(\frac{\tau_{\epsilon l}^{C_{13}}}{\tau_{\sigma l}^{C_{13}}} - 1\right)\partial_x v_x$

$\partial_t R_{zz}^{C_{13}} = -\frac{1}{\tau_{\sigma l}^{C_{13}}}R_{xx}^{C_{13}} - \frac{1}{\tau_{\sigma l}^{C_{13}}}C_{13}\left(\frac{\tau_{\epsilon l}^{C_{13}}}{\tau_{\sigma l}^{C_{13}}} - 1\right)\partial_z v_z$

$\partial_t R_{zz}^{C_{33}} = -\frac{1}{\tau_{\sigma l}^{C_{33}}}R_{xx}^{C_{33}} - \frac{1}{\tau_{\sigma l}^{C_{33}}}C_{33}\left(\frac{\tau_{\epsilon l}^{C_{33}}}{\tau_{\sigma l}^{C_{33}}} - 1\right)\partial_z v_z$

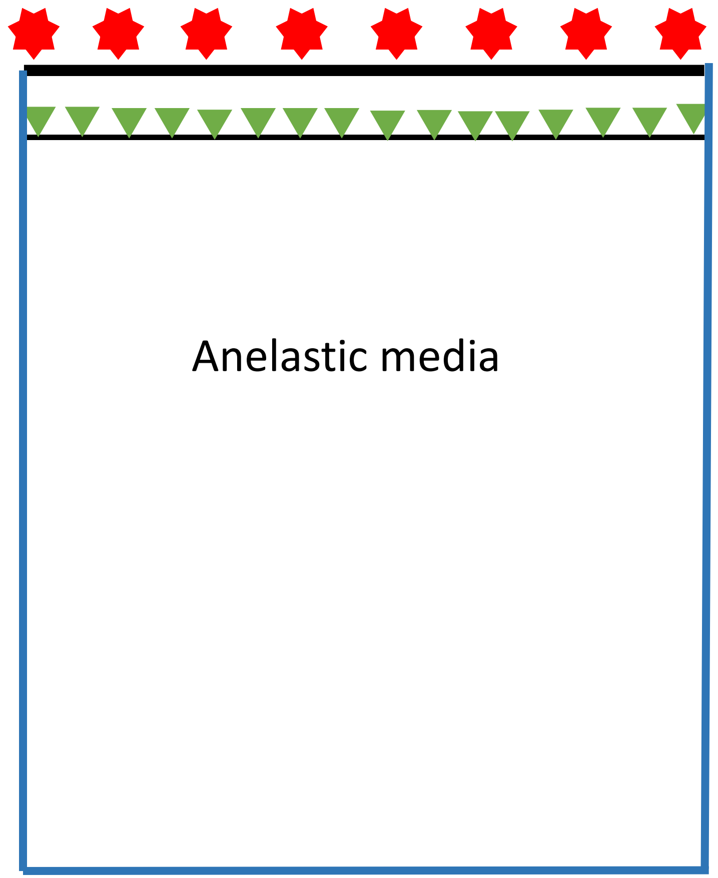
$\partial_t R_{xz}^{C_{44}} = -\frac{1}{\tau_{\sigma l}^{C_{44}}}R_{xz}^{C_{44}} - \frac{1}{\tau_{\sigma l}^{C_{44}}}C_{44}\left(\frac{\tau_{\epsilon l}^{C_{44}}}{\tau_{\sigma l}^{C_{44}}} - 1\right)(\partial_z v_x + \partial_x v_z)$

Update relaxation fields



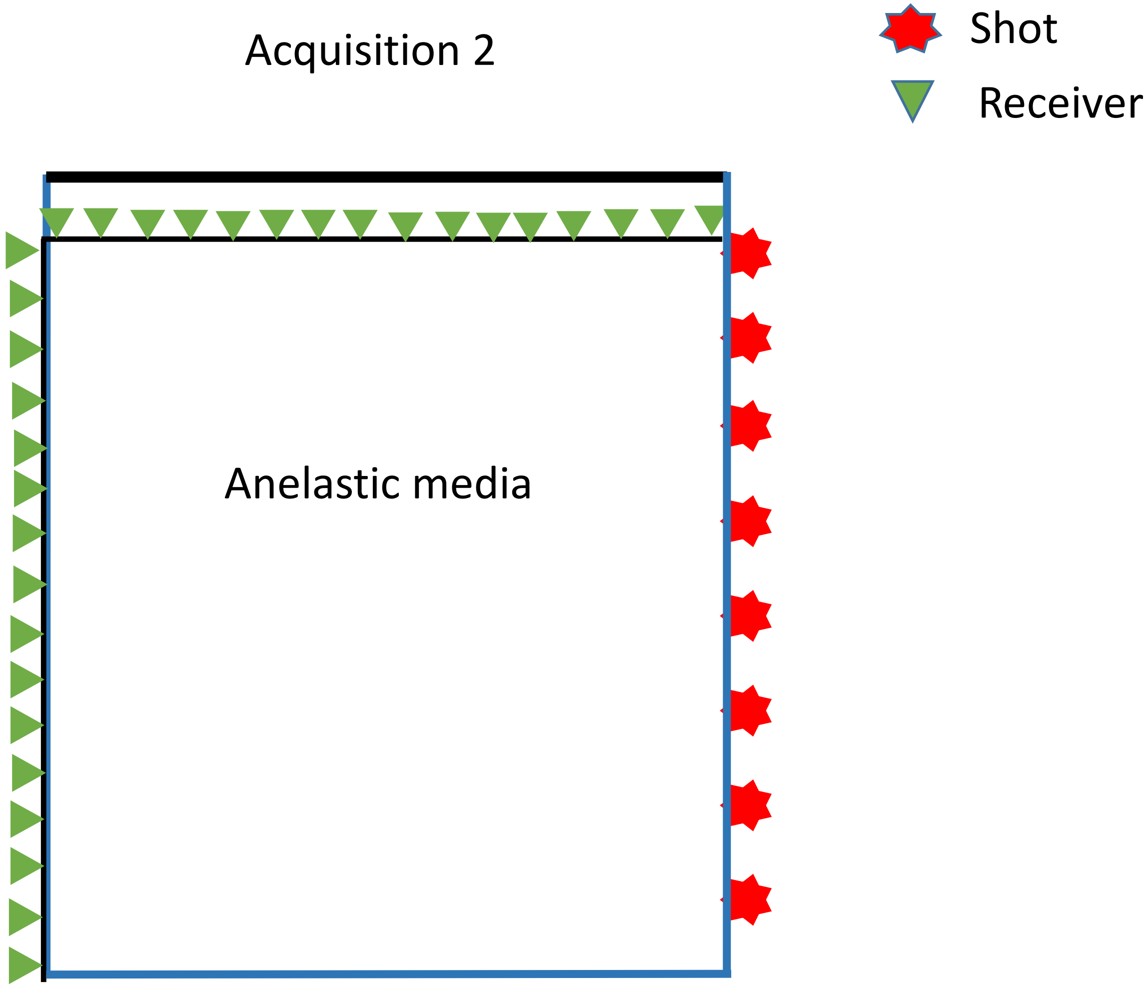
# Part TWO: Viscoelastic VTI full waveform inversion based on RNN

Acquisition 1




Surface acquisition

Acquisition 2




Cross well acquisition

# Part TWO: Viscoelastic VTI full waveform inversion based on RNN

High order TV regularization misfit

$$
\begin{aligned}
&\Phi_{l2}^{TV}(\mathbf{C}_{11},\mathbf{C}_{13},\mathbf{C}_{33},\mathbf{C}_{44},\mathbf{Qc}_{11},\mathbf{Qc}_{13},\mathbf{Qc}_{33},\mathbf{Qc}_{44},)= \\
&\frac{1}{2}\|\mathbf{D}_{syn}(\mathbf{C}_{11},\mathbf{C}_{13},\mathbf{C}_{33},\mathbf{C}_{44},\mathbf{Qc}_{11},\mathbf{Qc}_{13},\mathbf{Qc}_{33},\mathbf{Qc}_{44})-\mathbf{D}_{obs}\|_2^2+ \\
&\left.\begin{aligned}
&\alpha_1^{c11}\Theta_{TV}(\mathbf{C}_{11})+\alpha_1^{c13}\Theta_{TV}(\mathbf{C}_{13})+\alpha_1^{c33}\Theta_{TV}(\mathbf{C}_{33})+\alpha_1^{c44}\Theta_{TV}(\mathbf{C}_{44})+ \\
&\alpha_1^{Qc11}\Theta_{TV}(\mathbf{Qc}_{11})+\alpha_1^{Qc13}\Theta_{TV}(\mathbf{Qc}_{13})+\alpha_1^{Qc33}\Theta_{TV}(\mathbf{Qc}_{33})+\alpha_1^{Qc44}\Theta_{TV}(\mathbf{Qc}_{44})
\end{aligned}\right\}\text{Frist order TV regulation} \\
&\left.\begin{aligned}
&\alpha_2^{c11}\Upsilon_{TV}(\mathbf{C}_{11})+\alpha_2^{c13}\Upsilon_{TV}(\mathbf{C}_{13})+\alpha_2^{c33}\Upsilon_{TV}(\mathbf{C}_{33})+\alpha_2^{c44}\Upsilon_{TV}(\mathbf{C}_{44})+ \\
&\alpha_2^{Qc11}\Upsilon_{TV}(\mathbf{Qc}_{11})+\alpha_2^{Qc13}\Upsilon_{TV}(\mathbf{Qc}_{13})+\alpha_2^{Qc33}\Upsilon_{TV}(\mathbf{Qc}_{33})+\alpha_2^{Qc44}\Upsilon_{TV}(\mathbf{QC}_{44}),
\end{aligned}\right\}\text{Second order TV regulation}
\end{aligned}
\tag{9}
$$

Frist order TV regulation

$$TV_1((\mathbf{m}))=\sum_{i=1}^{n-1}\sum_{j=1}^{m-1}|M_{i+1,j}-M_{i,j}|+\sum_{i=1}^{n-1}\sum_{j=1}^{m-1}|M_{i,j+1}-M_{i,j}| \tag{10}$$

Second order TV regulation

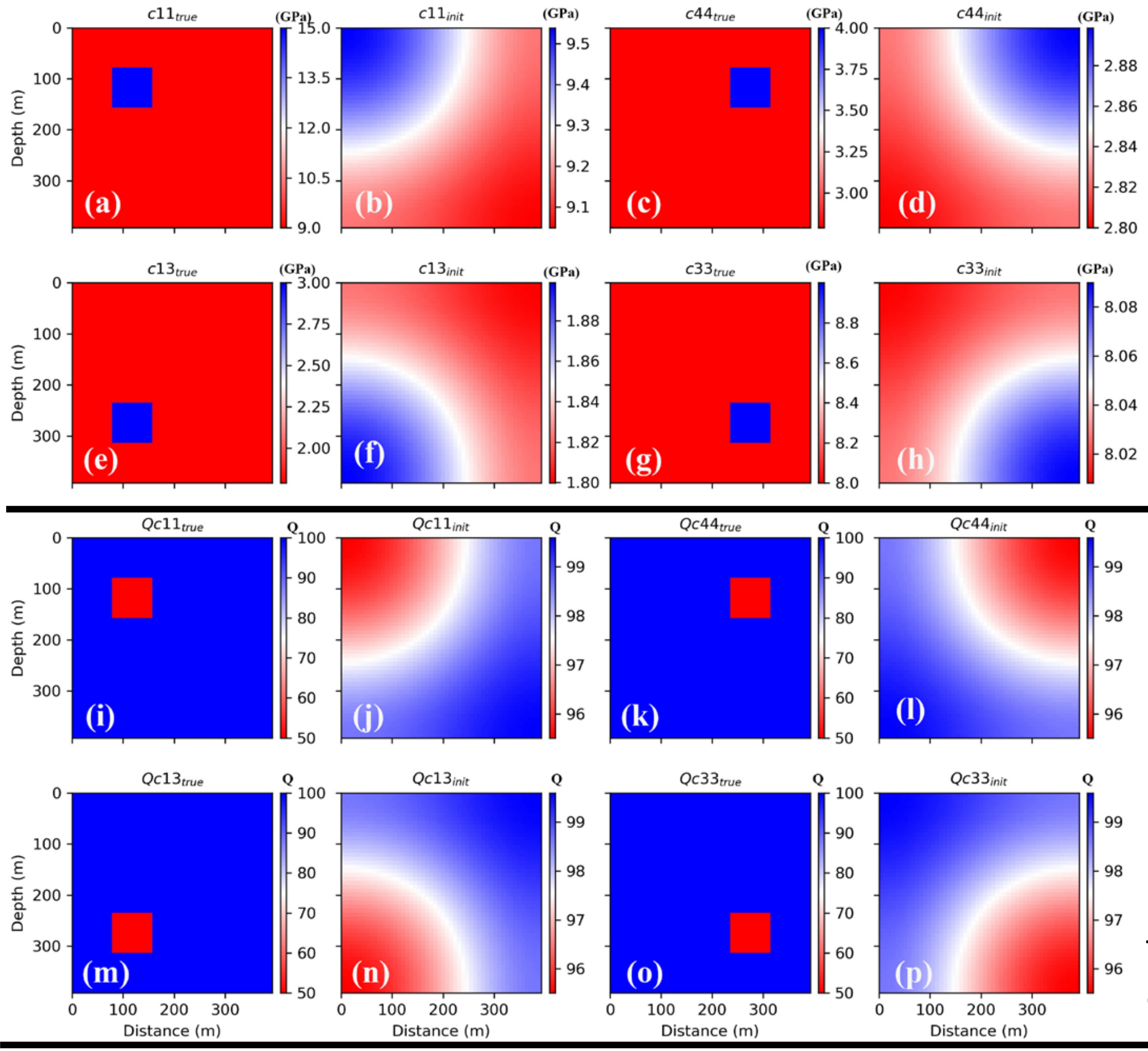
$$TV_2((\mathbf{m}))=\sum_{i=1}^{n-1}\sum_{j=1}^{m-1}|M_{i+1,j}-2M_{i,j}+M_{i-1,j}|+\sum_{i=1}^{n-1}\sum_{j=1}^{m-1}|M_{i,j+1}-2M_{i,j}+M_{i,j-1}| \tag{11}$$

# Part TWO: Viscoelastic VTI full waveform inversion based on RNN

Model size: 50*50    Maximum time: 0.7s

Wavelet: Ricker wavelet (Main frequency 20Hz)

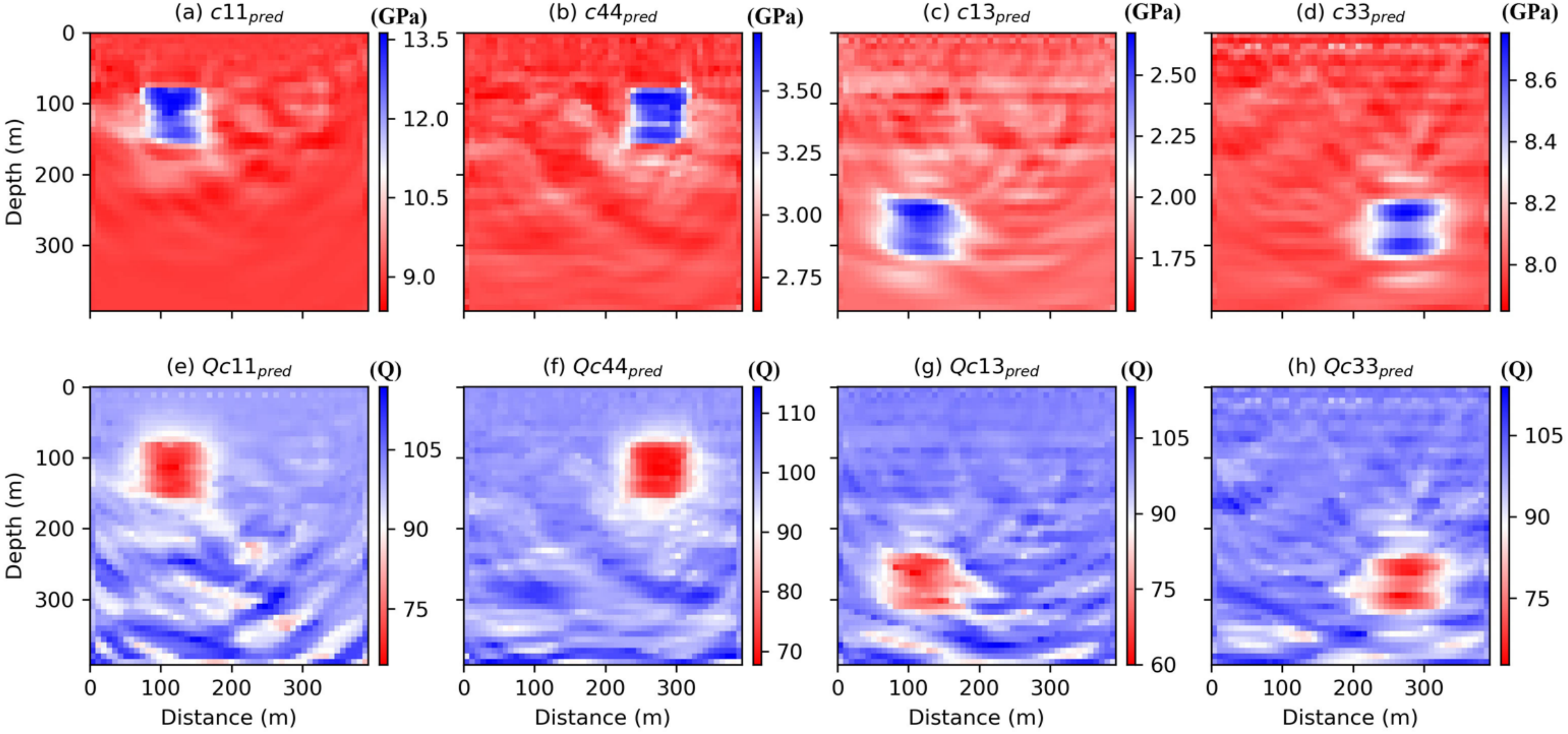
Dx=Dz=7m    Maximum iteration:100




True and initial models for elastic modulus C11, C13, C33, C44

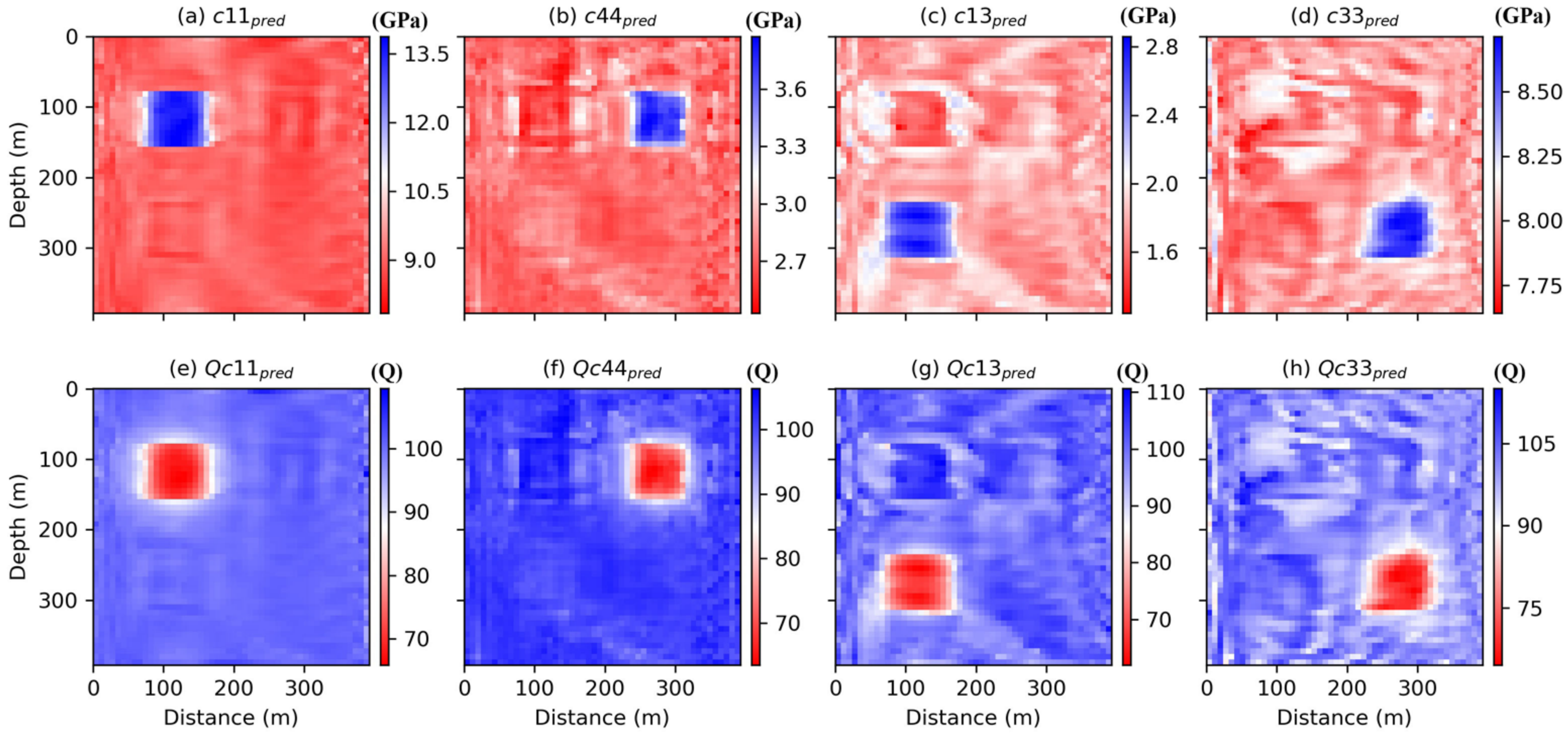
True and initial models for attenuation models Qc11, Qc13, Qc33, Qc44

# Part TWO: Viscoelastic VTI full waveform inversion based on RNN



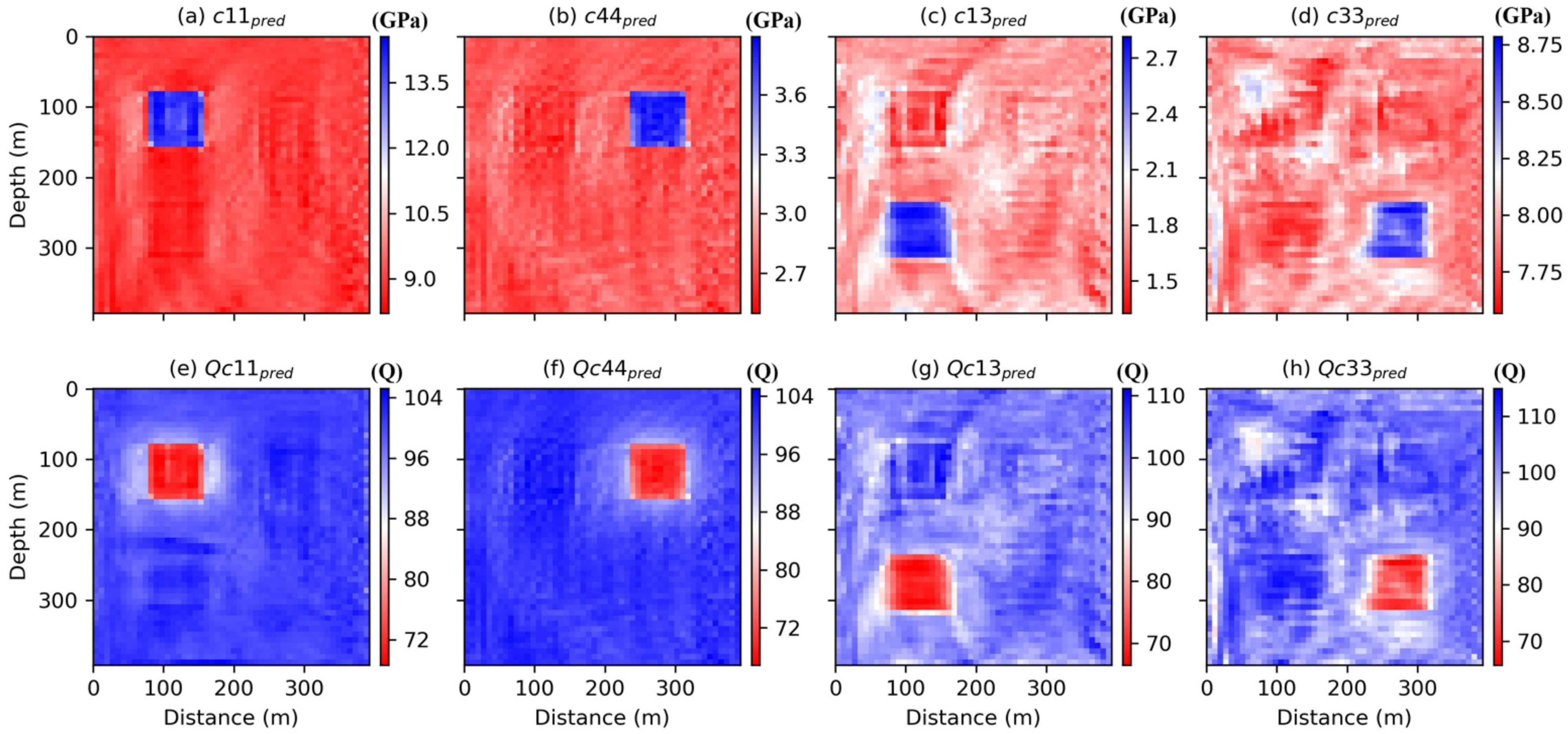

Surface acquisition inversion results

# Part TWO: Viscoelastic VTI full waveform inversion based on RNN



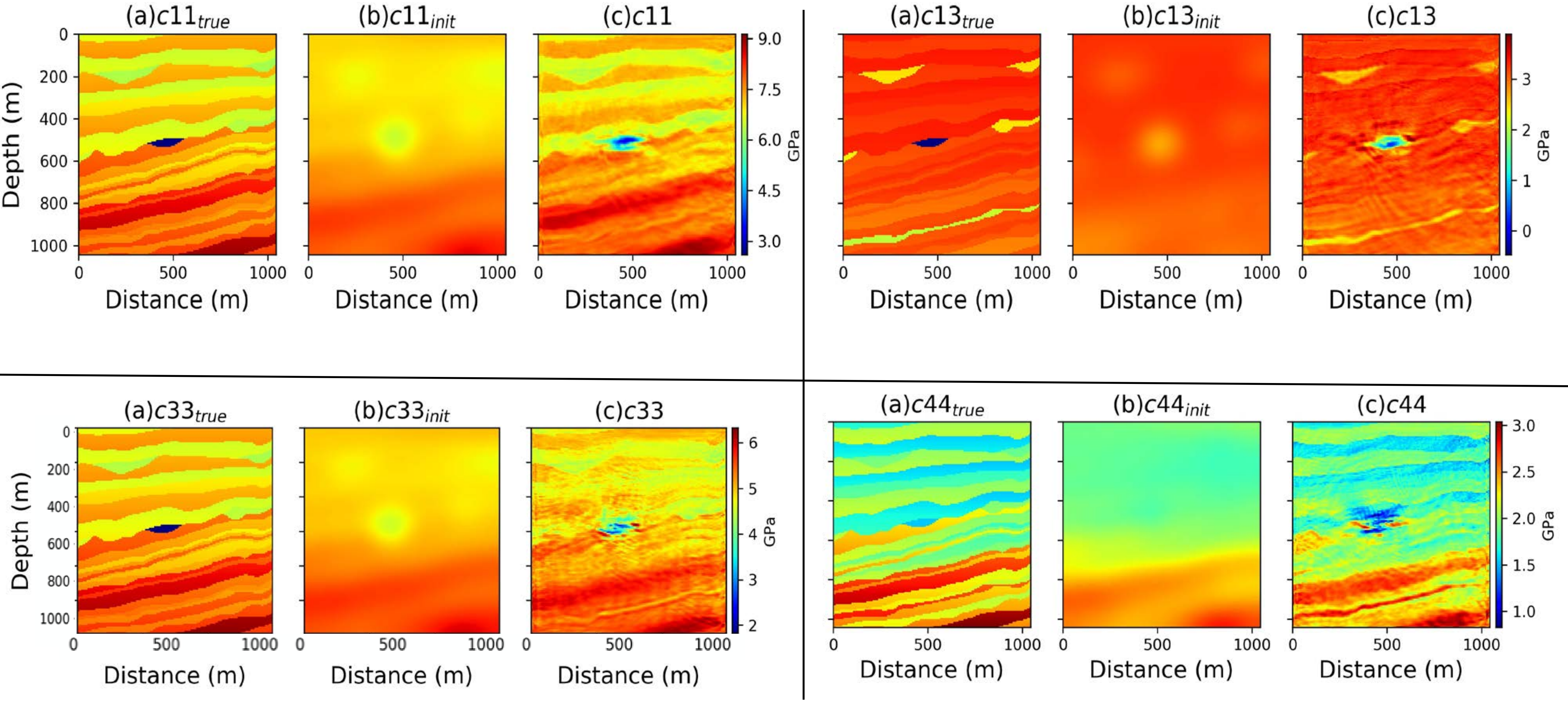

Cross well acquisition inversion results

# Part TWO: Viscoelastic VTI full waveform inversion based on RNN

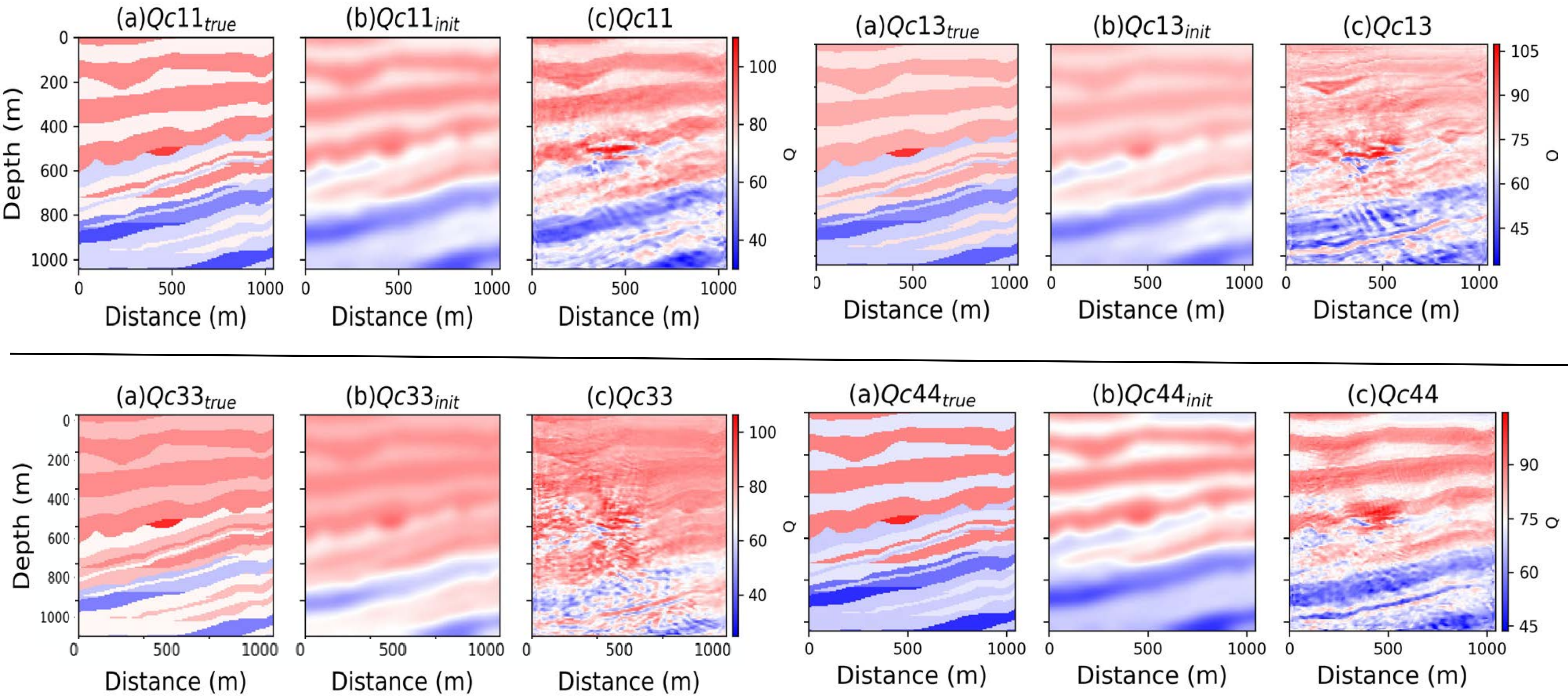



Cross well acquisition inversion results with high order TV regulations

# Part TWO: Viscoelastic VTI full waveform inversion based on RNN



# Part TWO: Viscoelastic VTI full waveform inversion based on RNN



# Part Three:
# TTI full waveform inversion based on RNN

# Part Three: TTI full waveform inversion

Most fractures are not vertically but with certain dips and azimuths, thus estimating the title angles along with the elastic parameters are important for accurately invert the parameters.

For 2D TTI, the orientation of the anisotropic symmetry axis can be described using only the tilt angle.

Using bond transformation, the elasticity coefficients of TTI medium with any azimuths and dips can be transformed from the constitutive coordinates to the Cartesian coordinates.

In Cartesian observation system, the elastic coefficient matrix of a TTI media can be expressed as:

$$\mathbf{C_{TTI}} = \mathbf{M}_\theta \mathbf{M}_\theta \mathbf{C_{VTI}} \mathbf{M}_\phi^{\mathbf{T}} \mathbf{M}_\phi^{\mathbf{T}}, \tag{12}$$

where $M_\theta$ and $M_\phi$ can be expressed as:

$$\mathbf{M}_\theta = \begin{bmatrix} cos^2\theta & 0 & sin^2\theta & 0 & -sin2\theta & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ sin^2\theta & 0 & cos^2\theta & 0 & sin2\theta & 0 \\ 0 & 0 & 0 & cos\theta & 0 & sin\theta \\ \frac{1}{2}sin2\theta & 0 & -\frac{1}{2}sin2\theta & 0 & cos2\theta & 0 \\ 0 & 0 & 0 & -sin\theta & 0 & cos\theta \end{bmatrix} \quad \mathbf{M}_\phi = \begin{bmatrix} cos^2\phi & sin^2\phi & 0 & 0 & 0 & -sin2\phi \\ sin^2\phi & cos^2\phi & 0 & 0 & 0 & sin2\phi \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & cos\phi & sin\phi & 0 \\ 0 & 0 & 0 & -cos\phi & cos\phi & 0 \\ -\frac{1}{2}sin2\phi & -\frac{1}{2}sin2\phi & 0 & 0 & 0 & cos2\phi \end{bmatrix} \tag{13}$$

# Part Three: TTI full waveform inversion

The stress velocity relationship can be expressed as:

$$\begin{aligned}\sigma_{xx} = &[R_{11}(R_{11}C_{11} + R_{13}C_{13}) + R_{13}(R_{11}C_{13} + R_{13}C_{33}) + R_{15}C_{55}R_{15}]v_{xx} \\ &+[(R_{11}C_{11} + R_{13}C_{13})R_{31} + (R_{11}C_{13} + R_{13}C_{33})R_{33} + R_{15}C_{55}R_{35}]v_{zz} \\ &+[(R_{11}C_{11} + R_{13}C_{13})R_{51} + (R_{11}C_{13} + R_{13}C_{53})R_{53} + R_{15}C_{55}R_{55}]v_{xz}\end{aligned} \tag{14}$$

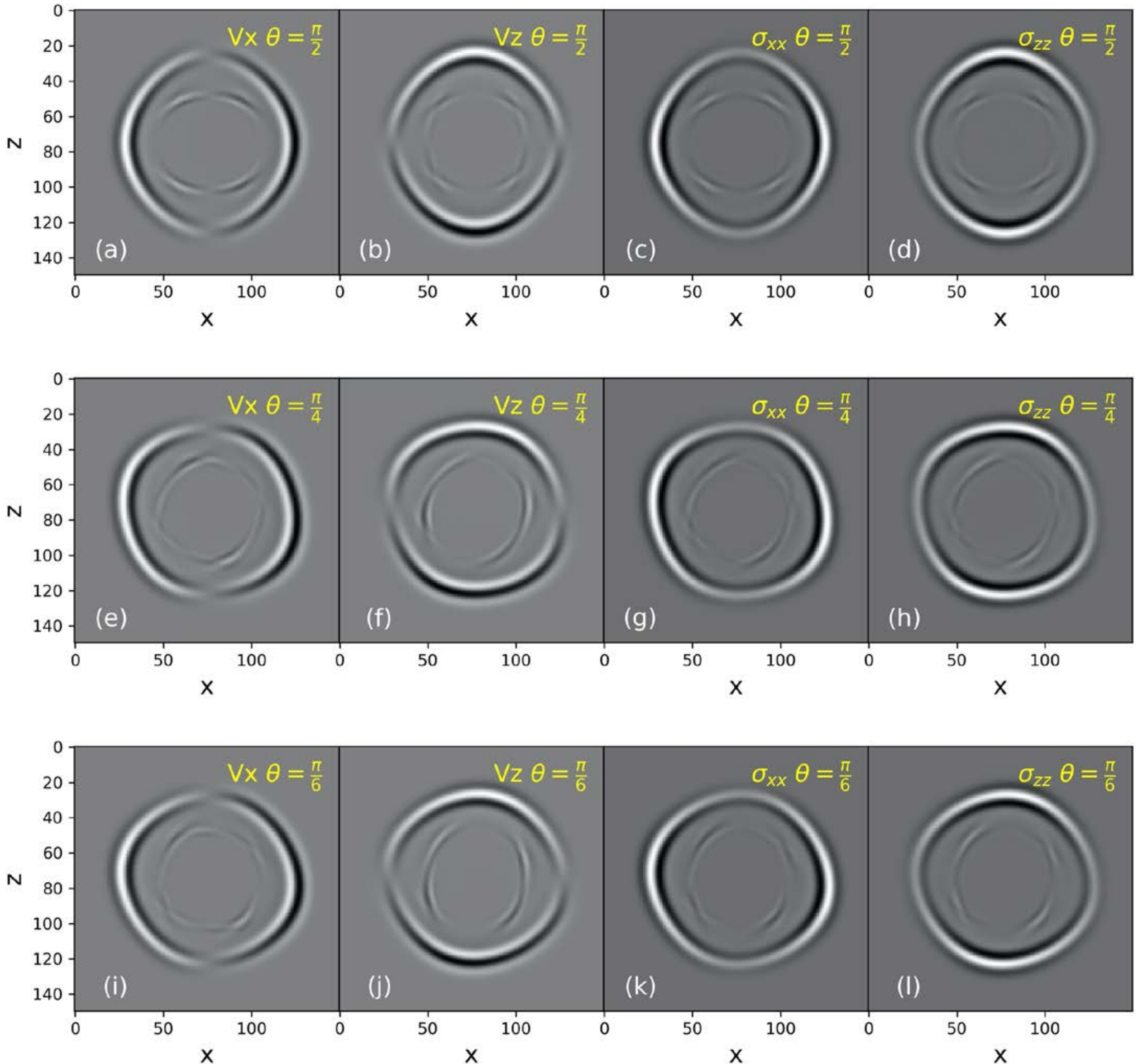
$$\begin{aligned}\sigma_{zz} = &[(R_{31}C_{11} + R_{13}C_{13})R_{11} + (R_{31}C_{13} + R_{33}C_{33})R_{13} + R_{35}C_{55}R_{15}]v_{xx} \\ &+[(R_{31}C_{11} + R_{33}C_{13})R_{31} + (R_{31}C_{13} + R_{33}C_{33})R_{33} + R_{35}C_{55}R_{35}]v_{zz} \\ &+[(R_{31}C_{11} + R_{33}C_{13})R_{51} + (R_{31}C_{13} + R_{33}C_{33})R_{53} + R_{35}C_{55}R_{55}]v_{xz}\end{aligned} \tag{15}$$

$$\begin{aligned}\sigma_{xz} = &[(R_{51}C_{11} + R_{53}C_{13})R_{11} + (R_{51}C_{13} + R_{53}C_{33})R_{13} + R_{55}C_{55}R_{15}]v_{xx} \\ &+[(R_{51}C_{11} + R_{53}C_{13})R_{31} + (R_{51}C_{13} + R_{53}C_{33})R_{33} + R_{55}C_{55}R_{35}]v_{zz} \\ &+[(R_{51}C_{11} + R_{53}C_{13})R_{51} + (R_{51}C_{13} + R_{53}C_{33})R_{53} + R_{55}C_{55}R_{55}]v_{xz}\end{aligned} \tag{16}$$

R is the rotation matrix and each element of R can be expressed as:

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{13} & R_{15} \\ R_{31} & R_{33} & R_{35} \\ R_{51} & R_{53} & R_{55} \end{bmatrix} = \begin{bmatrix} cos^2\phi cos^2\theta & cos^2\phi sin^2\theta & cos^2\phi sin2\theta \\ sin^2\theta & cos^2\theta & sin2\theta \\ \frac{cos\phi sin2\theta}{2} & -\frac{cos\phi sin2\theta}{2} & cos\phi cos2\theta \end{bmatrix}. \tag{17}$$

# Part Three: TTI full waveform inversion

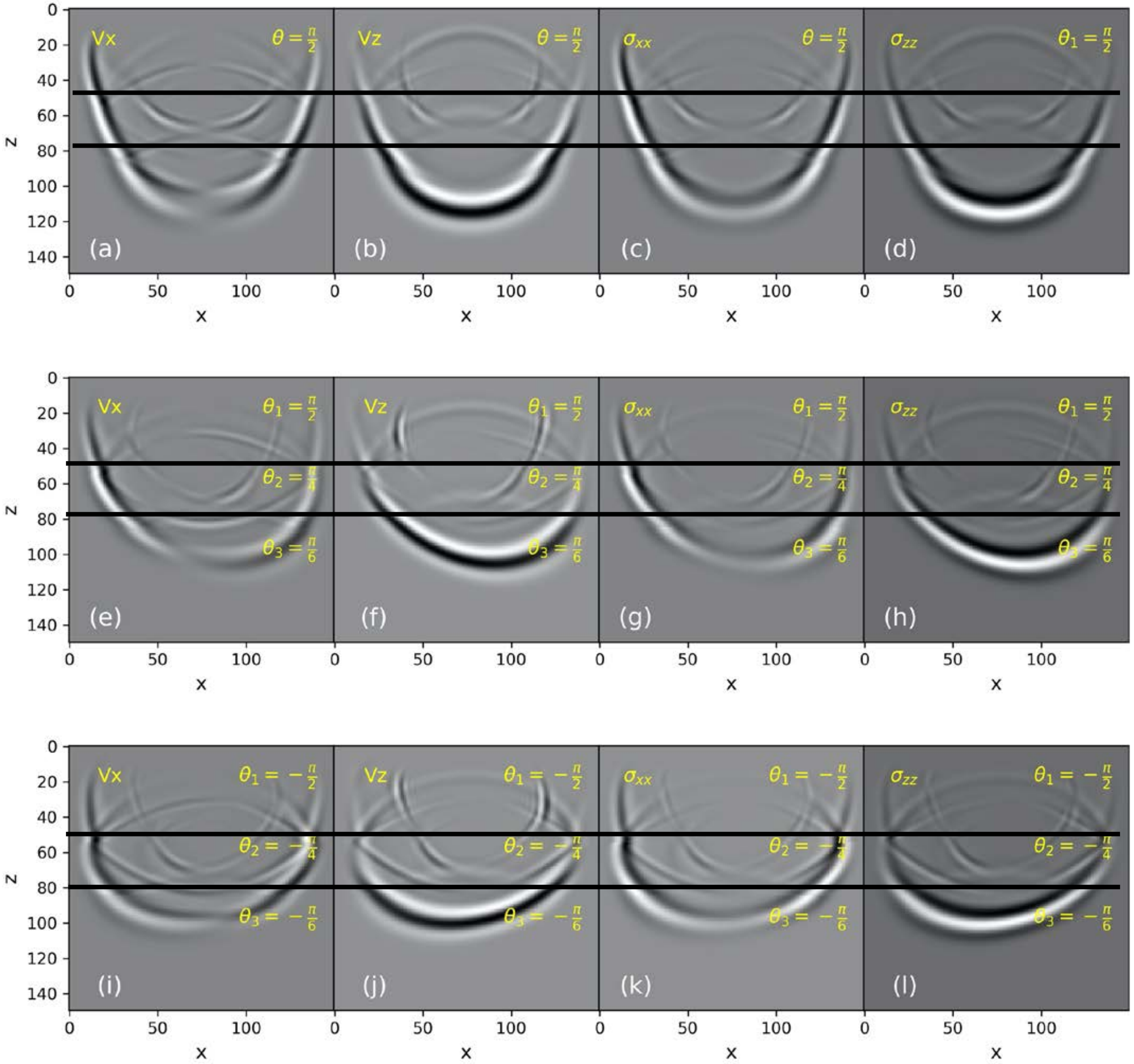



C11 = 9 GPa,
C13 = 1.79 Gpa
C33= 8 GPa ,
C44 = 2.79 GPa.

Source : Ricker's wavelet 20Hz located at Center of the model
Dx = Dz = 7m
Model size :50*50

How title can influence the shape of wavefields

# Part Three: TTI full waveform inversion



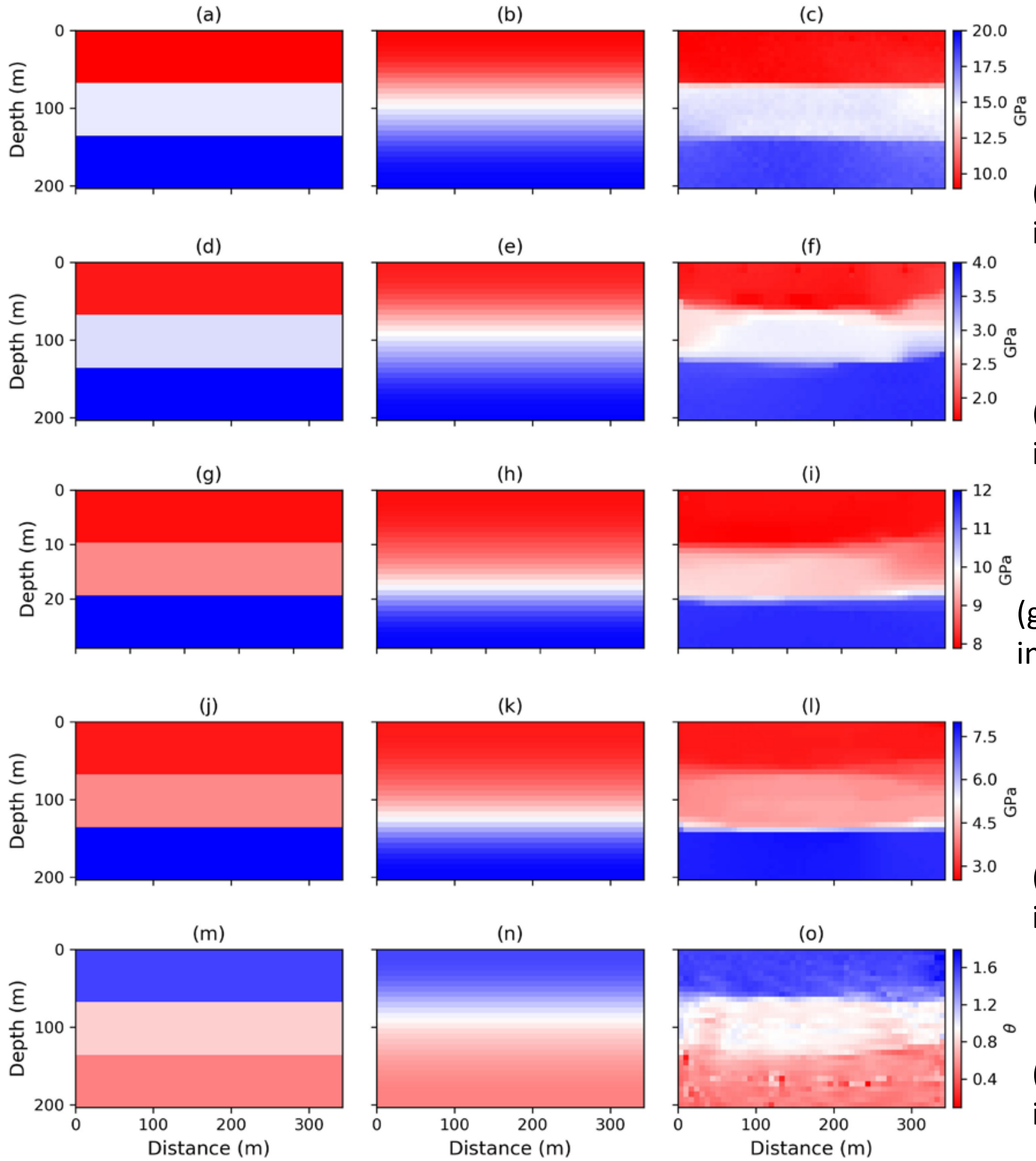

Source : Ricker's wavelet 20Hz

Dx = Dz = 7m

Model size :50*50

How title can influence the shape of wavefields

# Part Three: TTI full waveform inversion




(a),(b),(c) True, initial and inversion result for C11

(d),(e),(f) True, initial and inversion results for C13

(g),(h),(i)True, initial and inversion results for C33

(j),(k),(l)True, initial and inversion results for C44

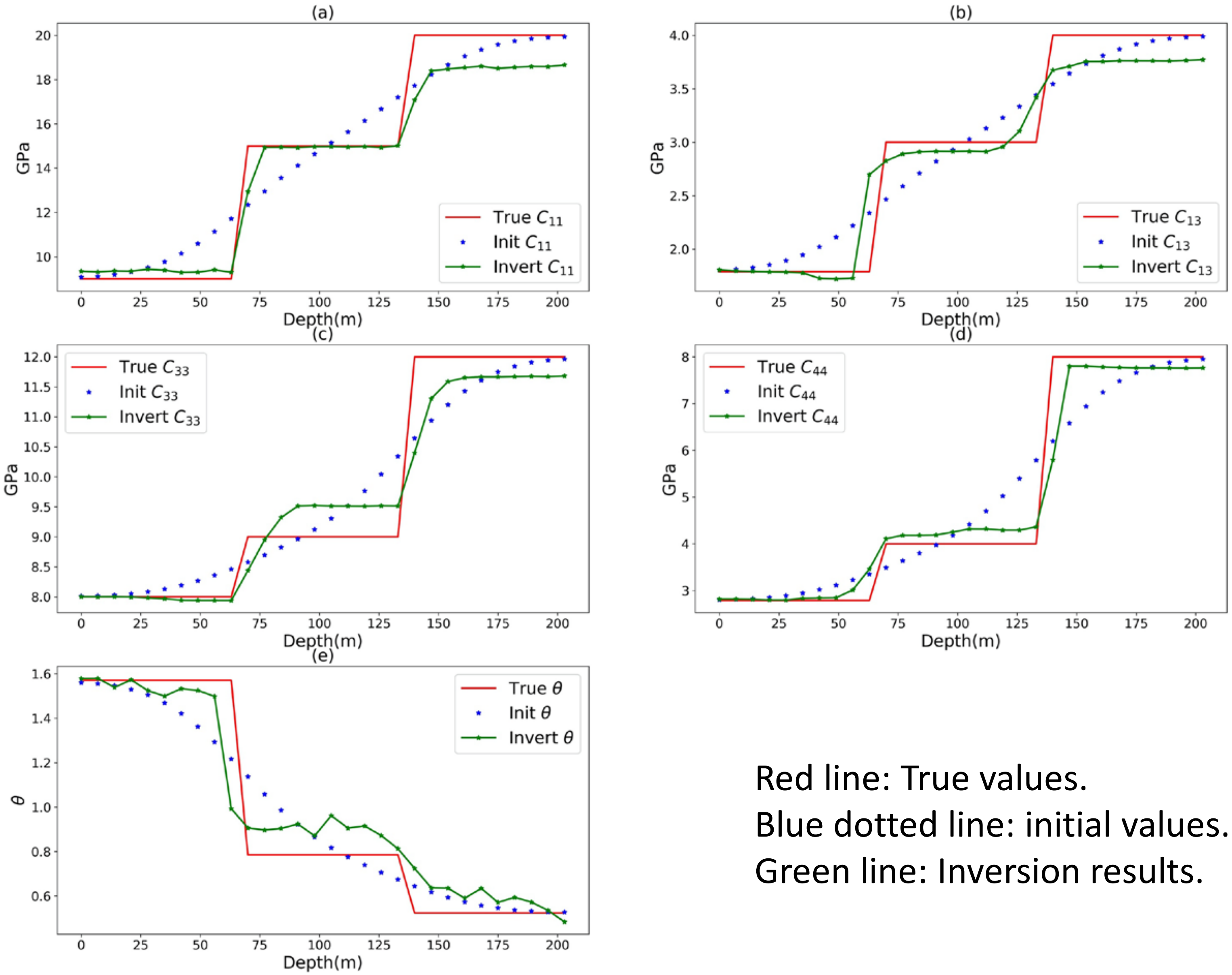
(m)(n)(o)True, initial and inversion results for title angle

Maximum time: 0.7s
dt = 0.001

Wavelet: Ricker wavelet (Main frequency 20Hz)

Maximum iteration:100

# Part Three: TTI full waveform inversion




Maximum time: 0.7s
dt = 0.001

Wavelet: Ricker wavelet
(Main frequency 20Hz)

Maximum iteration:100

Red line: True values.
Blue dotted line: initial values.
Green line: Inversion results.

# Conclusions and future study

**Conclusions :**

- RNN is a powerful tool for seismic inversion problem.
- Attenuation, azimuth title angle have influence in the inversion results and should be considered into full waveform inversion
- With automatic gradient method, gradient based on complex misfits can be calculated

**Future study:**

- Incorporate more data drive methods to mitigate modeling error problems.
- As we can now calculate more complex misfits for the inversion, we will search for more suitable misfits more FWI that may suffer less from issues like local minimum.
- Finding new parameterizations with less cross talk

# Conclusions and future study

- **CREWES sponsors, staff and students**
- **CSC (China Scholarship Council )**

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