Constraining model parameters in full waveform inversion

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Full waveform inversion

- FWI: match data by minimizing ϕ with respect to m
- Use first and second derivatives of ϕ to find a descent direction d
- Calculate step-length α which minimizes ϕ in that direction

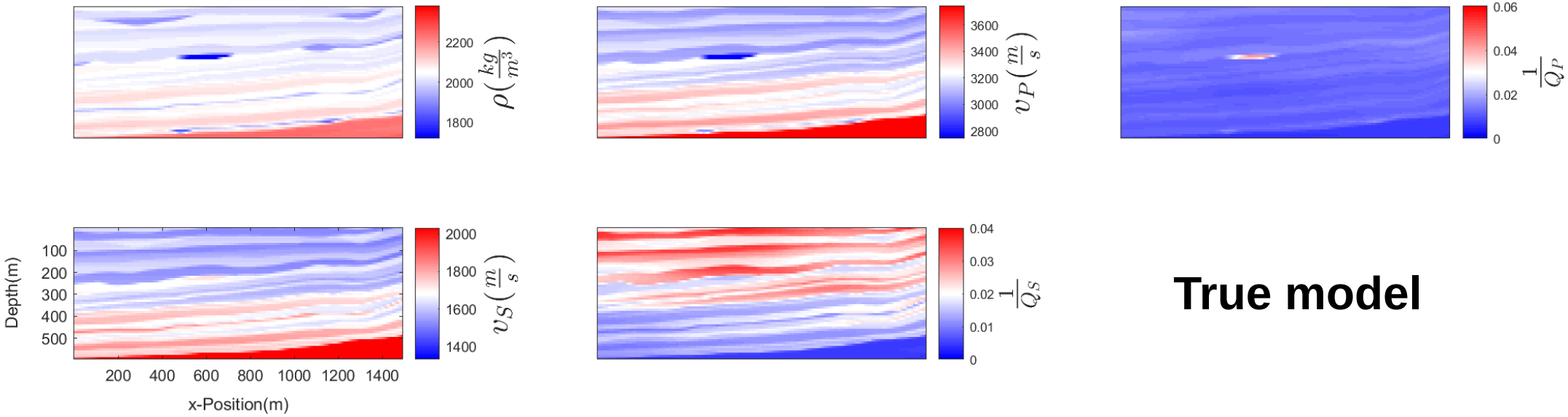
$$m_{k+1} = m_k + \alpha d(\phi, m_k)$$

Model constraint

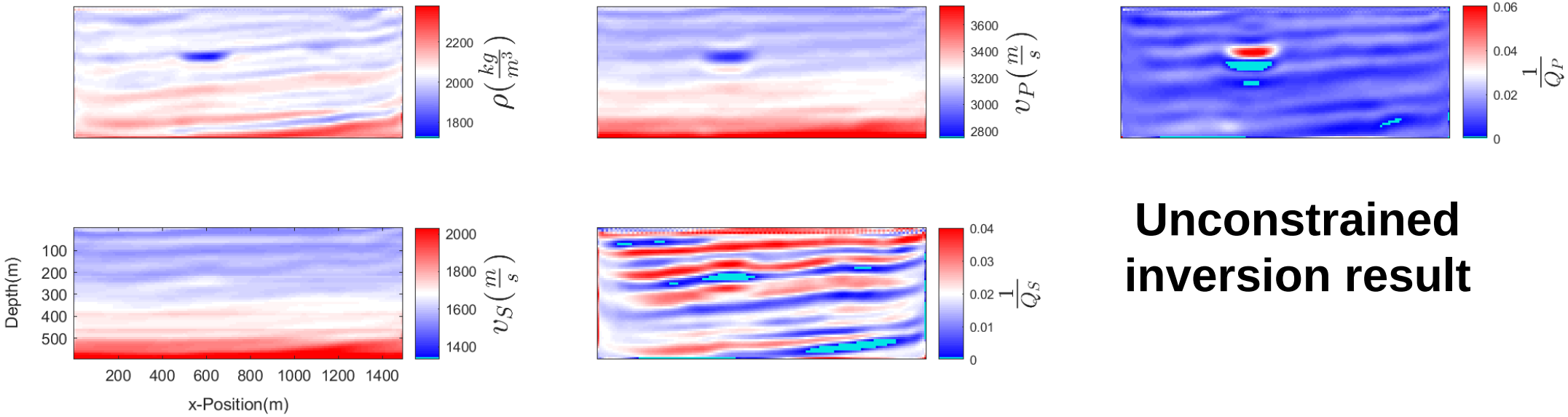
- Properties like Q, $V_{P,}$ and porosity have physical constraints
- A naïve inversion will not always satisfy these constraints

How can we make our inversion satisfy our constraints?

Unconstrained inversion



True model



Unconstrained inversion result

Strategies for constraining optimization

We consider five broad approaches to constraining the inversion:

1. Step-length restriction
2. Projection of updates / update directions
3. Variable transforms
4. Hard penalty terms
5. Soft penalty terms

$$m_{k+1} = m_k + \alpha d(\phi, m_k)$$

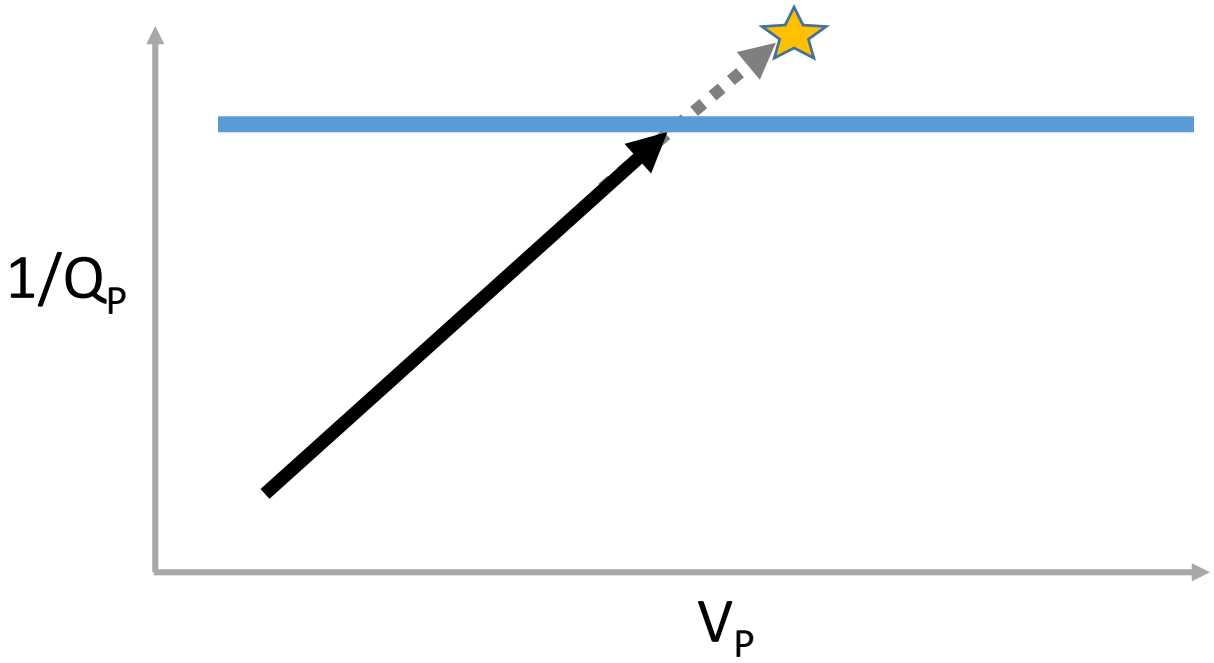
Cap step-lengths to prevent constraint violation

Advantages:

- Very simple to implement
- Exactly enforces constraint

Disadvantages:

- Often leads to cripplingly slow convergence or stalling



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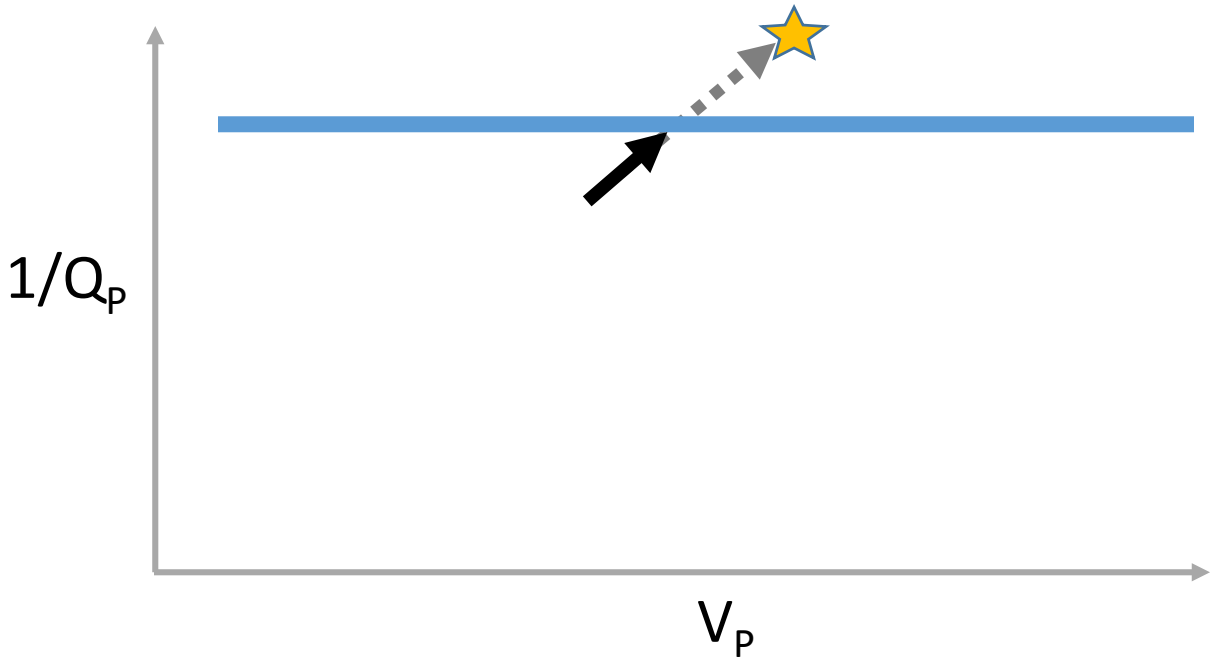

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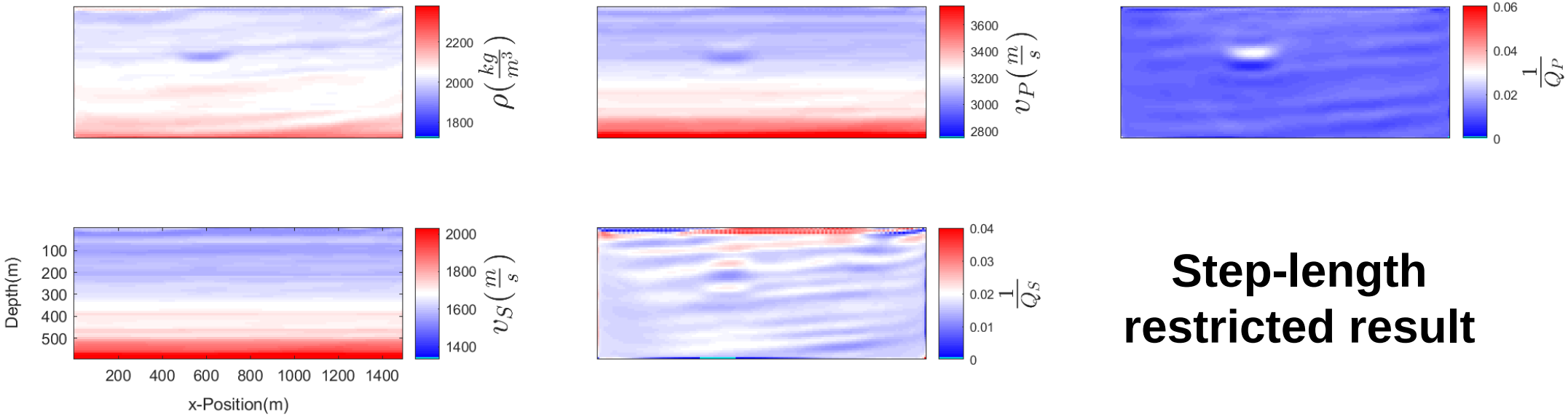
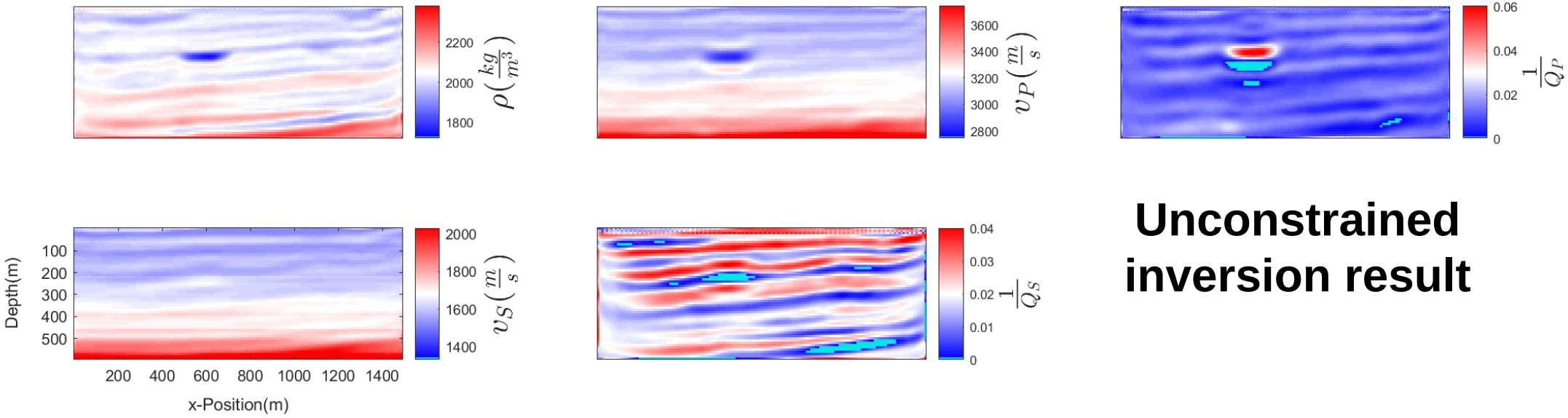
Step-length restriction



Step-length restricted result



Unconstrained inversion result

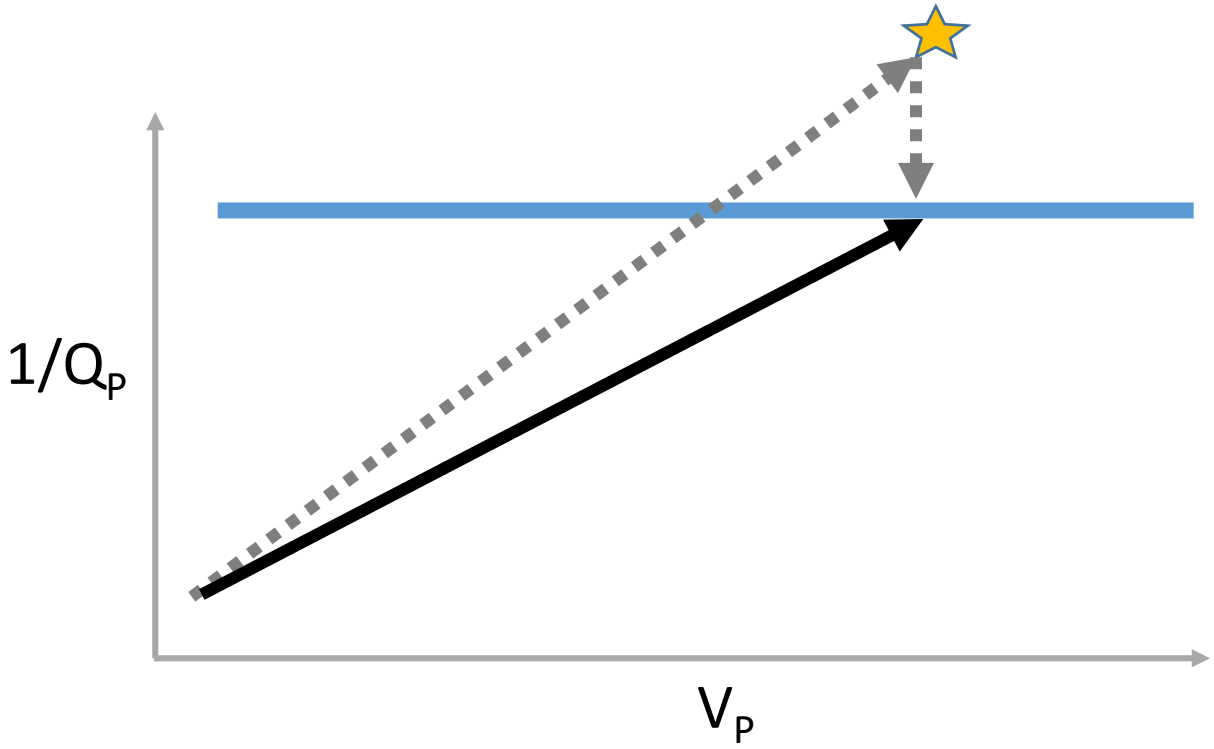
Line-search past constraint, project back to allowed region

Advantages:

- Very simple to implement
- Exactly enforces constraint
- Avoids very small or zero length steps

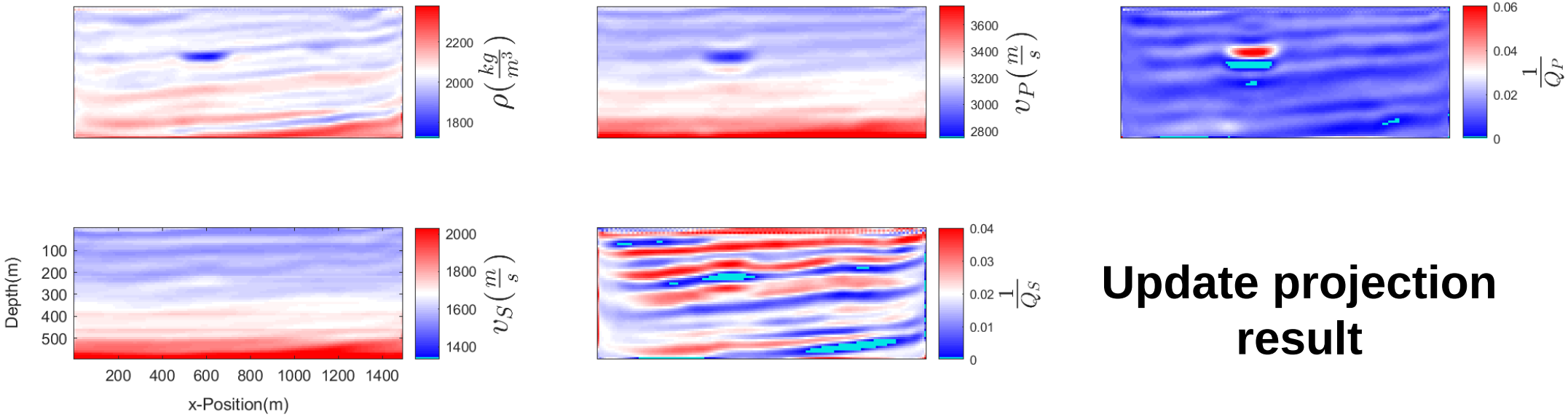
Disadvantages:

- Doesn't always decrease objective function
- Blind to barrier

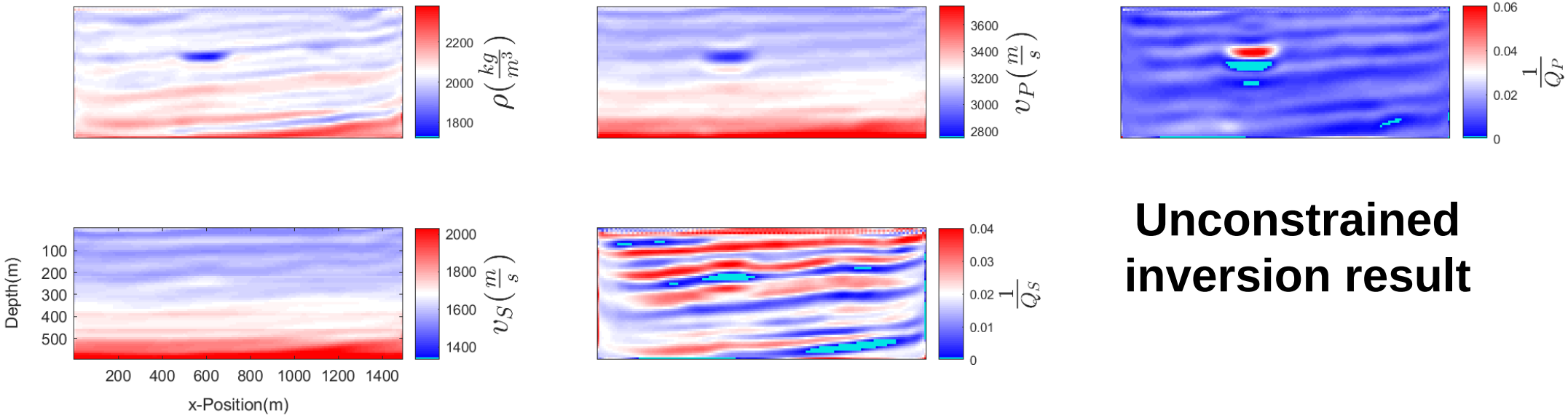


$$m_{k+1} = m_k + \alpha d(\phi, m_k)$$

Projection of updates



Update projection result



Unconstrained inversion result

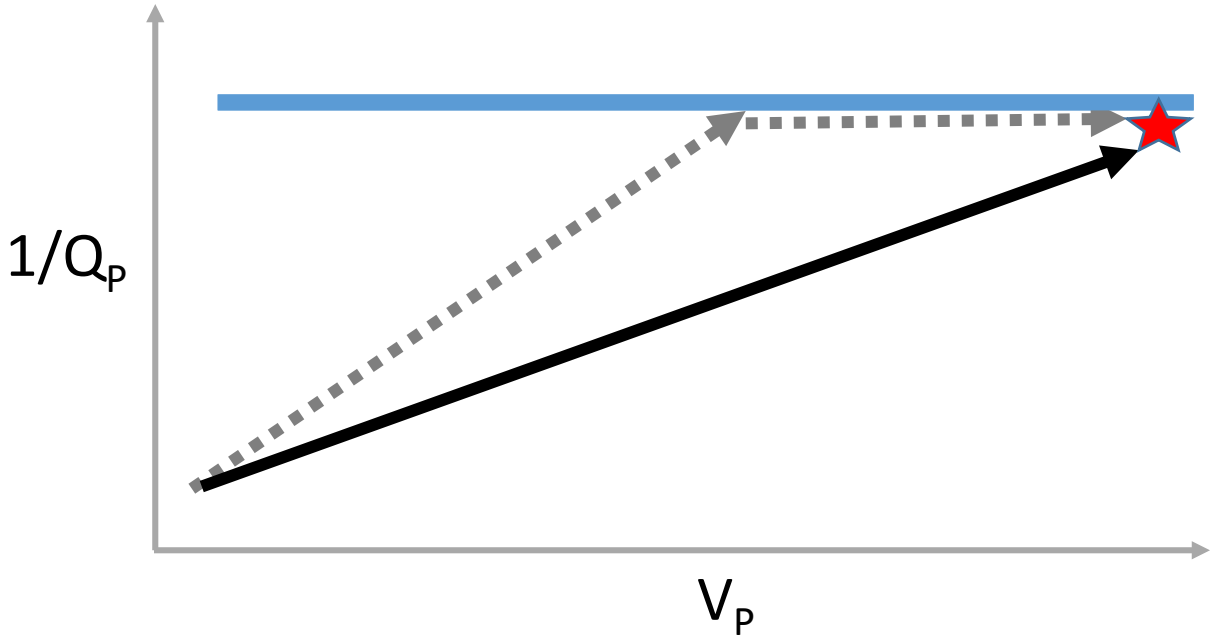
Project search direction onto constraints reached

Advantages:

- **Exactly enforces constraint**
- **Avoids very small or zero length steps**
- **Only considers valid models in line-search**

Disadvantages:

- **Hugs, but ignores, the boundary**
- **Makes second-derivative information misleading**



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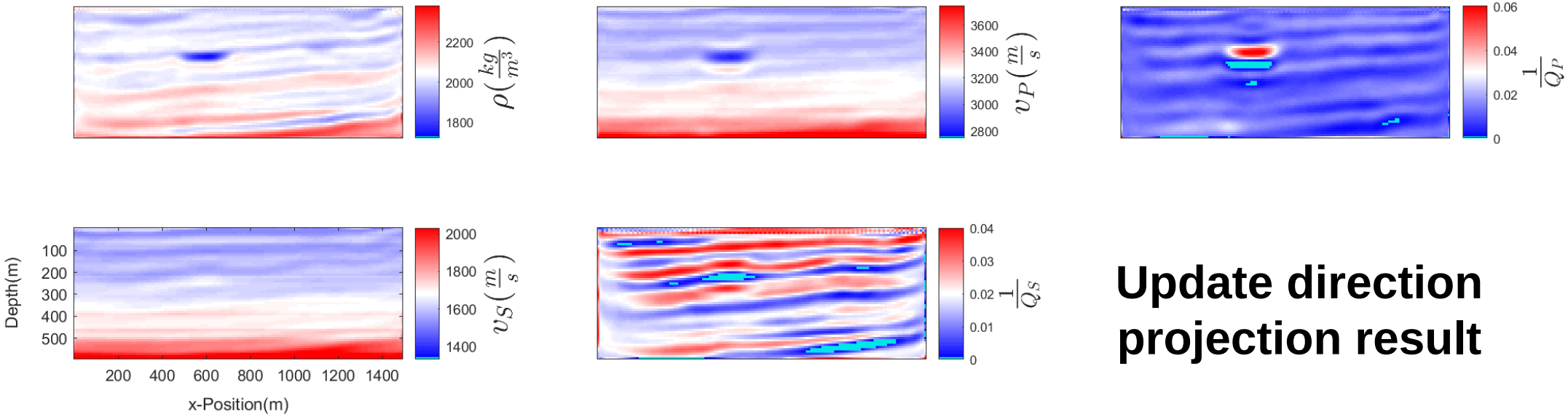
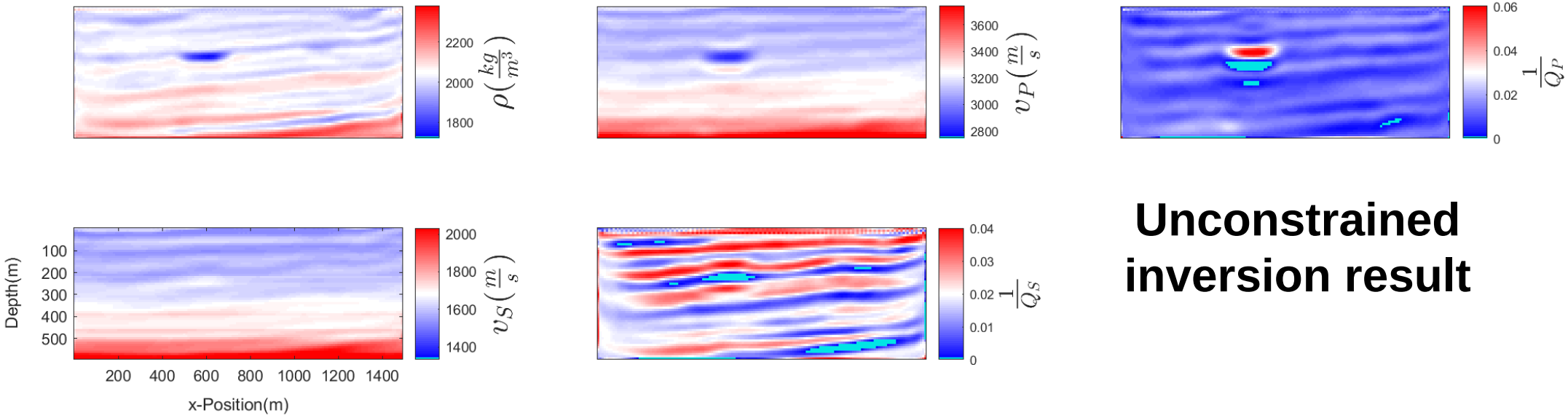
Projection of update directions



Update direction projection result



Unconstrained inversion result

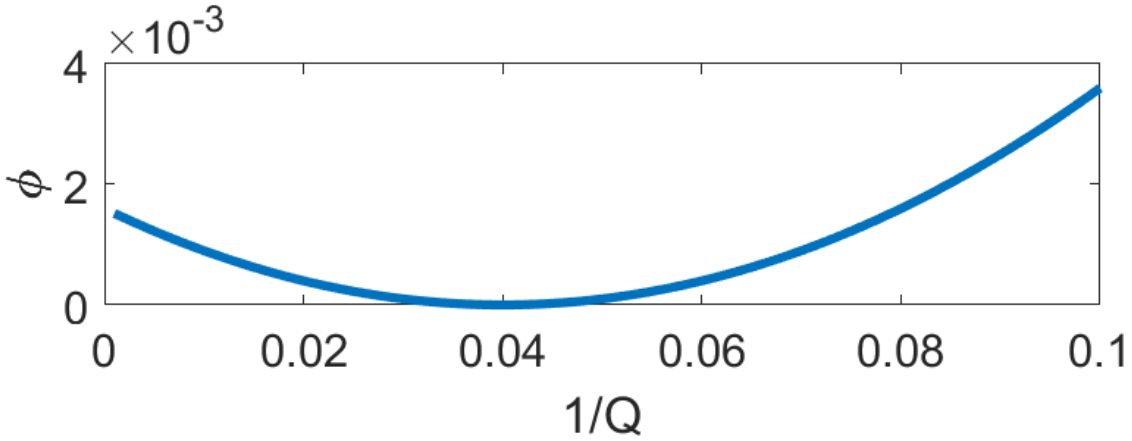
Choose variables which cannot violate constraints

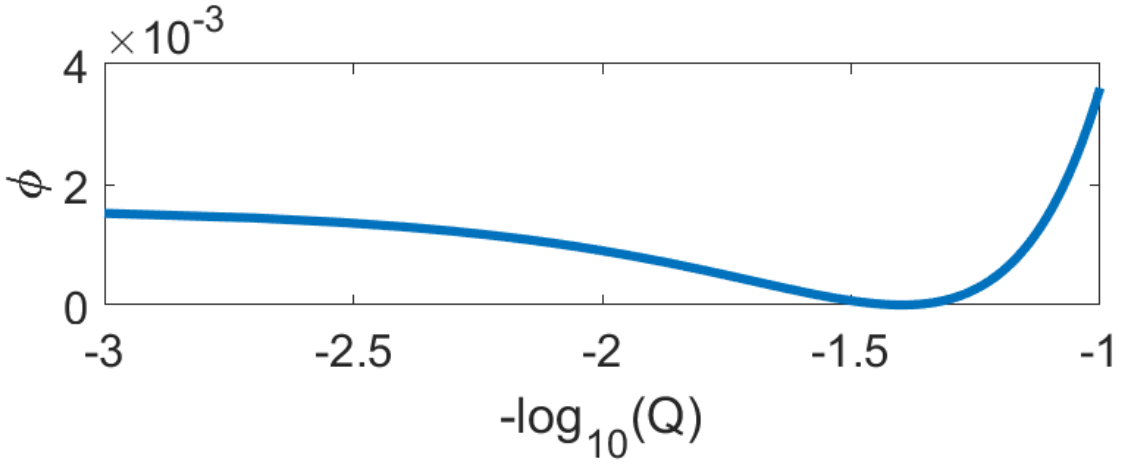
Advantages:

- Exactly enforces constraint
- Optimization approach can recognize constraint

Disadvantages:

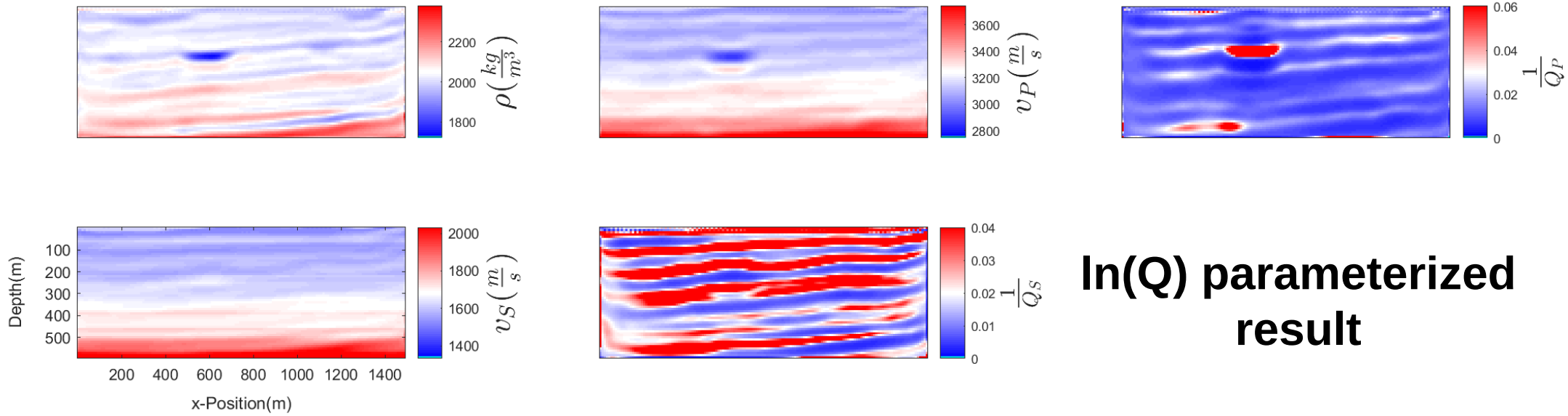
- If original problem was close to linear, transformed version will be very nonlinear



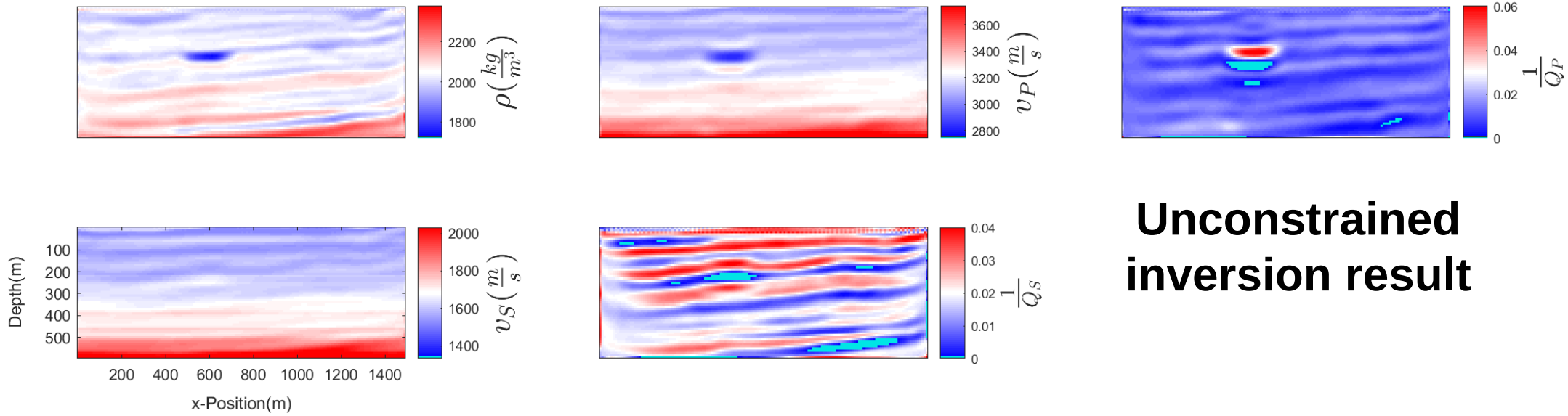


$$m_{k+1} = m_k + \alpha d(\phi, m_k)$$

Variable transformations



In(Q) parameterized result



Unconstrained inversion result

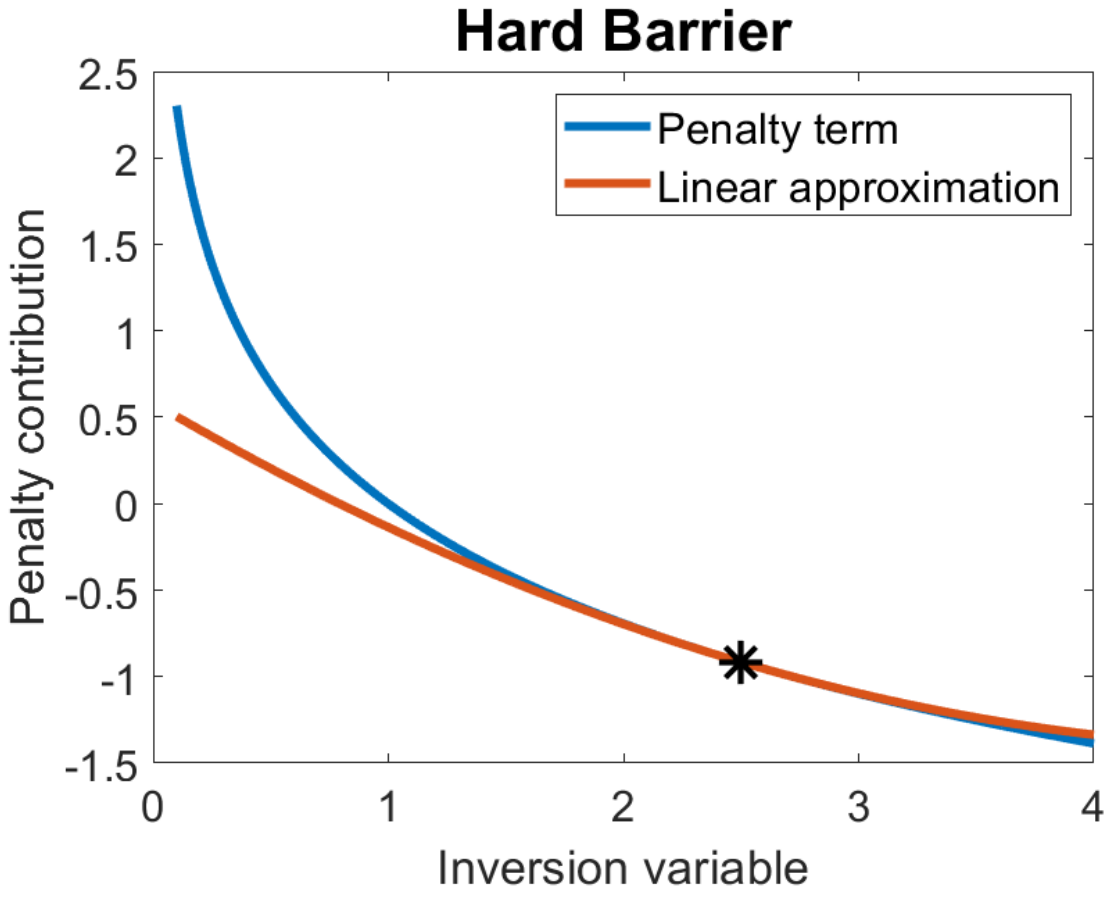
Infinite penalty at constraint

Advantages:

- Exactly enforces constraint
- Optimization approach can recognize constraint

Disadvantages:

- Barriers can be traps - easy to approach, difficult to get away from



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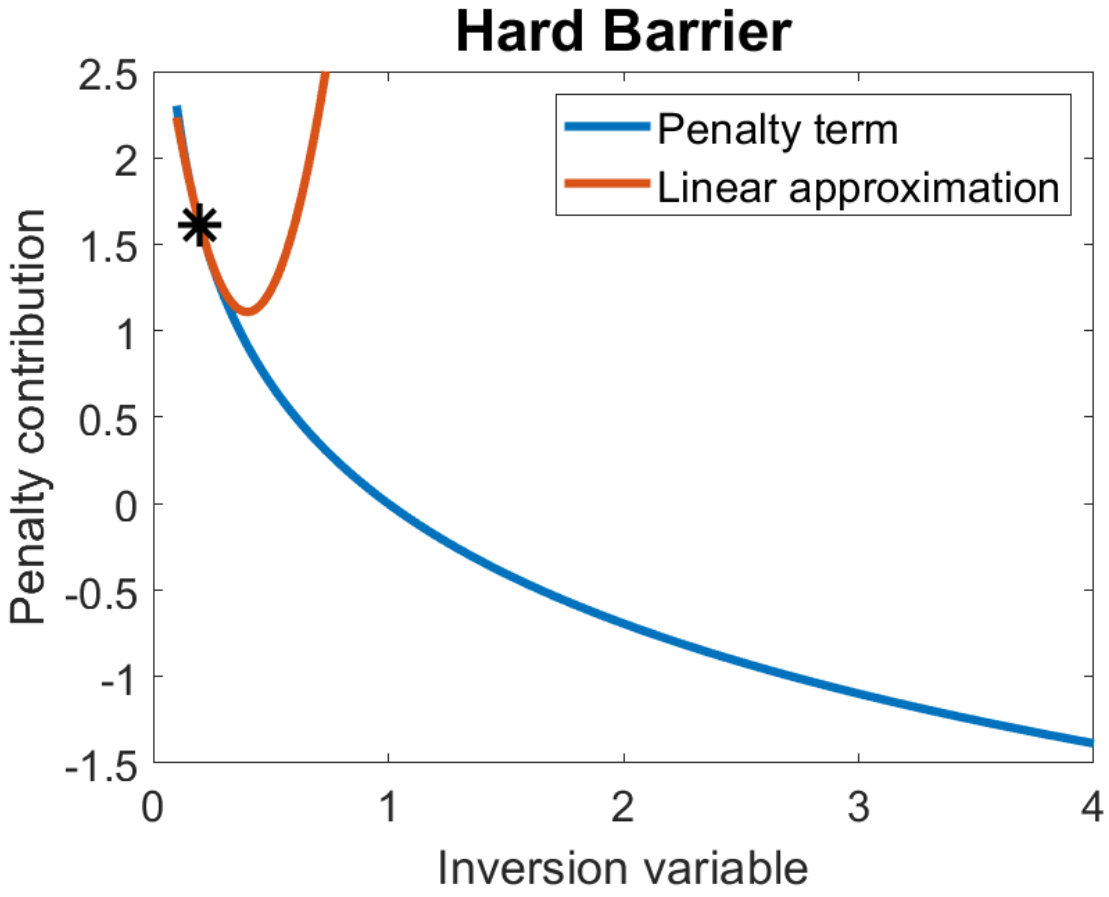

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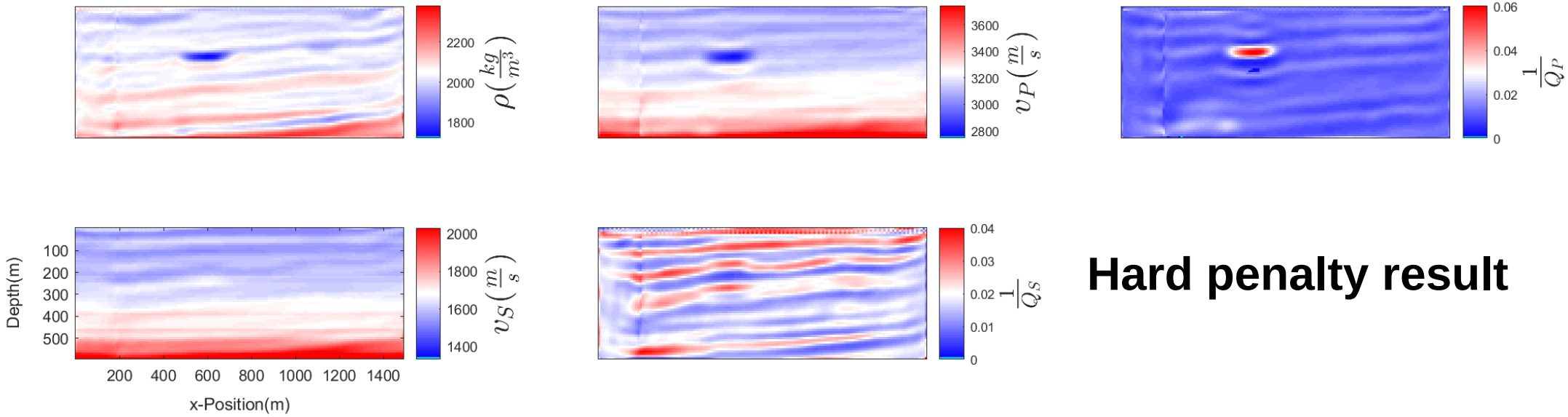
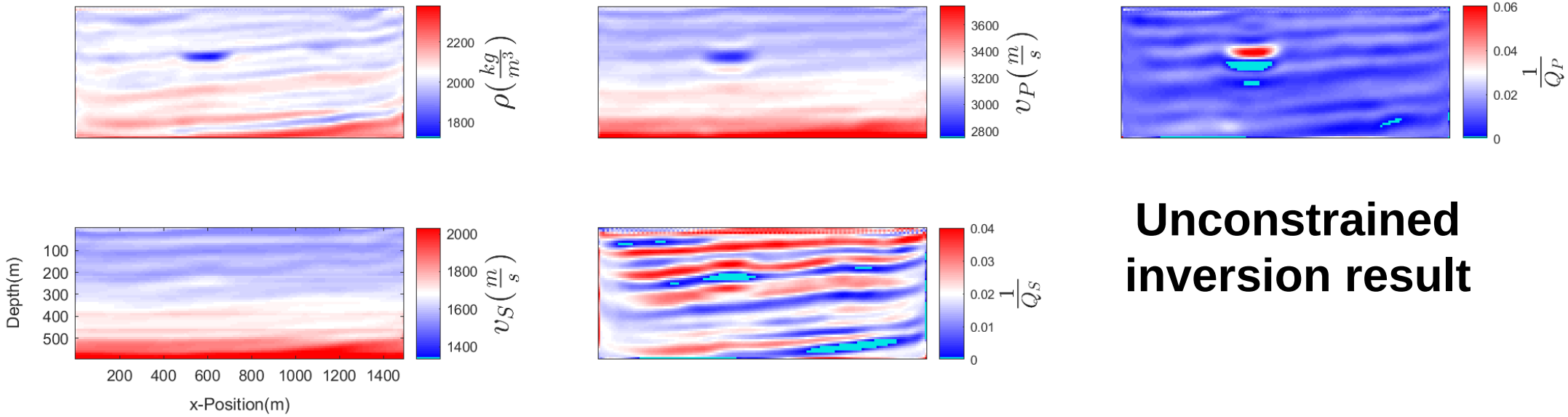
Hard penalty terms



Hard penalty result



Unconstrained inversion result

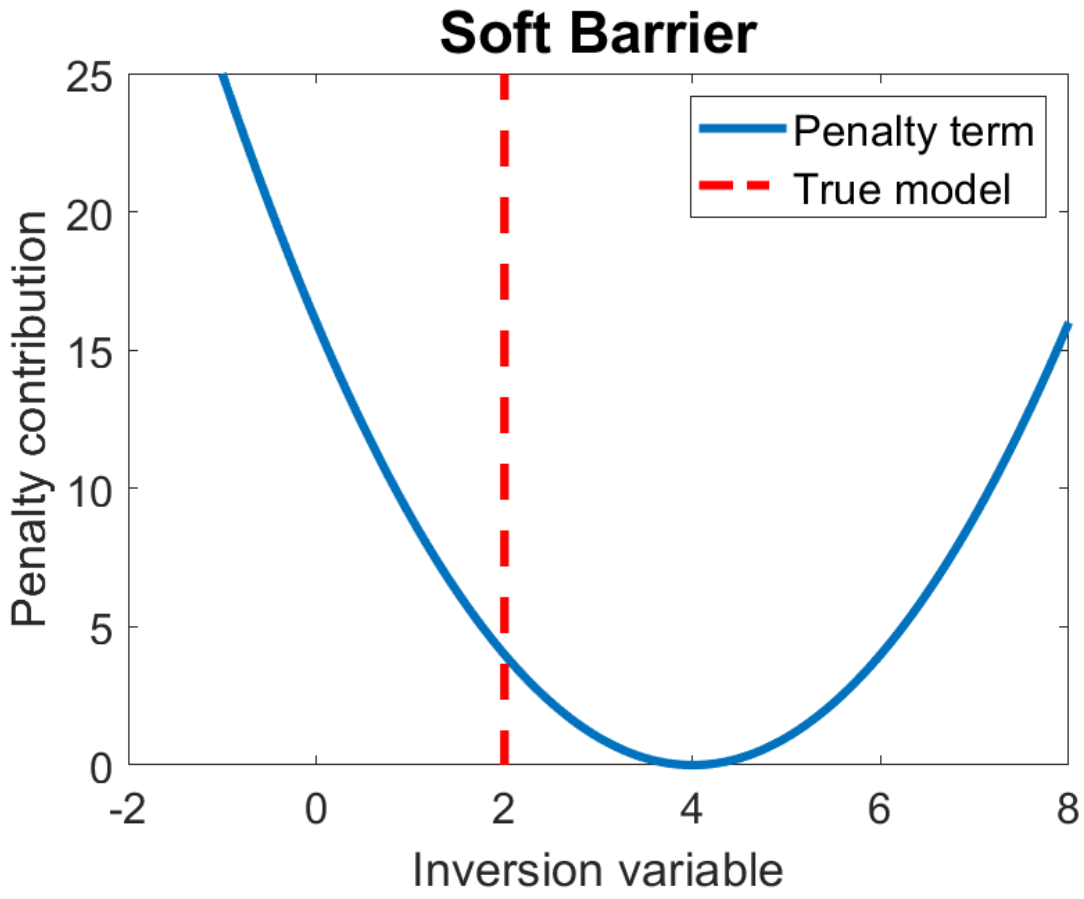
Finite linear penalty

Advantages:

- **Optimization approach anticipates penalty everywhere**

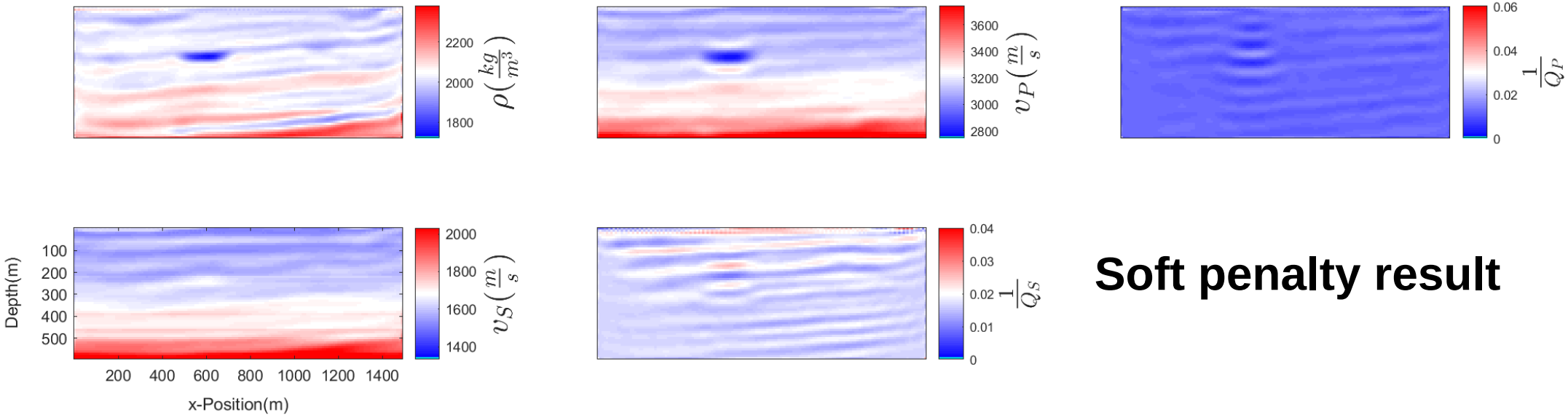
Disadvantages:

- **Doesn't exactly enforce constraint**
- **Linear penalty terms push strongly on models even far from constraint**

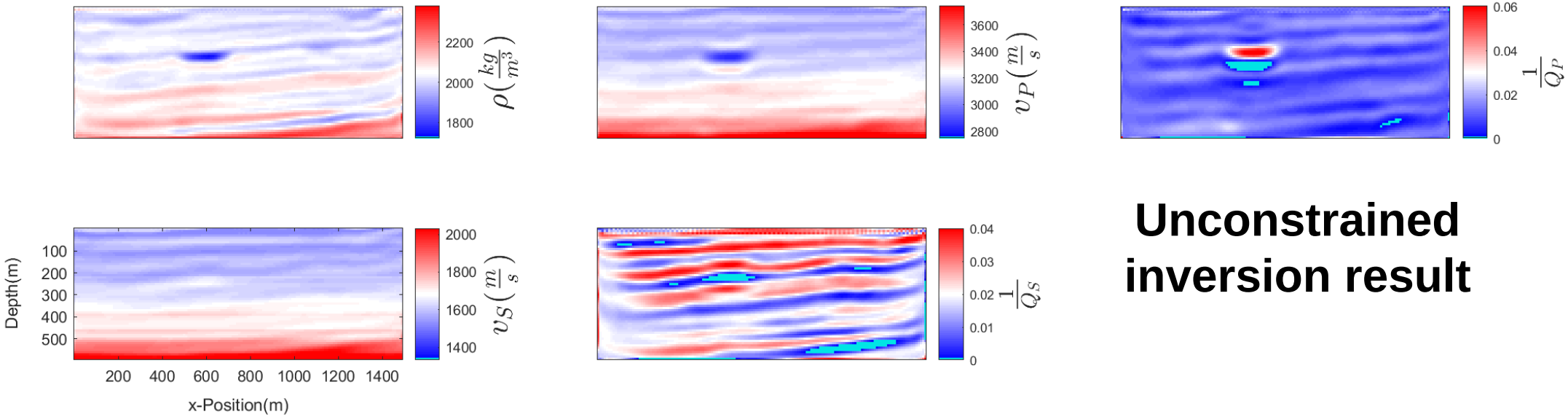


$$m_{k+1} = m_k + \alpha d(\phi, m_k)$$

Soft penalty terms



Soft penalty result



Unconstrained inversion result

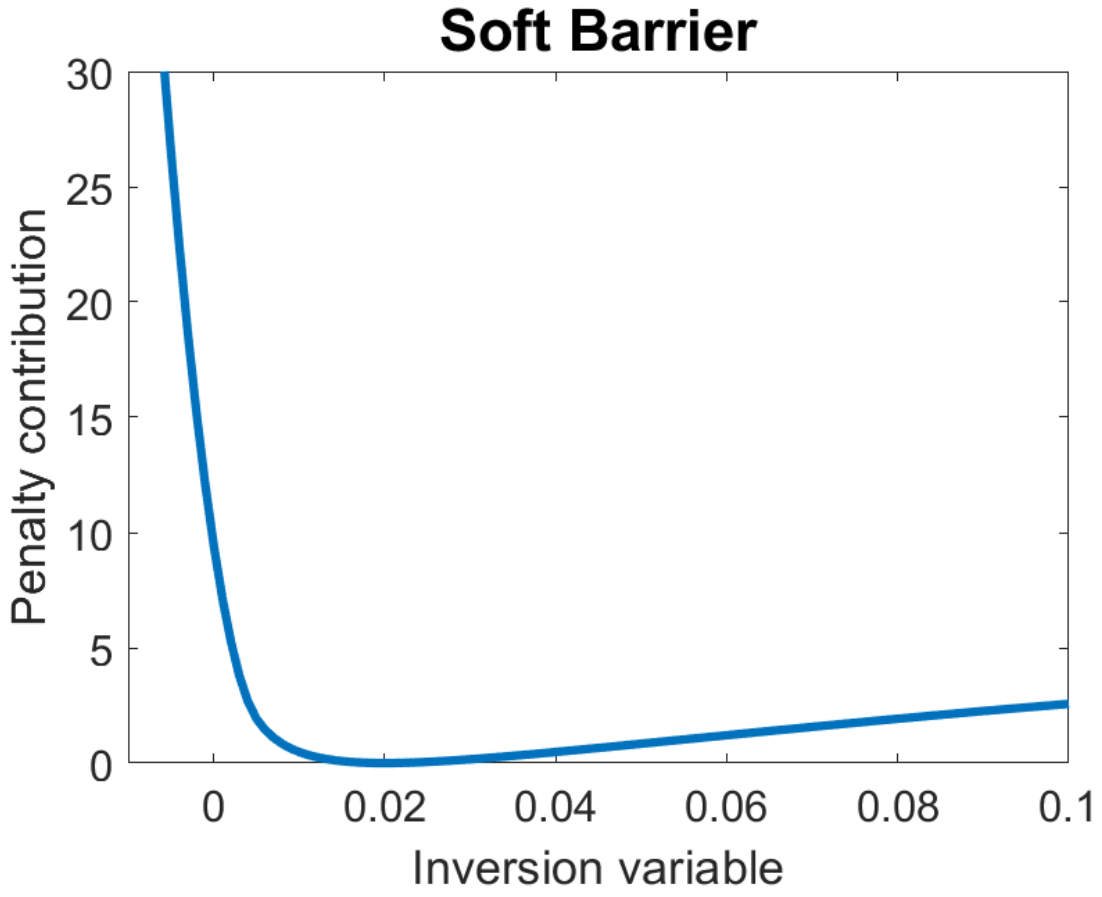
Linear-penalty to mild-penalty transition

Advantages:

- Anticipates penalty close to constraint
- Little to no forcing away from constraint

Disadvantages:

- Doesn't exactly enforce constraint
- Struggles to anticipate penalty away from constraint



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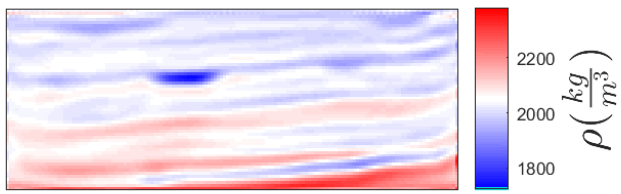
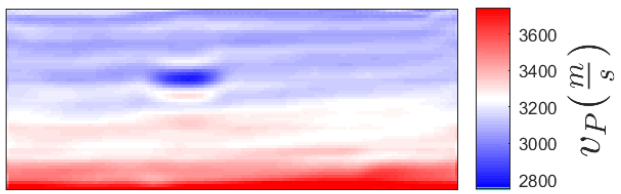
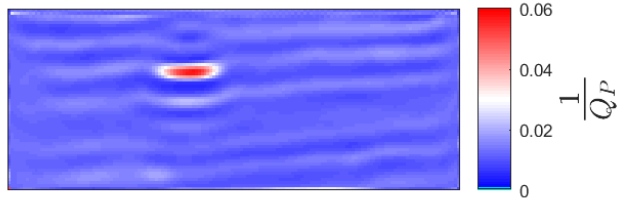
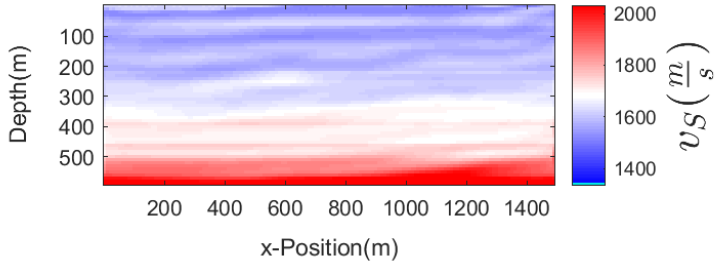
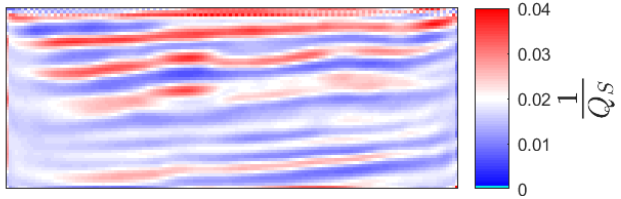
Piecewise soft penalty terms



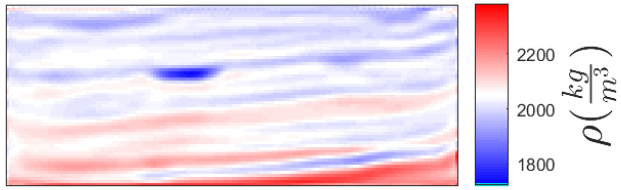


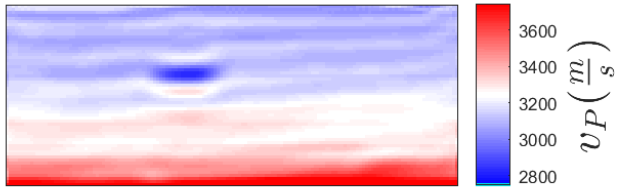


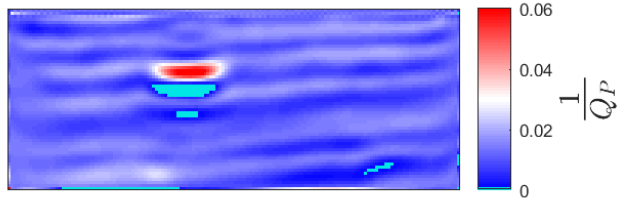


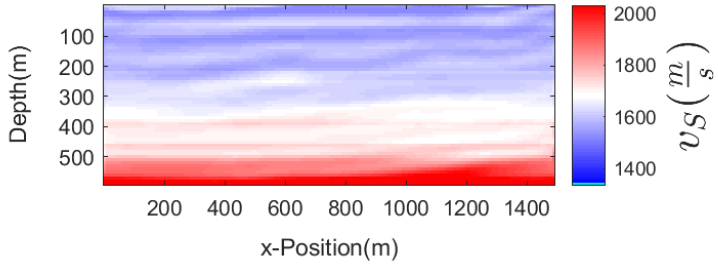


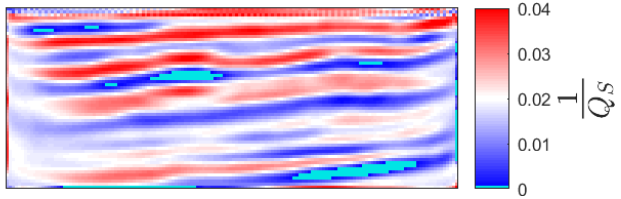
Piecewise soft penalty result











Unconstrained inversion result

Conclusions

- There are many ways to impose constraints in the FWI problem
- Hard constraints introduce substantial nonlinearity, which is problematic from an optimization standpoint
- Soft penalty terms may be more effective in constraining the inversion
- Piecewise penalty terms can create escapable barriers without overly biasing results

Acknowledgements

- CREWES sponsors, staff and students
- SEG and CSEGF

CREWES

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