# Amplitude preserving Kirchhoff migration: A traveltime based strategy

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**Abstract**

Amplitude versus offset information is a key feature to seismic reservoir characterization. Therefore amplitude preserving migration was developed to obtain this information from seismic reflection data. For complex 3-D media, however, this process is computationally expensive. In this paper we present an efficient traveltime based strategy for amplitude preserving migration of the Kirchhoff type. Its foundation is the generation of first and later arrival traveltime tables using a wavefront-oriented ray-tracing technique and a generalized moveout relation for 3-D heterogeneous media. All required quantities for the amplitude preserving migration are computed from coarse gridded traveltime tables. The migration includes the interpolation from the coarse gridded input traveltimes onto the fine migration grid, the computation of amplitude preserving weight functions, and, optionally, the evaluation of an optimized migration aperture. Since ray tracing is employed for the traveltime computation the input velocity model needs to be smooth, i.e., spatial velocity variations below the wavelength of the considered reflection signals are removed. Numerical examples on simple generic models validate the technique and an application to the Marmousi model demonstrates its potential to complex media. The major advantages of the traveltime based strategy consist of its computational efficiency by maintaining sufficient accuracy. Considerable savings in storage space ($10^5$ and more for 3-D data) are achieved. The computational time for the stack is substantially reduced (up to 90% in 3-D) if the optimized migration aperture is used.

## Introduction

The exploration industry faces targets of ever increasing complexity since the simple reservoirs are already found and exploited. Pre-stack depth migration (PSDM) became a standard tool to image the subsurface with seismic reflection data from 3-D complex media. Most currently existing implementations are based on a Kirchhoff type migration, which is also considered in this contribution. In Kirchhoff type migration, the subsurface is represented on a reasonably discretized *migration grid*, where every grid point corresponds to a point scatterer. The discretization interval depends on the resolution of the reflection data under consideration, i.e., the discretization step should be far below the dimension of the first Fresnel volume. This usually leads to a spacing of 5-25 m for the migration grid (also called *fine grid* in this paper). For each scatterer of this discretized model diffraction traveltimes are computed for sources and receivers at the Earth's surface. The amplitudes of the recorded seismic traces are then stacked along the diffraction time curves. Only if the scatterer corresponds to a reflector a non-vanishing contribution is obtained in the stacked section. This contribution appears at the correct spatial location in the migrated image, if the diffraction traveltimes are computed for the correct velocity model (for a good introduction to Kirchhoff type migration please see Bleistein 1999). The velocity model usually corresponds to a smooth velocity distribution without discontinuous changes.

In exploration the goal is not just to image the subsurface but also to characterize the reservoir, i.e., its lithology, fluid content, porosity, and others. Shear properties are a key feature in this investigation. Amplitude versus offset (AVO) studies turned out to be a powerful tool to obtain estimates on the shear properties of the reservoir without measuring shear waves directly (most industry data are vertical component P-wave observations). For AVO investigations the reflection coefficient has to be recovered from the seismic data. Amplitude preserving migration is a special implementation of Kirchhoff type migration where the amplitude of the migrated depth image corresponds to the reflection coefficient of the reflector under consideration (if transmission losses at interfaces above the target reflector are properly applied). In this process the geometrical spreading in the seismic data is removed by applying an appropriate weight function during the stacking process (for an introduction to amplitude preserving migration please see Schleicher et al. 1993).

Thus, amplitude preserving migration (APM) is separated into three major tasks: (1) the computation of diffraction traveltimes, (2) the determination of proper weight functions, and (3) the stacking of the traces along the diffraction time surface. The summation stack is usually carried out over the whole aperture of the experiment, which is a very time-consuming process. Thus, the computational costs of the third task can be significantly reduced if only those traces are included in the stack that really contribute. These traces define the size of an optimized migration aperture, which can be determined using the method outlined in this paper. For the computation of traveltimes either Finite Difference Eikonal solvers (FDES) or ray tracing is used. FDES (see, e.g., Vidale 1988, 1990) represent robust and fast tools to compute first arrival traveltimes. The inability to compute later arrivals of triplicated wavefronts is, however, a major drawback of these techniques Geoltrain and Brac (1993). Moreover, for accuracy reasons FDES do not work on coarse grids but must use a discretization in the dimension of the migration grid. To save storage space, especially in 3-D, diffraction traveltimes are usually stored on *coarse grids* with discretizations of several ten to hundred meters. Therefore FDES traveltimes need to be re-sampled to the coarse grid size for storage, i.e., a lot of computational effort is thrown away. Traveltimes on the fine grid are obtained during migration by interpolation, usually of bi-linear or tri-linear type. Therefore, APM actually consists of *four* major tasks: (1) computation of diffraction traveltimes on coarse grids, (2) traveltime interpolation from coarse storage grids to the fine migration grids, (3) computation of weight functions, and (4) the stacking process using the optimized migration aperture.

Several implementations of ray tracing are available. The classical ray shooting Červený et al. 1977 propagates a single ray through the medium to the receiver by two-point ray tracing Červený and Pšenčík (1984). Considering single rays usually leads to illumination problems, i.e., some grid points are not hit by rays. Techniques of the wavefront construction (WFC) type Vinje et al. (1993); Ettrich and Gajewski (1996); Lambaré et al. (1996); Bulant (1999); Coman and Gajewski (2001) propagate several rays at the same time to forward a complete wavefront through the medium. The wavefront at every time step should be sampled by a sufficient number of rays. In case of poor ray coverage (or poor sampling of the wavefront) virtual sources are introduced on the wavefront and new rays are traced from these positions. This leads to a sufficient illumination of the whole medium. Thus, WFC is better suited for the computation of traveltime grids than ray shooting. The introduction of new rays, however, may lead to inaccuracies since the initial conditions of these rays, i.e., coordinates of the virtual source on the wavefront and slownesses, are interpolated from neighboring rays Bulant (1999); Coman and Gajewski (2001). These errors increase if further rays have to be introduced since the information obtained from the neighboring ray is already erroneous.

In this paper we present innovations related to all four major tasks for APM by combining a ray tracing procedure with a traveltime based strategy for the computation of migration weights. For the computation of diffraction times we introduce the *wavefront-oriented ray-tracing* (WRT) technique which represents an alternative implementation of WFC. Here the new rays are not interpolated on the wavefront but are computed directly from the source. Thus the accuracy of the ray tracing is maintained even for the newly introduced rays. We also present a new method to determine traveltimes at grid points from the traveltimes of the closest wavefronts. The interpolation from the coarse grid to the fine migration grid uses a new technique based on a hyperbolic expansion of traveltimes which acknowledges the local curvature of the wavefront. Owing to this feature, this interpolation is superior to the commonly used bi- or tri-linear interpolation. Moreover, since this method allows for the interpolation of source positions, also a contribution to the computation of diffraction traveltimes is provided by this technique. Finally, we introduce a new method to compute migration weights and optionally an optimized migration aperture, which considerably reduces the computational time for the stack. Only coarse gridded diffraction traveltimes are needed as input information for the method. This permits a ray tracing in purely kinematic mode, since no dynamic ray tracing is required. These new techniques are the essential ingredients of the traveltime based strategy for amplitude preserving migration.

In the next section the methods behind the four ingredients are described where we start with the description of the wave front oriented ray tracing. After that we introduce a generalized moveout formula for 3-D heterogeneous media and explain its application to traveltime interpolation and the computation of migration weights. It also provides the foundation for the optimization of the migration aperture. Numerical examples on simple models where analytical solutions exist are used to verify the introduced methods and to visualize their strengths and weaknesses. Applications to the Marmousi model illustrate the potential of the traveltime based strategy for complex models. The section on smoothing of velocity models discusses the necessary requirements for the application of high frequency methods and its relation to wave propagation in the real Earth. The Conclusion and Outlook section ends the description of the traveltime based strategy for amplitude preserving migration.

# Method

APM consists of four major tasks: computation of diffraction traveltimes on coarse grids, traveltime interpolation from coarse grids to the fine migration grids, computation of weight functions, and optimization of the migration aperture. In this paper we introduce innovations related to each of these four tasks. The ingredients of the innovations are described in the following sections.

## Wavefront Oriented Ray Tracing

As mentioned in the introduction the WFC methods are best suited for the computation of transmitted (diffraction) traveltimes in complex models. Several implementations of WFC were published during the last years (e.g., Vinje et al. 1993; Ettrich and Gajewski 1996; Lambaré et al. 1996; Vinje et al. 1996). They all have in common, that they interpolate new rays on the wavefront. This interpolation procedure should increase the efficiency of the WFC method, but the interpolation introduces errors Bulant (1999); Coman and Gajewski (2001). These errors may even increase if further rays need to be included since traveltimes, slownesses and dynamic ray tracing results of neighbouring rays are already erroneous.

To avoid these inaccuracies we suggest to trace the new ray directly from the source. The initial direction of the new ray is given by the bisector of the angle between the neighbouring rays at the source. Thus, the accuracy of ray tracing is maintained. This accuracy is controlled by the technique used to solve the differential equations of the kinematic ray tracing system. We use a Runge-Kutta procedure of 4th order. It may happen that the newly introduced ray does not hit the desired location. For these situations, an iterative procedure is used to to find the appropriate position. In the examples presented below the simple averaging led the ray to the desired position in almost all cases and no iterations were necessary. Variations of initial conditions at the source (i.e., take off angles of rays) may be very small if strong divergence is present. It is therefore suggested to assign quantities related to initial conditions in double precision. Since we have combined wavefront construction and single ray shooting in this technique we call it *wavefront-oriented ray tracing* (WRT).

Extended and spatially localized low velocity bodies (like lenses) may lead to poor illumination using our technique. If the spatial extension of a low velocity zone exceeds the dimension of a *ray cell*, classical WFC techniques are applied (i.e., introduction of virtual sources and interpolation of initial conditions). In 2-D a ray cell corresponds to the spatial region which is separated by two neighboring rays and two neighboring wavefronts (in 3-D accordingly). The spatial separation of wavefronts is determined by the time step used to integrate the kinematic ray tracing system. It should be chosen such that the separation is smaller than the smallest spatial variation of the velocity model. A wavenumber analysis and the smallest velocity present in the model under consideration provide the necessary insight to properly choose the time step. Similar rules apply to the separation of rays of a ray cell. Their spatial separation also should not exceed the smallest spatial variation of the velocity model. The above mentioned issue of low velocity bodies is of minor concern for migration applications. For steep angle reflections as used in exploration these zones are less critical than for wide angle reflections or refracted waves which propagate in more horizontal directions. Steep angle reflections in many cases penetrate low velocity zones whereas wide angle reflections or refracted waves travel around the low velocity body, propagating in the high velocity areas around the low velocity body.

So far we have described how the continuous illumination is realized using the WRT technique. The interpolation from the traveltimes available at the corners of the ray cells to the grid points of the coarse traveltime grid is another important issue in any wavefront construction method. We suggest the following procedure: To allow a high accuracy in this process the wavefront curvature is taken into account. It is approximately determined by Finite Differences of the slownesses at the corners of the ray cell similar to the procedure suggested in Gajewski and Pšenčík (1987). The slowness vectors are obtained as a by-product of the solution of the kinematic ray tracing system. Then traveltimes from all corner points to the grid point under consideration are computed using an expansion which is exact up to second order (i.e., slope and curvature of the traveltime curve are considered). The final traveltime at the grid point is then obtained by a weighted average of these traveltimes. More details on the WRT technique are described in Coman and Gajewski (2001).

## A Generalized Moveout Formula

In the previous section the WRT technique was introduced to compute traveltime tables on coarse grids. This concludes the first of the four major tasks of APM. The three remaining tasks, the interpolation to the fine migration grid and the determination of the weight functions and the optimum aperture are both related to a generalized moveout formula. We expand the traveltime up to the second order. Since we are inspired by traveltime hyperbolae in exploration seismics, we actually do not expand the traveltime, $\tau$, but the square of the traveltime, $\tau^2$. For the 3-D case, the Taylor expansion has to be carried out in six variables, namely, the three components of the source position vector, $\hat{\boldsymbol{s}}$, and those of the receiver position vector, $\hat{\boldsymbol{g}}$, which in our case corresponds to the grid-point position vector. The resulting hyperbolic traveltime expansion reads

$$\begin{aligned}\tau^2(\hat{\boldsymbol{s}},\hat{\boldsymbol{g}}) \quad &= \quad (\tau_0 - \hat{\boldsymbol{p}}_0^\top \Delta\hat{\boldsymbol{s}} + \hat{\boldsymbol{q}}_0^\top \Delta\hat{\boldsymbol{g}})^2 + \tau_0 \Big( -2\,\Delta\hat{\boldsymbol{s}}^\top \underline{\hat{\mathsf{N}}}\,\Delta\hat{\boldsymbol{g}} \\ &\qquad -\Delta\hat{\boldsymbol{s}}^\top \underline{\hat{\mathsf{S}}}\,\Delta\hat{\boldsymbol{s}} + \Delta\hat{\boldsymbol{g}}^\top \underline{\hat{\mathsf{G}}}\,\Delta\hat{\boldsymbol{g}} \Big) + \mathcal{O}(3) \end{aligned} \tag{1}$$

The coefficients in the above equation are the first and second derivatives of traveltimes. The first order derivatives with respect to the source position yield the components of the slowness at the source, $p_{i_0}$. Similarly, $q_{i_0}$ are the components of the slowness vector at the receiver:

$$p_{i_0} = -\left.\frac{\partial \tau}{\partial s_i}\right|_{\hat{\boldsymbol{s}}_0,\hat{\boldsymbol{g}}_0} \quad , \qquad q_{i_0} = \left.\frac{\partial \tau}{\partial g_i}\right|_{\hat{\boldsymbol{s}}_0,\hat{\boldsymbol{g}}_0} \quad . \tag{2}$$

The second order derivatives with respect to source and receiver coordinates lead to the matrices $\underline{\hat{\mathsf{S}}}$ and $\underline{\hat{\mathsf{G}}}$, and the mixed second order derivative matrix $\underline{\hat{\mathsf{N}}}$

$$\begin{aligned}
S_{ij} &= -\left.\frac{\partial^2 \tau}{\partial s_i \partial s_j}\right|_{\hat{\boldsymbol{s}}_0,\hat{\boldsymbol{g}}_0} = S_{ji} \quad , \\
G_{ij} &= \left.\frac{\partial^2 \tau}{\partial g_i \partial g_j}\right|_{\hat{\boldsymbol{s}}_0,\hat{\boldsymbol{g}}_0} = G_{ji} \quad , \\
N_{ij} &= -\left.\frac{\partial^2 \tau}{\partial s_i \partial g_j}\right|_{\hat{\boldsymbol{s}}_0,\hat{\boldsymbol{g}}_0} \neq N_{ji}
\end{aligned} \tag{3}$$

Physically the expansion Eq. 1 corresponds to a local approximation of the traveltime curve by a segment of a curve of second order, e.g., a hyperbola. Or, in other words, the wavefront is locally approximated by a surface of second order, e.g., a sphere. Since the coefficients are expressed in terms of traveltime derivatives the expansion applies to any 3-D heterogeneous model, including anisotropic media, and any wave type.

Suppose that you know the coefficients involved in the expansion, then you already have an interpolation formula which is exact up to the second order. On the other hand, if a sufficient number of traveltimes is available, the above equation can be solved for the coefficients. This was already indicated by Bortfeld (1989) in connection with a parabolic traveltime expansion. Since a large number of traveltimes to every grid point for many sources need to be available for the migration to determine the diffraction time surfaces we can in fact solve for these coefficients. The number of coefficients in Eq. 1 can be reduced by using the eikonal equation Vanelle and Gajewski (2002b).

The coefficients are the key feature of the traveltime based strategy for APM. They are used for the traveltime interpolation, for the computation of migration weights and for the estimation of the optimized migration aperture. Eq. 1 represents the most general form of a moveout relation with an accuracy up to second order. It can be considered as a generalization of the well known $T^2 - X^2$ method to 3-D heterogeneous media Gajewski and Vanelle (2001). As for any expansion the accuracy of the interpolated quantity decreases with the distance from the expansion point. The distances with respect to the point of expansion are always small, because the determination of the coefficients of Eq. 1 is strictly local (i.e., spacing of the coarse grid). This leads to a high accuracy of the coefficients (see also the numerical examples below). The relation of the generalized moveout formula Eq. 1 to other published moveout formulas, e.g., the Common Reflection Surface stack (CRS, Jäger et al. 2001; Zhang et al. 2001) is discussed in Gajewski and Vanelle (2001) and Vanelle (2002). In the next sections we present applications of the generalized moveout formula which are of importance to APM.

**Traveltime Interpolation**

It was already mentioned that, if the coefficients are known, Eq. 1 represents an appropriate relation to interpolate traveltimes from the coarse traveltime grid to the fine migration grid. Explicit expressions for the coefficients and a detailed investigation on their accuracy are presented in Vanelle (2002). If the coefficients are determined from the coarse input traveltime grid, the interpolation to the fine migration grid is a straightforward application of Eq. 1. A more detailed description of the procedure and an error analysis is given by Vanelle and Gajewski (2002b). In the next section the relation of the migration weights of APM to the coefficients of Eq. 1 is presented. It is important to note, that Eq. 1 also allows to interpolate for sources, which is considerably faster than the computation with FDES or WRT.

### Migration Weights

The application of migration weights during the stacking process removes the geometrical spreading from the reflection data. This is the key feature of APM to reconstruct the reflection coefficient. The geometrical spreading and the migration weights can be expressed in terms of traveltime derivatives, i.e., in terms of coefficients of the generalized moveout equation which we already determined for the traveltime interpolation. The migration weights read:

$$W_{3\mathrm{D}}(\boldsymbol{\xi}, M) = \frac{1}{v_1}\sqrt{\cos\alpha_1\ \cos\alpha_2}\,\frac{|\det(\underline{\mathsf{N}}_1^\top\underline{\boldsymbol{\Sigma}} + \underline{\mathsf{N}}_2^\top\underline{\boldsymbol{\Gamma}})|}{\sqrt{|\det\underline{\mathsf{N}}_1|\,|\det\underline{\mathsf{N}}_2|}}\,\mathrm{e}^{-i\frac{\pi}{2}(\kappa_1+\kappa_2)} \quad . \tag{4}$$

To construct the stacking surface we split the diffraction time into the traveltime from the source to the image point $M$ and that from the receiver to $M$. Quantities related to these two traveltime branches are denoted by the indices 1 and 2 in Eq. 4, as, e.g., the matrices $\underline{\mathsf{N}}_i$. All involved quantities of Eq. 4 are known. The matrices $\underline{\mathsf{N}}_i$ are obtained from the second-order mixed derivative matrices $\hat{\underline{\mathsf{N}}}_i$ (obtained from Eq. 1) by rotating the latter into the tangent plane of the candidate reflector at the image point. The velocity $v_1$ is that at the source point. The angle $\alpha_1$ is the emergence angle from the source and $\alpha_2$ is the incidence angle at the receiver. Both angles are related to the slowness vectors, i.e. first derivatives of traveltimes, and are also obtained from the coefficients of Eq. 1. The KMAH indices $\kappa_i$ are provided by the WRT. The KMAH index is characteristic to each traveltime table for the phases of a triplicated wavefront. Finally, the matrices $\underline{\boldsymbol{\Gamma}}$ and $\underline{\boldsymbol{\Sigma}}$ describe the measurement configuration of the experiment, e.g., common shot. Thus, the weight function is computed from the very same coefficients already used for the interpolation. No additional computational effort (except for the evaluation of Eq. 4) is needed. To properly apply the migration weights a priori information on the structure under investigation is required, i.e., reflector location and dip. This can be either derived from a previous migration (e.g., an unweighted diffraction stack) or, for simpler media, from the velocity model used to compute diffraction traveltimes. The a priori knowledge is not specific to the traveltime based strategy of APM but a necessity to every APM. In the next section the optimized aperture is applied to further reduce the computational effort of APM.

### Optimized Aperture

Usually the weighted stack is carried out over the whole aperture of the experiment, i.e., the full spread used during recording. It was, however, previously shown that only the traces constructively contribute to the stack, where the diffraction time differs by less than the signal length from the traveltime of the stationary ray Schleicher et al. (1997); Hubral et al. (1993). The locations in the recording surface where the difference between diffraction and reflection traveltime equals the signal length define the optimum migration aperture. Since this difference can be approximated in terms of the coefficients of Eq. 1, the optimum aperture can be determined Vanelle and Gajewski (2002a). Again, only the coefficients already determined for the traveltime interpolation are needed to obtain the aperture.

Similar to the application of the coefficients for the migration weights, a priori information is required to determine the stationary reflection which forms the center of the optimized aperture. If the optimized aperture is applied, considerable savings in CPU time are achieved since much less traces need to be

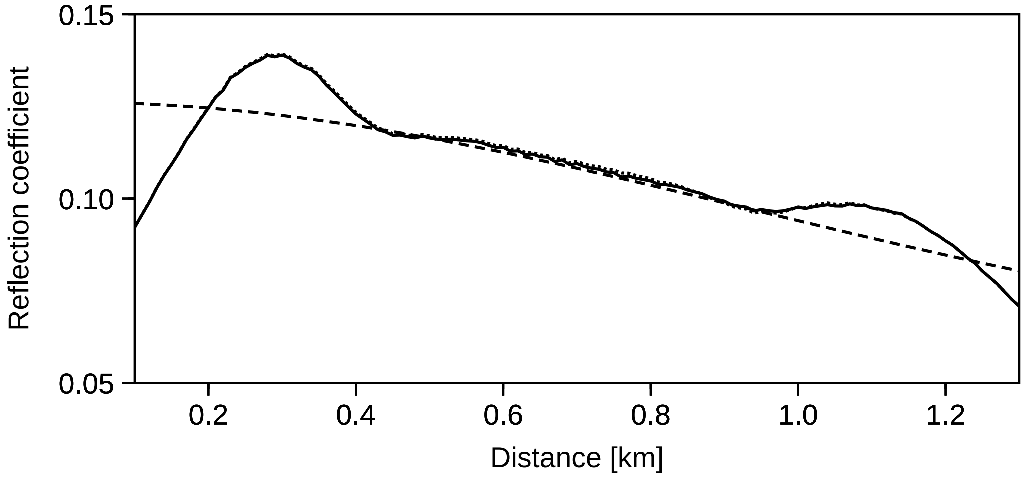


Figure 1: Recovered reflection coefficients from noise-free data for a horizontal reflector. Dashed line: analytic reflection coefficients. Solid line: recovered coefficients if the optimum aperture is used. Dotted line: recovered coefficient if the whole recording spread is used. The curves for both apertures virtually coincide. Despite boundary effects (peaks at 0.3 and 1.1km) caused by the limited extent of the spread the differences between results are very small.

stacked. Optimized apertures are often in the order of a few hundred meters whereas the recording spread is in the order of a few kilometers. Moreover, also the signal to noise ratio of the migrated image is improved owing to not stacking amplitudes which do not contribute to the reflected signal. We will now apply the traveltime based strategy to various models. First we will investigate a simple generic model where analytic solutions to the involved quantities are available. This example serves to validate the new strategy for traveltime based APM.

## Numerical Example

In a first example we apply the traveltime based strategy to a simple 2-D model. It is a two layer model separated by a horizontal interface. The P-wave velocity $v_p$ is 5 km/s in the first layer and 6 km/s in the second layer. The $v_p/v_s$ -ratio is $\sqrt{3}$ for both layers and the density $\rho$ is evaluated using the relation $\rho = 1.7 + 0.2v_p$. For this model, traveltimes and their first and second spatial derivatives (i.e., the coefficients of Eq. 1) and all quantities required for APM with optimized aperture can be computed analytically and compared to the results for coefficients determined from the coarse grid traveltime tables. Ray synthetic seismograms were computed for a receiver line consisting of 300 equidistantly positioned receivers with a spacing of 10m and the first receiver 10m away from the source. Synthetics were generated using the SEIS package Červený and Pšenčík (1984).

For this section an amplitude preserving shot migration was carried out. Coarse grid analytic traveltime tables with a spacing of 50m for each direction were used to determine the coefficients. Diffraction surfaces were interpolated for a migration grid with a spacing of 2m in z-direction and 10m in x-direction. Optimized aperture and migration weights were computed from the coefficients. The reconstructed reflection coefficient is shown in Fig. 1 and compared to the analytically obtained results.

The amplitude versus offset behavior is successfully recovered by the migrated results as well as the form of the source signal (not shown here). This applies to the migration using the full spread

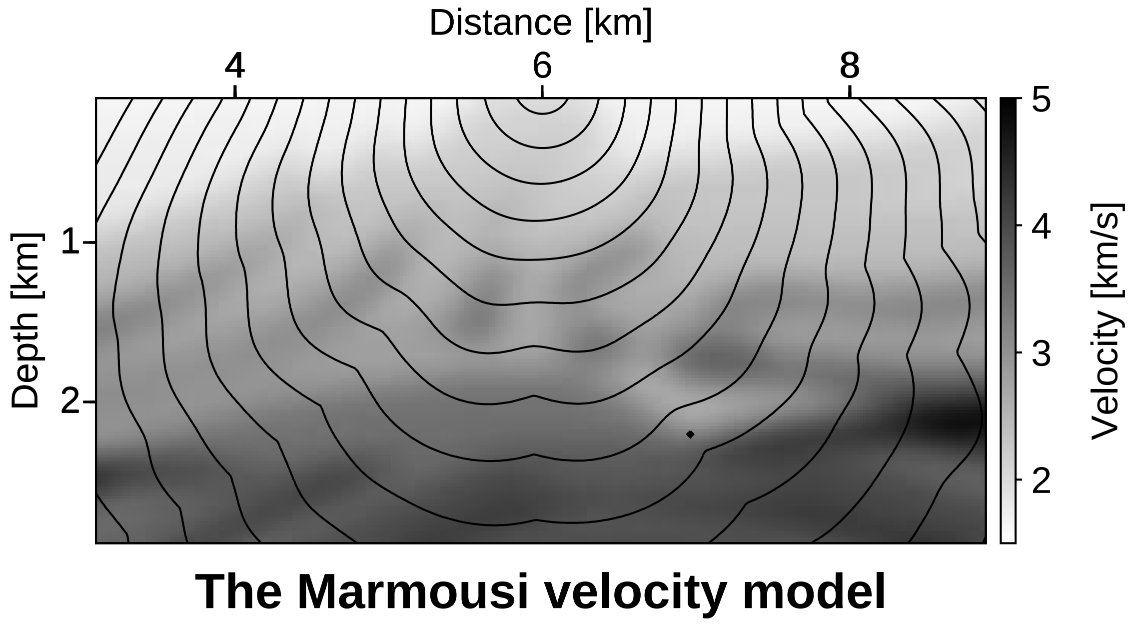


Figure 2: Marmousi model with smoothing applied. Wave numbers below 200m are removed. First arrival isochrones for a source on top of the model at x=6km are also given.

and the optimized aperture. Please note, that the latter requires less CPU time since it stacks much less traces (about 80% less) than for the full spread. If noise is present in the data, the migrated image resulting from the optimized aperture also displays an improved S/N-ratio. The CPU savings are even more important for 3-D media (more than 90% depending on the model, signal length, spread length and reflector depth). Please note, that the optimized aperture determined from the coefficients is an approximation to the exact optimum aperture. For steep dips the approximation may not cover the range required to reconstruct the reflection coefficient but will nevertheless recover the AVO trend Vanelle (2002); Vanelle and Gajewski (2002a).

The simple example has proved that the traveltime based strategy to APM is in fact able to reconstruct the reflection coefficient. In the next example we will investigate a complex 2-D model where the traveltimes need to be computed numerically by the above described WRT technique. Here, analytical results are not available. The coefficients, however, can be determined in the same way as above and traveltimes are interpolated from the coarse grid to the fine migration grid. These interpolated traveltimes are compared to directly computed fine grid traveltimes using the WRT technique in a parameter setting for high accuracy. The errors are an indication for the performance of the traveltime based strategy to APM in complex media since the very same coefficients used for the interpolation are also used to compute migration weights and optimized aperture.

The Marmousi model Versteeg and Grau (1991) is shown in Fig. 2. Since the foundation of exploration seismics as well as the theory of APM is based on geometrical optics (i.e., high frequency asymptotics) the model is smoothed such that spatial wave numbers of the velocity below 200m are removed (for a more detailed discussion on smoothing see below in the section on smoothing). The filter characteristic of the smoothing operator is shown in Fig. 3.

A spacing of 200m is used for each spatial direction of the coarse grid traveltime tables. The

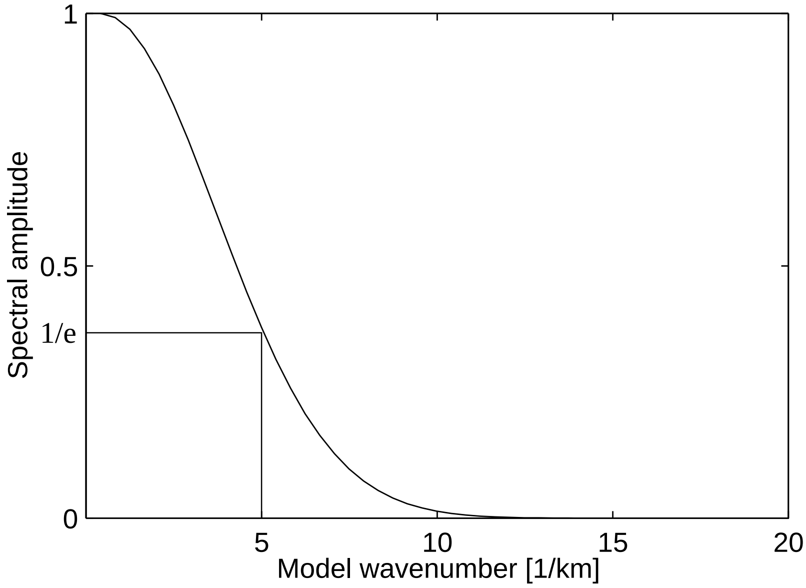


Figure 3: Impulse response of the low pass filter used to smooth the Marmousi model. The upper boundary of the low pass wavenumber corresponds to 5km, i.e., spatial variations below 200m are removed.

separation of shots at the surface is also 200m. Traveltimes are computed using the WRT technique on the coarse grid as well as for the fine migration grid with a spacing of 20m. The data on the fine grid serve as reference data.

The coefficients of a 2-D version of Eq. 1 are determined using the coarse grid traveltime tables. They are then applied to interpolate traveltimes to the fine migration grid. The interpolated traveltimes are compared to the reference traveltimes. The relative errors are shown in Fig. 4.

The overall performance of the interpolation is very satisfactory even close to the source, where the wavefront curvature is strongest. In regions with strong wavefront curvature the hyperbolic interpolation is far superior to the commonly used bi-linear interpolation. Please note, that in complex models, strong curvature may occur even very far away from the source. A visual inspection of the isochrones in Fig. 4, e.g., at about x=8km and z=1.5km, illustrates this statement. The largest errors are correlated with kinks in the wavefronts. These kinks in first arrival traveltimes indicate the presence of triplicated wavefronts. In these regions different seismic phases were mixed together when determining the coefficients, leading to erroneous values. The inclusion of later arrivals overcomes this problems. The WRT technique is able to compute first and later arrivals. The implementation of our current program version to determine the coefficients, however, does not consider later arrivals yet.

It was mentioned above, that the technique presented in this paper allows to interpolate also for sources. This is impractical with tri-linear interpolation, i.e., much more shots need to be computed. The direct computation of shots is more time consuming than the hyperbolic interpolation: savings of more than 85 % in CPU time per shot are possible. In Fig. 5 the relative error for the interpolation of traveltimes for an interpolated source are displayed. The agreement of directly computed traveltimes using the WRT technique and the interpolated traveltimes is satisfactory since the relative errors stay far below 1% in most parts of the model except zones with kinks in the wavefronts (see above). Further applications of traveltime interpolation to 3-D models are presented in Vanelle and Gajewski (2002b).

Fig. 6 shows the geometrical spreading computed from the coefficients of the hyperbolic expansion.

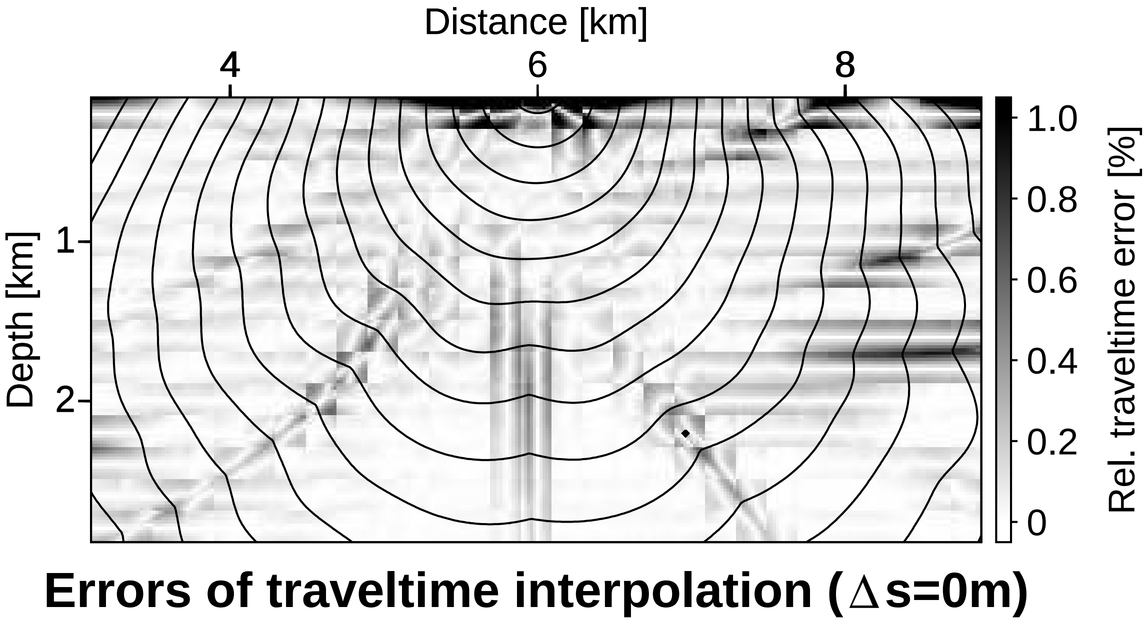


Figure 4: Gray scaled relative errors of interpolated and reference traveltimes for the smoothed Marmousi model. The overall errors are small. There is an obvious correlation of the error with “kinks” in the isochrones (see text).

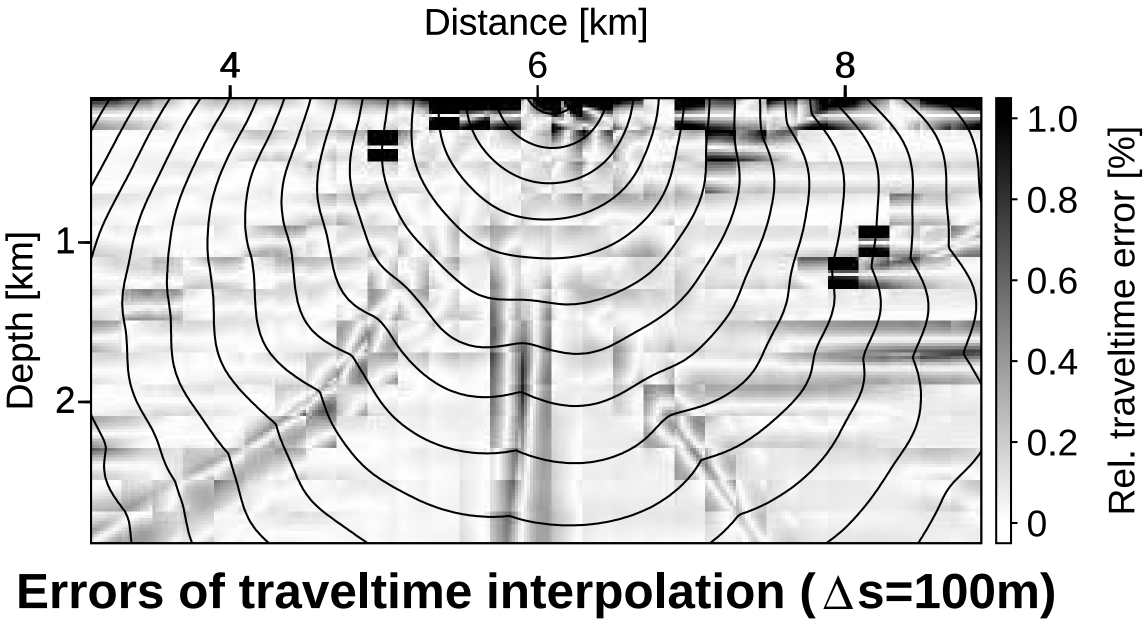


Figure 5: Interpolation of traveltimes for a source position shifted 100m in x-direction. Relative errors are given. The overall performance is similar to the interpolation of grid positions only (see Fig. 4).

As expected, the behavior of the spreading follows the curvature of the wavefront. The computation of migration weights is carried out with the same coefficients as for the traveltime interpolation and the determination of geometrical spreading. No reference solution was available for the Marmousi model to estimate the errors of the method for the determination of geometrical spreading. We have, however, computed geometrical spreading from traveltimes for models where an analytical solution exists (homogeneous medium, constant velocity gradient model, see Vanelle and Gajewski 1999). The resulting relative errors of the spreading for these models are of a magnitude of 0.05%. The computation of migration weights is carried out with the same coefficients as the traveltime interpolation and the determination of geometrical spreading. Therefore we expect similar accuracy for the determination of the migration weights as for the spreading.

To apply high frequency (HF) techniques certain limitations apply. In order to satisfy the conditions of applicability of HF methods Červený et al. (1977); Červený (2001) usually requires smoothing of the models determined by velocity model building tools or obtained from velocity logs. Smoothing is often considered to be harmful to imaging. Some considerations on smoothing are discussed in the following section.

## Some Comments on Smoothing

The foundation of applied seismics including exploration seismics is based on high frequency asymptotics. Seismic phases, traveltimes and rays serve as powerful tools to understand problems in reflection seismics. Classical concepts like NMO, DMO, CMP- or CRS-stack and Kirchhoff migration are firmly based on HF assumptions. This assumption also applies to the whole theory of amplitude preserving

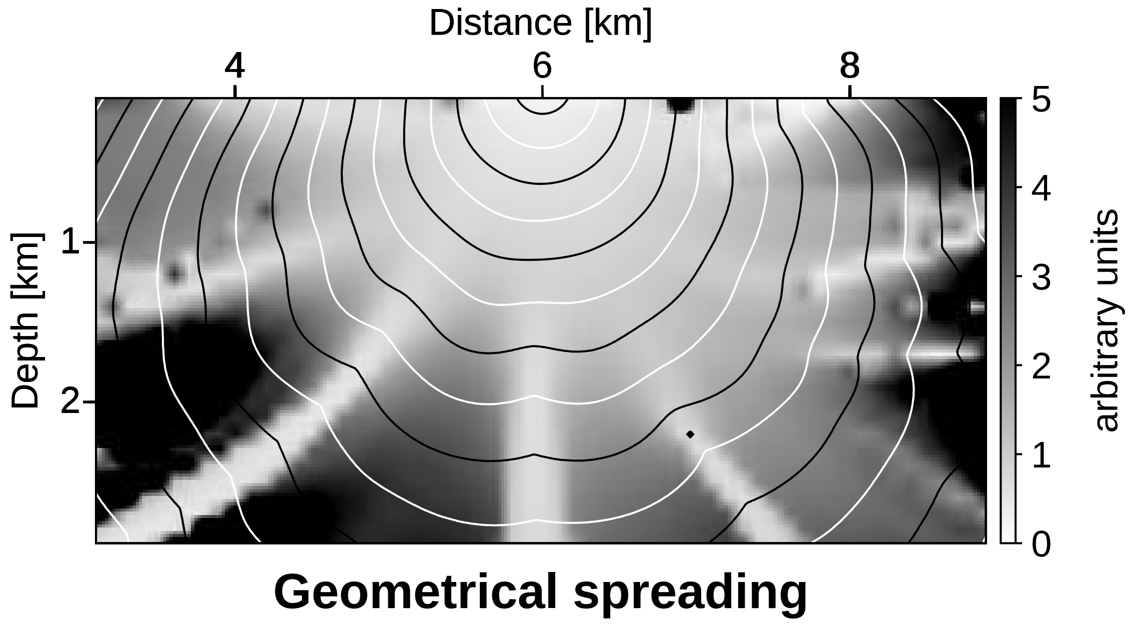


Figure 6: Geometrical spreading for the Marmousi model computed from the coefficients of the hyperbolic traveltime expansion.

migration. Also the imaging conditions of Finite Difference migration (often also called wave field migration) are based on the ray concept. Here the wavefield continuation is based on a one way wave equation whereas the imaging condition relies on HF-methods. To apply any kind of HF method certain conditions of applicability must be satisfied Červený et al. (1977); Červený (2001). The spatial variations of the involved quantities (velocity and its derivatives, amplitude, slowness, polarization etc.) should be small with respect to the wavelength of the signal under consideration. Usually this requires smoothing of the model.

Borehole logs are obtained on a completely different scale than reflection seismic data. This often leads to miss-ties, since inappropriate smoothing was used to match the different scales of the experiments. Smoothing is a process which occurs in the propagation of real seismic waves. In the far field we observe band limited signals with limited resolution since the Earth serves as a low pass filter during propagation. Thus, reasonable smoothing appears to be a natural process and unlikely to be harmful to imaging. However, the smoothing tools currently available are not advanced enough to simulate the smoothing happening in the real Earth. The smoothing usually applied these days is a global smoothing of velocity models by spatially applying a smoothing operator. Bulant (2002) describes a new smoothing method which is based on the minimization of the Sobolev norm. The technique is particularly suited for applications in ray tracing and Kirchhoff type migration. Another sophisticated style of smoothing could be based on directional averaging within the first Fresnel volume of each source receiver connection to obtain something in a way of a local effective medium at each step of the propagation. This effective local medium would be anisotropic and frequency dependent.

Since these tools are not available yet, we try to construct a global smoothing procedure with current tools, which are most close to the physics under consideration. The procedure to choose an appropriate smoothing parameter for the application of the WRT technique and the traveltime based strategy for

APM could look as follows: A frequency analysis of the reflection data provides an average of the prevailing frequency in the data set. Then the lowest occurring velocity in the velocity model under consideration is determined. The ratio of lowest occurring velocity and prevailing frequency provides the critical spatial measure for the smoothing and the choice of the WRT parameters, particularly the spatial dimension of ray cells (spatial separation of wavefronts, i.e., time step for the integration of the ray tracing equations, and the spatial separation of rays; see the section on wavefront-oriented ray tracing). The global smoothing of the velocity model is carried out such that all wave numbers below the critical spatial measure are removed. This procedure helps to satisfy the condition of applicability of HF methods to the model and data under consideration and to optimally choose the WRT parameters for computational efficiency and sufficient accuracy

## Conclusions and Outlook

The combination of a wavefront-oriented ray-tracing technique with a hyperbolic traveltime interpolation leads to a new strategy for amplitude preserving migration of the Kirchhoff type for complex media. This strategy is entirely based on traveltimes which are in any event needed to carry out the diffraction stack. The fast computation of first and later arrival traveltimes is based on a wavefront oriented ray tracing technique. The WRT technique provides a continuous illumination of the subsurface with increased accuracy compared to classical wavefront construction approaches by propagating new ray directly from the source. The new ray is traced without interpolation pertaining the accuracy of the solver used to integrate the kinematic ray tracing system. An FD procedure provides the curvature of the wavefront which allows an improved interpolation of traveltimes from the corners of ray cells to the coarse traveltime grid. The ability of ray tracing techniques to compute traveltimes with sufficient accuracy on coarse grids is a major advantage against FD eikonal solvers which compute first arrivals only, and only work on fine grids.

A hyperbolic expansion is used to determine first and second spatial derivatives of traveltimes from the coarse gridded traveltime tables computed by WRT for the velocity model under consideration. These derivatives (or coefficients of the expansion) are the foundation of an efficient traveltime interpolation from the coarse grid to the fine migration grid. This interpolation also works for source positions and is therefore particularly efficient with respect to computer storage (only coarse traveltime grids need to be stored) and CPU time (less shots need to be computed and stored). Choosing a ratio of 10 between fine and coarse grid, savings in computer storage of $10^5$ are achieved for a 3-D model ($10^3$ grid points for every shot and $10^2$ less shots, where we have assumed that shot and receiver spacing are the same). The CPU time needed for $10^2$ shots is saved minus the CPU time required to perform the determination of the interpolation coefficients and the interpolation of shots. The total CPU savings are in the order of 85% per shot.

The very same coefficients used for the interpolation are also used to compute migration weights and the optimized migration aperture which makes this procedure for APM even more computationally efficient. In this step, however, as for any other available APM technique a priori knowledge of the structure is required. It can be obtained from the velocity model and/or a previous (e.g., unweighted) migration. The traveltime based approach to determine migration weights can be combined with any tool that provides traveltimes. Therefore, it is also a tool for first arrival amplitude preserving migration using Finite Difference eikonal solvers which do not provide geometrical spreading or migration weights directly.

The range of applicability of the hyperbolic expansion is broader than presented in this paper. It is also applicable to 3-D heterogeneous anisotropic media (the assumption for the expansion is local smoothness of the quantity to be expanded, no assumption on the model is made). The interpolation of traveltimes is straightforward and should be of even larger value than in isotropic media. The computation of traveltimes in anisotropic media using ray tracing needs about 5-10 times more CPU time than in isotropic media. The adaption of the WRT to anisotropic media just requires the exchange of routines concerning the computation of right hand sides of the kinematic ray tracing system and some model related routines. The procedures to ensure continuous illumination and the determination of traveltimes at grid points of the coarse grid remain unchanged. The relations of the coefficients to geometrical spreading and the migration weights in anisotropic media still need further investigation.

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